2025 SCEC Project Final Report

SCEC Identifier	25156
Project Title	*Adjoint ruptures with slip-weakening friction for dynamic source inversions and sensitivity analysis*
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SCEC Science Milestones Addressed	C1,2,3-1, C1-2, C2-2, C3-1

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Adjoint method and applications in rupture dynamics

The adjoint method is used for optimization (inverse problems), sensitivity analysis, and uncertainty quantification. It provides a computationally efficient means of calculating the gradient of a scalar objective function $\phi(m)$ with respect to model parameters m. There are many possible objective functions and model parameters, depending on the science or engineering question of interest. For example, in a dynamic rupture inversion, the objective function might be the L$_2$ norm over time of a particle velocity seismogram at a set of receivers,

$$\phi(m) = \frac{1}{2}\sum_n \|\dot{u}(x_n, t) - d_n(t)\|_2^2,$$

where n denotes the receiver and $d_n(t)$ is the data, and the model parameters might include the frictional parameters and initial stress on the fault. Alternatively, for investigation of fundamental rupture dynamics processes, the objective function might be the arrival time of the rupture front (or the rupture velocity, or slip, or other field) at some point on the fault, the final rupture extent, or the seismic moment. For sensitivity analysis and uncertainty quantification, the objective function would be an engineering-relevant measure of ground motion (e.g., PGV, PGA, spectral acceleration), and we seek to understand how uncertainty in friction, stress, or elastic properties propagates through the rupture process and wave propagation to uncertainty in ground motion.

The brute force approach to this class of problems involves running many (hundreds or thousands or even more) forward models, each with different model parameters. For inversions, sampling can be done with Bayesian methods, as originally done for dynamic rupture models by Peyrat & Olsen (2004). Several groups are pursuing this now for dynamic rupture inversions (Gallovič et al., 2019a, 2019b) and for a combination of dynamic rupture models and earthquake sequence models to also capture afterslip (Premus et al., 2022; Schliwa et al., 2024).

The adjoint method provides the gradient or sensitivity kernel for an arbitrary number of model parameters with only two simulations: a forward simulation and an adjoint simulation. The adjoint simulation, solved in a time-reversed manner, back-propagates the residual from the receivers to portions of the fault where the rupture process requires modification to better fit the data (or more generally, to minimize the objective function). The sensitivity kernel directly provides the information needed for uncertainty quantification. The gradient can be used in a gradient-based optimization algorithm (e.g., L-BFGS, which goes beyond gradient descent by iteratively providing low-rank updates to an approximate inverse Hessian). Alternatively, the gradient can be used to accelerate Monte Carlo sampling in Bayesian inversions by guiding the sampler to avoid parts of model parameter space that are likely to be rejected. One popular algorithm is Hamiltonian Monte Carlo and there have been advances in full waveform inversion to augment this with approximate Hessian information (Fichtner et al., 2021; Zunino et al., 2023).

Our group has developed the theory for adjoint rupture dynamics with rate-state friction (Stiernström et al., 2024), building upon work by Kano and coworkers for a specific quasi-dynamic BEM rate-state model for afterslip and slow slip (Kano et al., 2013, 2015, 2020). We also did proof-of-concept dynamic rupture inversions using a MATLAB 2D antiplane shear code.

In this SCEC-funded project, we accomplished the following:

- Derived the adjoint equations for slip-weakening friction (in 3D, allowing for rake rotation). The 3D extension built on a derivation for the 2D problem with PhD student Rikuto Fukushima and collaborators Martin Almquist and Vidar Stiernström.
- Implemented the adjoint method (for both rate-state and slip-weakening friction laws) in Wenqiang Zhang's 3D dynamic rupture code drdg3d (Zhang et al., 2023) and verified the adjoint gradient by comparison to a finite difference gradient.
- Extended adjoint method to fully coupled acoustic-elastic models with gravity, which simulate ocean acoustic waves and tsunami in addition to dynamic rupture.
- Derived adjoint sources for various data types: displacement and velocity seismograms, surface displacement snapshots (e.g., InSAR), strain and strain rate (e.g., optical fiber distributed acoustic sensing or DAS), and ocean bottom data (e.g., ocean bottom seismometers, hydrophones, pressure gauges).
- Performed proof-of-concept inversions on synthetic test problems.

The drdg3d code is open-source, verified on SCEC/USGS benchmarks (Harris et al., 2018) and CRESCENT tsunami generation benchmarks (Kutschera et al., 2026), capable of handling arbitrarily complex fault network geometries, and produces accurate results even on relatively low quality meshes (facilitating the ease of application)—making it an excellent choice for our project. It is based on a nodal discontinuous Galerkin (DG) finite element method using a novel mixed flux scheme (Zhang et al., 2023). This scheme suppresses high-frequency oscillations and spatial spikes in fault normal stress, seen when using upwind fluxes around the fault. The GPU-accelerated code scales on supercomputers; the GPU version will be publicly available soon.

Proof-of-concept inversion

Figure 1 shows results from an inversion with rate-state friction. Rupture occurs on a thrust fault and velocity seismograms on the surface are inverted to determine the direct effect parameter a (which, because b is fixed, determines whether the fault is velocity-weakening (VW) or velocity-strengthening(VS)). Starting with a spatially uniform VW initial guess, the inversion quickly (~10 iterations) maps out the VW and VS regions. Subsequent iterations fine-tune the spatial distribute and reduce error around the edges of the fault, which create arrivals deeper in the seismograms. While this is a synthetic, noise-free test for a single parameter, it demonstrates that the implementation can be used for inversions.

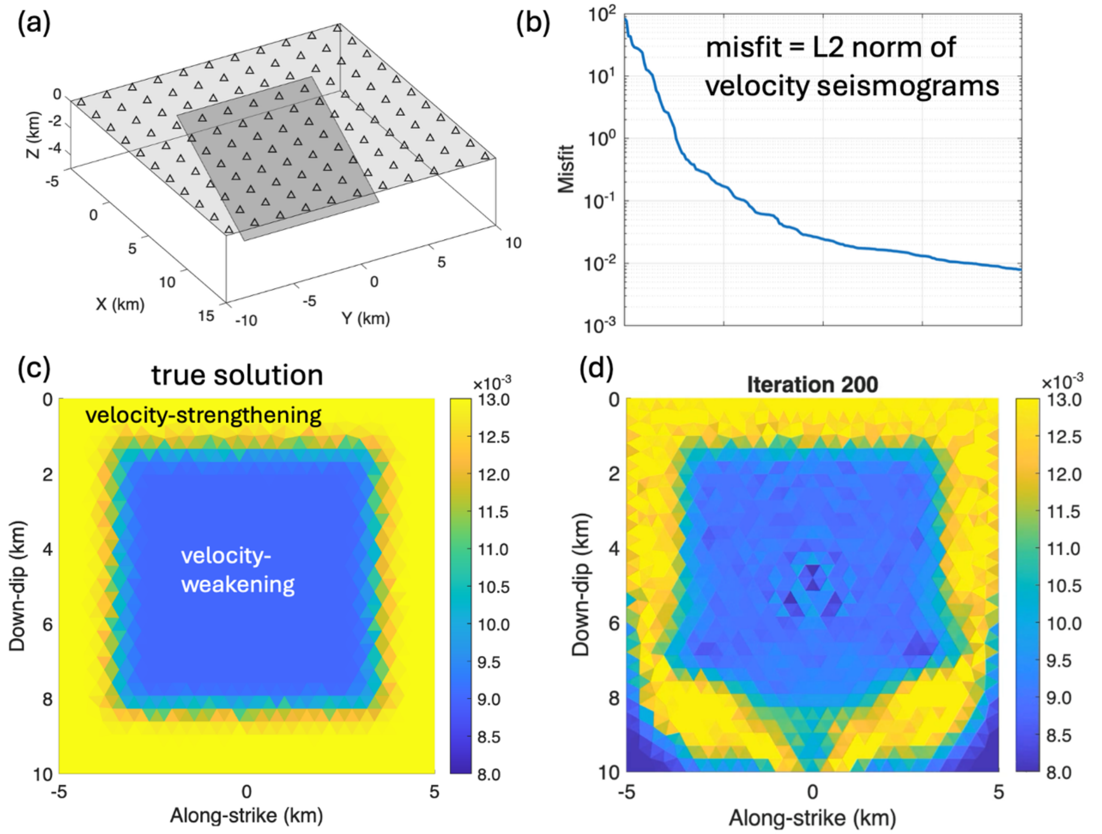

Figure 1. Adjoint-based inversion for rate-state direct effect parameter a using 3D rate-state dynamic rupture simulations. (a) Thrust fault with surface stations. (b) Decrease in misfit showing convergence of the inversion. (c) True distribution of rate-state direct effect parameter a, chosen so that rupture occurs within a central velocity-weakening (VW) region and is arrested by a surrounding velocity-strengthening (VS) region. (d) Direct effect a, after 200 iterations starting from uniform initial guess $a = 0.008$, using L-BFGS optimization. The seismograms lack arrivals from the bottom corners of the fault, preventing the inversion from being able to adjust parameter values there. Otherwise, the inversion successfully delimits the VW/VS regions with reasonably correct parameter values.

Extension to slip-weakening friction

While rate-state friction is being increasingly used in earthquake modeling, slip-weakening friction remains the most commonly used friction law in single event dynamic rupture modeling (Harris et al., 2018). This is because slip-weakening friction has fewer parameters and a well-defined fracture energy, making it relatively easy for modelers to adjust parameters to achieve some desired rupture history. We also understand which parameter combinations can be well resolved and which lie within the null space (Fukushima & Dunham, 2025). Specifically, fracture energy and dynamic stress drop are best recovered, while data are over an order of magnitude less sensitive to the slip-weakening rate. This matches intuition, experience from linear elastic fracture mechanics, and previous studies (e.g., Guatteri & Spudich, 2000).

Instead of the stating the 3D adjoint slip-weakening equations here, we highlight some interesting features of the adjoint problem. This discussion will assume constant normal stress, though our derivation and implementation account for variable normal stress (which requires nonzero fault opening in the adjoint problem, Stiernström et al., 2024). Consider a slip-weakening law in which shear strength τ_{str} depends on slip δ through a differentiable function $\tau_{str}(\delta)$. The weakening rate $d\tau_{str}/d\delta$ is piecewise constant for linear slip-weakening, which we assume hereafter. Complicating matters is the transition between locked and slipping states that is determined by the inequality constraints (e.g., Day et al., 2005): $\tau \leq \tau_{str}, V \geq 0, (\tau - \tau_{str})V = 0$. There is also the transition between the slip-weakening cohesive zone at the rupture front and the region where sliding occurs at constant residual strength behind this. The fault can also lock and then reslip.

These transitions are abrupt and delimit regions in space-time. The adjoint friction law is linear in the adjoint fields but must be defined piecewise in these space-time regions. Locked parts of the fault in the forward problem remain locked in the adjoint problem and slipping parts of the fault will obey a condition similar to a linear slip-dependent friction law relating adjoint shear traction $\tau^{\dagger}$ and adjoint slip $\delta^{\dagger}$, but expressed as $\dot{\tau}^{\dagger} = \left(\frac{d\tau_{str}}{d\delta}\right)_0 \delta^{\dagger}$. While in 2D this can be integrated in time to give a linear relation between $\tau^{\dagger}$ and $\delta^{\dagger}$, this cannot be done in 3D due to rake rotation introducing a dependence on slip velocity. The implementation of the adjoint problem is consequently more similar to a rate-state law than a slip-weakening law. Furthermore, while in the forward problem, slip velocity and shear traction are parallel (resistance directly opposes sliding), this is no longer true in the adjoint problem.

Verification of the adjoint gradient for slip-weakening friction and adjoint sensitivity to different receiver configurations and data types

We verify that our implementation of the adjoint method is correct by computing the gradient with finite differences. In addition to the original forward model, we run M additional forward models, where M is the number of model parameters (e.g., the number of elements on the fault if friction parameters at each element are used for the model parameterization). We compare the finite difference gradient to the adjoint gradient, finding excellent agreement.

This brings us to the point where we can start doing inversions with our method and code. However, geophysical inversions almost always have null spaces arising from parameter trade-offs and limited resolution. Before we apply the method to real data, we need to understand these trade-offs and resolution constraints. As a very preliminary exploration of the latter, we calculate the adjoint gradient (also known as the sensitivity kernel) with respect to the direct effect parameter for two station configurations (Figure 2). The central part of the fault around the hypocenter is resolved, but the surrounding region is not. Of course, the sensitivity kernel depends also on the forward model, and in an inversion the model parameters are updated at each iteration, so the sensitivity kernel also changes. We've seen in several cases that the sensitivity kernel for the first few iterations is highly concentrated around the hypocenter, which partially explains what is seen here. However, the comparison between the two station configurations clearly shows how adding additional stations affects resolution. Understanding these sensitivity kernels will be a major focus of our future work.

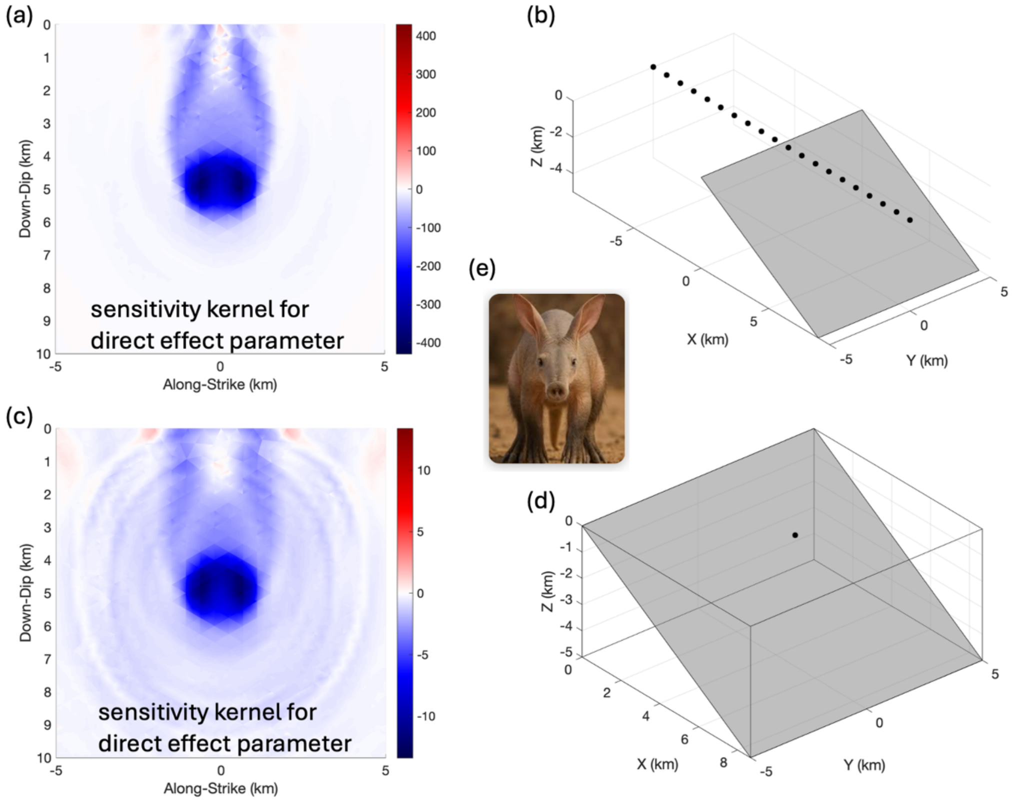

Figure 2. Comparison of sensitivity kernels of velocity seismogram misfit to direct effect parameter at one iteration in a dynamic rupture inversion. Shown for single station (a) and (b), and a linear array of stations (c) and (d). (e) Full waveform inversion sensitivity kernels look like bananas and donuts, but this kernel looks more like an aardvark to us. Understanding how the station distribution affects sensitivity to parameters of interest will be a major focus of future work.

Relevance to SCEC Milestones

Our work is most directly related to 8.C. Developing advanced modeling frameworks. The project team is active in dynamic rupture code verification activities (year 2 milestone C2-1). The open-source code that we used for this project, drdg3d, is developed by postdoc Wenqiang Zhang, who participates in the dynamic rupture benchmarks. The new modeling capabilities will help connect simulation data and real-world data, which is one of the goals of milestone C1-2. By making our code and adjoint modeling/inversion workflows available to the community, we will contribute to milestone C2-2. Finally, there is an indirect connection of our proposed work to milestone C3-1 that aims to create "simulations of coupled evolution of earthquakes, faults, and tectonic deformation." While our single event dynamic rupture simulations focus exclusively on the coseismic rupture, the stress changes that occur during ruptures contribute to the evolution of the damage zone and perhaps even the evolving fault structure. Critical to the success of this endeavor is ensuring that our dynamic rupture simulations are realistic, which means connecting them to real-world data. The adjoint method, which will make dynamic rupture inversions faster, more reliable, and automatic, will help with this. Furthermore, real faults are geometrically complex, multiscale and multisegment systems—which is challenging for rupture dynamics modeling. The drdg3d code, which combines a high-order-accurate discontinuous Galerkin method with the flexibility of unstructured meshes, is ideally suited for this challenge. This makes it one of the only codes that can be used for the advanced modeling capabilities that are needed to meet these SCEC milestones.

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