FNOs for Modeling Earthquake Cycles and Accelerating Aseismic and Seismic Deformation

1. Introduction

Earthquake cycle simulations are essential for understanding the long-term evolution of fault slip, stress accumulation, rupture nucleation, dynamic rupture, and postseismic relaxation. However, these simulations are computationally expensive because they must resolve both slow aseismic deformation over long time scales and rapid seismic rupture over very short time scales. Traditional physics-based solvers can accurately model these processes, but their computational cost increases significantly for long-duration simulations, fine spatial resolution, and more complex fault behavior. Neural operators provide a promising surrogate modeling approach because they learn mappings between input and output function spaces and can generate predictions much faster once trained. In this study, we develop a multi-phase Fourier Neural Operator framework to accelerate earthquake cycle simulations by separating the cycle into aseismic, nucleation, seismic, and postseismic phases. A distinct FNO is trained for each phase and connected sequentially by passing the slip rate, state variable, and shear stress between phases. This framework is first evaluated for periodic earthquake cycles and is further extended to nonperiodic events using both hybrid FNO and full FNO workflows, allowing the trade-off between accuracy, stability, and computational efficiency.

2. Methodology

2.1 Earthquake Cycle Dataset

The training dataset is generated using a hybrid finite element–spectral boundary integral solver. The fault is modeled as a planar fault embedded in a two-dimensional elastic medium. The frictional behavior follows a rate-and-state friction law, with a velocity-weakening region located near the center of the fault and velocity-strengthening regions surrounding it.

The simulation includes both quasi-dynamic and fully dynamic behavior. During aseismic deformation, the inertia term is approximated using radiation damping, producing a quasi-

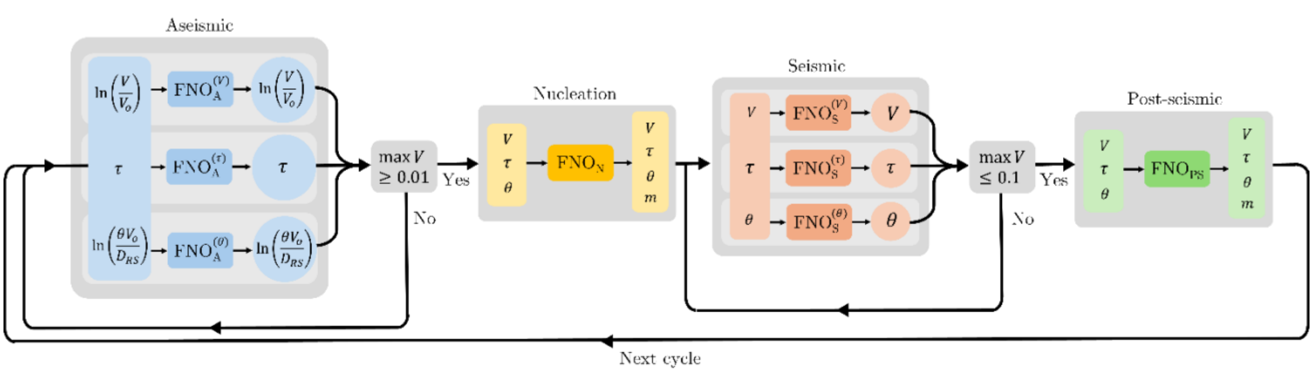

Figure 1 Schematic diagram of the hybrid FNO framework for modeling different phases in an earthquake cycle.

dynamic formulation. During seismic rupture, the fully dynamic elastodynamic equations are used. The transition from quasi-dynamic to dynamic behavior occurs when the maximum slip rate along the fault exceeds 10^{-2} m/s. The transition back to the quasi-dynamic regime occurs when the maximum slip rate falls below 0.5×10^{-2} m/s. The primary variables used in the FNO framework are the slip rate V, the state variable θ, and the shear stress τ. These variables describe the evolution of fault slip and frictional resistance throughout the earthquake cycle.

2.2 Multi-Phase FNO Framework

The proposed framework divides the earthquake cycle into four phases including aseismic, nucleation, seismic, and postseismic. A separate FNO model is trained for each phase. The aseismic model, FNO_A, predicts the slow interseismic evolution of the system. The nucleation model, FNO_N predicts the transition from aseismic slip to dynamic rupture. The seismic model, FNO_S, predicts the rapid evolution of slip during rupture. The postseismic model, FNO_{PS} , predicts the transition from dynamic rupture back to slow deformation.

The models are connected sequentially. At each phase transition, the predicted distributions of V, θ, and τ are passed to the next FNO as initial conditions. This allows the complete workflow to simulate multiple earthquake cycles continuously.

2.3 Training Strategy for Aseismic and Seismic Phases

The aseismic and seismic phases are trained using a recursive prediction strategy. For each phase, the time series is divided into sliding windows. The input to the FNO is the initial state of a window, and the output is the evolution of the target variable over the following time steps. The predicted output can then be used as the input for the next prediction window.

For the aseismic phase, adaptive time stepping is used because the slip rate changes slowly during most of the interseismic period but accelerates as the system approaches nucleation. The adaptive time step is computed based on the characteristic slip distance and the maximum slip rate from the previous step. The input variables for the aseismic FNO are transformed as $ln(V/V_0)$, τ, $ln(\theta V_0/L)$. The logarithmic transformation helps the model learn variables that vary across several orders of magnitude.

For the seismic phase, constant time stepping is used based on the Courant–Friedrichs–Lewy condition. Unlike the aseismic phase, the seismic model uses the physical variables V, θ, and τ directly. Separate FNO models are trained to predict each variable. Later, we found that training on logarithmic transformation improves stability and accuracy of the events. We apply logarithmic transformation to predict nonperiod events.

2.4 Training Strategy for Nucleation and Postseismic Phases

The nucleation and postseismic phases are modeled using one-shot prediction. Instead of recursively predicting many time windows, these FNOs directly predict the final state at a prescribed slip-rate threshold.

For the nucleation phase, FNO_N takes the system state near the end of the aseismic phase and predicts the state at the nucleation threshold, defined as $V_{th}^{(n)}$= 0.25 m/s. For the postseismic phase, FNO_{PS} predicts the state at the postseismic threshold, defined as $V_{th}^{(ps)}$= 0.005 m/s

Both one-shot models predict four output channels: slip rate, shear stress, state variable, and a feature map representing the time to the target threshold. The time feature map allows scalar time information to be included in a spatial FNO output format.

To improve robustness, the nucleation and postseismic models are trained using both ground-truth states and predicted states from the previous FNO model. For example, FNO_N is trained using true aseismic states and recursive predictions from FNO_A. This helps the model correct errors accumulated during recursive prediction.

3. Results and Discussion

The proposed multi-phase Fourier Neural Operator framework accurately captures the main behavior of earthquake cycle simulations across aseismic, nucleation, seismic, and postseismic phases. For the aseismic phase, the model successfully predicts the slow evolution of slip rate, shear stress, and state variable, with relative L_2 errors for $\ln V/V_0$ and τ generally below 5% and 0.7%, respectively. These results suggest that the logarithmic transformation helps the model learn variables that vary across several orders of magnitude. For the seismic phase, the model captures the rapid evolution of dynamic rupture, with shear-stress errors below 1% and slip-rate errors generally below 9%. Although the state variable has the largest error, more than 90% of the samples remain below 5% error. Small mismatches appear near sharp rupture-front gradients, likely due to Gibbs oscillations from the Fourier representation, but the global rupture behavior is still well captured. The nucleation and postseismic FNOs also perform well in modeling phase transitions. In one nucleation example, the predicted time to nucleation is 0.02209 s, compared with the ground truth of 0.02146 s, giving a relative error of 2.97%. For the

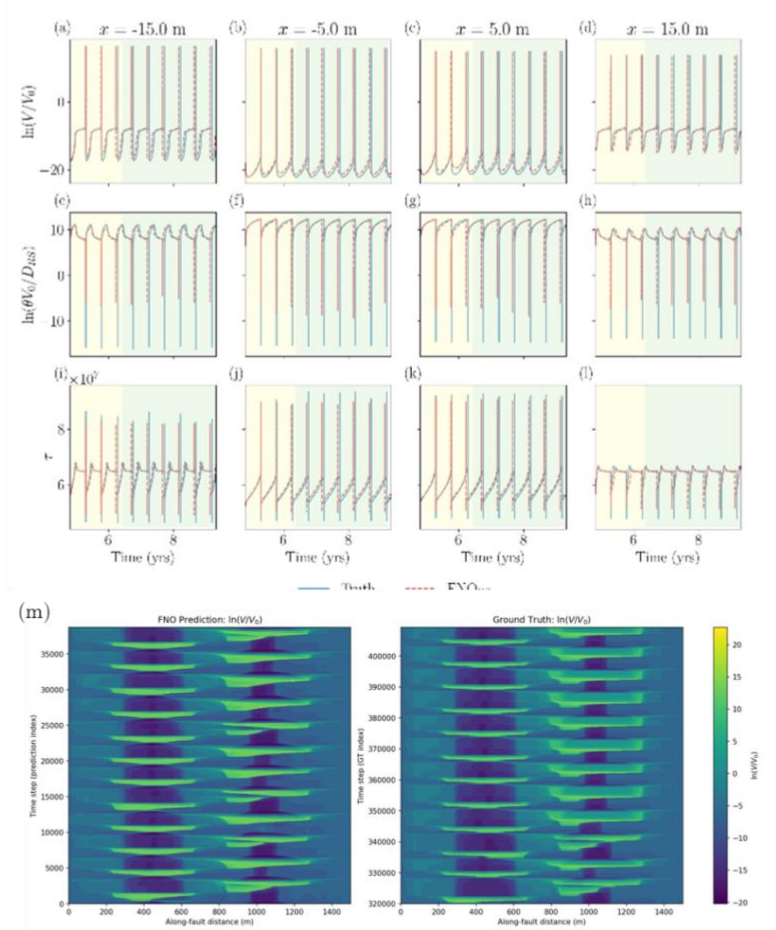

Figure 2 (a)-(l) Predictions of full earthquake cycle simulations on periodic events obtained from the integrated workflow combining FNO_A, FNO_S, FNO_N, and FNO_{PS}. (m) Contours of predictions and ground truth of nonperiodic earthquake events.

postseismic phase, more than 80% of samples have errors below 5% for slip rate, 0.15% for shear stress, and 0.8% for the state variable, while the predicted postseismic time shows a representative error of 4.37%. These results indicate that the one-shot FNO strategy can recover both spatial variables and transition timing.

The four trained FNOs are then combined into a full earthquake cycle workflow. The model progresses from aseismic deformation to nucleation, seismic rupture, postseismic relaxation, and back to aseismic deformation. The predicted histories of slip rate, state variable, and shear stress agree well with the numerical solution. Small timing shifts occur, but the total shift is less than 0.01 years, corresponding to about 2% error relative to the aseismic duration. The model also remains stable beyond the testing interval, suggesting potential for long-term earthquake cycle prediction.

The framework provides a significant computational speedup. Over a testing interval containing three seismic events, the numerical solver requires 31,865 s, while the FNO workflow requires only 2.64 s on a single NVIDIA A100 GPU. This corresponds to a speedup of approximately 12,070 times. Overall, the proposed multi-phase FNO framework provides an accurate and efficient surrogate model for earthquake cycle simulations. Future work may extend this approach to nonperiodic events, heterogeneous friction, complex fault geometries, and physics-based error correction using high-fidelity solvers.

To further explore more complex earthquake cycle behavior, we also apply the full FNO workflow to nonperiodic events. Unlike periodic or quasi-periodic cycles, nonperiodic events can exhibit variations in recurrence interval, rupture intensity, and postseismic recovery. These variations make long-term prediction more challenging because small errors in event timing or state variables can accumulate over multiple cycles. Nevertheless, the results show that FNOs can predict multiple nonperiodic cycles and reproduce meaningful statistical features of the system.

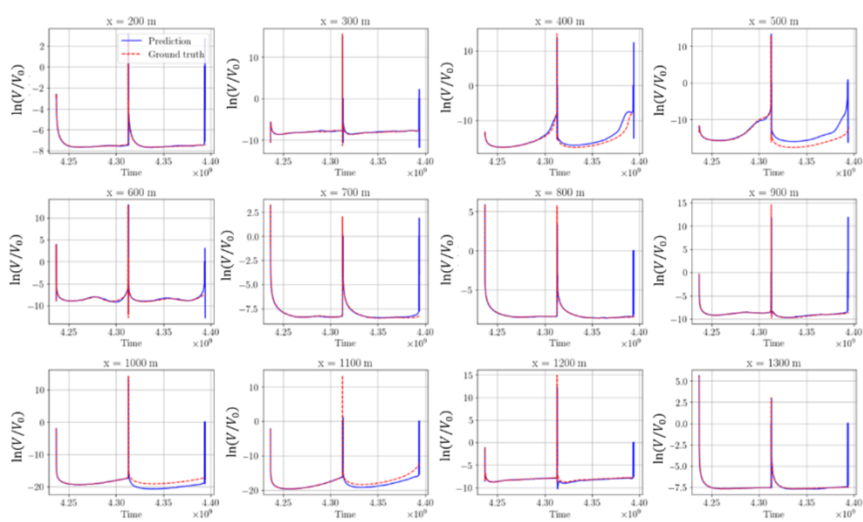

Figure 3 Comparison between the hybrid FEBE-FNO prediction and the ground-truth solution for nonperiodic earthquake cycle simulation at different fault locations.

To improve the accuracy, we further introduce a hybrid FEBE–FNO workflow. While the full FNO workflow provides maximum computational efficiency, nonperiodic cycles may involve stronger variations in recurrence interval, rupture intensity, and postseismic recovery, which can make prediction errors accumulate over time. The hybrid framework addresses this issue by combining FNO predictions with the high-fidelity FEBE solver during critical transition periods. In this workflow, the aseismic FNO predicts the slow deformation phase until the maximum slip rate reaches a prescribed threshold. The simulation then switches to FEBE to resolve nucleation more accurately until the seismic-start threshold is reached. Afterward, the seismic FNO predicts the dynamic rupture phase until the slip rate decreases to the seismic-end threshold, and FEBE is used again to capture the postseismic transition. This hybrid strategy is designed to retain the computational efficiency of FNOs during the main aseismic and seismic phases while improving accuracy and stability near phase transitions.