SCEC Award 21017: A Bayesian Framework for the Joint Estimation of Corner Frequency and Rupture Directivity for Southern California Earthquakes

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I. Project Overview and Objectives

For more fifty years, seismologists have used the spectra of earthquake waveforms to gain insight into the physics of earthquake rupture (Brune, 1970). In broad terms, the spectral amplitude at low frequencies is controlled by the overall size or moment M_0 of the earthquake. At higher frequencies, source spectral amplitudes decay in a power law fashion at frequencies higher than a transitional "corner frequency" f_c that depends on overall source duration. Taken together, these source spectral parameters – moment M_0, corner frequency f_c, and high-frequency falloff rate n – are widely used all across earthquake science, from fundamental studies of the rupture energy budget to more applied projects in ground motion and hazard estimation.

Unfortunately, the analysis and interpretation of earthquake source spectra is notoriously difficult and fraught with uncertainties (Abercrombie, 2015, 2021; Baltay et al., 2011; Bindi et al., 2020; Cotton et al., 2013; Shearer & Abercrombie, 2021), to the extent that different studies quite commonly come to opposing conclusions when tackling the same problem (Pennington et al., 2021; Shearer et al., 2019; Shible et al., 2022). While troubling, it is easy to spot a number of good reasons for such discrepancies. A central challenge in source spectral analysis is the need to correct the observed spectrum for path and site effects and thus obtain a clear picture of source's frequency content. This task, while simple in principle, can be quite difficult in practice. For large and exceptionally well-recorded datasets, it is sometimes possible to isolate the source spectrum using a spectral decomposition (Chen & Shearer, 2011; Shearer et al., 2006; Trugman & Savvaidis, 2021; Trugman & Shearer, 2017) or generalized inversion framework (Ameri et al., 2011; Bindi et al., 2021; Oth, 2013). More common is the practice to use small events that occur nearby a larger target event as "empirical Green's functions" or EGF and form spectral ratios in which common path and site effects will cancel (Abercrombie et al., 2017; Boyd et al., 2017; Hough, 1997; Huang et al., 2017; Ruhl et al., 2017). Both decomposition and spectral ratio workflows, while useful, are severely limited in their capacity to properly track data and modeling uncertainties, motivating a more nuanced approach.

An equally important issue, and one often neglected, is the need to consider variation in source spectral content across the focal sphere. Even for the commonly assumed paradigm of a symmetric circular source, the observed corner frequency and high-frequency falloff rate will vary markedly depending on the source-receiver azimuth and takeoff angle (Kaneko & Shearer, 2014). For real earthquakes, the problem will be more pronounced, as dynamic effects like rupture directivity will enhance these angular variations (Kaneko & Shearer, 2015). During a unilateral rupture, for example, apparent pulse durations will be shorter in the forward direction, resulting in elevated corner frequencies and corresponding larger high-frequency ground motions (Calderoni et al., 2017; Kurzon et al., 2014; Somerville et al., 1997). Because of real-world effects like these, developing new techniques that characterize rupture directivity parameters alongside traditional source parameters (and their uncertainties) is of utmost importance.

That, in a nutshell is the task that this project set out to tackle. In this work, we developed an inversion framework based on probabilistic programming and Bayesian inference to analyze earthquake spectra and extract key source parameters, measures of rupture directivity, and associated uncertainties. This approach, while more complex and computationally intensive than is conventional, is appealing for several reasons. The Bayesian framework allows one to rigorous track parameter uncertainties throughout the inversion process, including any tradeoffs between parameters that are commonly fixed during classical inversion approaches (such as high-frequency falloff rate or EGF corner frequency). Moreover, our technique allows one to encode known physical bounds or prior knowledge directly into the problem statement, which both stabilizes the inversion and optimally balance the relative information content of prior knowledge with the available data. Well-recorded earthquakes will therefore have finer parameter resolution that those without adequate sampling of the focal sphere.

This report documents an initial pilot study of the technique to examine the source properties of 14 target earthquakes of magnitude 5 occurring in southern California since 2005 that span a wide range of tectonic regimes and fault systems. These prominent earthquakes, while comparable in size, exhibit marked diversity in their source properties and directivity, with clear spatial patterns, depth-dependent trends, and a preference for unilateral directivity. These coherent spatial variations source properties suggest that regional differences in tectonic setting, hypocentral depth or fault zone characteristics may drive variability in rupture processes, with important implications for our understanding of earthquake physics and its relation to hazard. An early version of this work was presented at the Fall 2021 SCEC conference and is now in the late stages of the peer review process in the *Journal of Geophysical Research – Solid Earth*. Moreover, this initial effort has set the stage for even more ambitious follow-on projects that apply the method at scale in different faulting environments, some of which are already underway. Following SCEC's commitment to open science, the codes are publicly available on GitHub (with Zenodo archiving, doi: 10.5281/zenodo.5965588) and it our sincere hope that this effort will spur future collaborative efforts at the cutting edge of observational earthquake science.

II. Summary of Technical Approach

In this study, we assume earthquake spectra can be adequately modeled using a generalized form of the classic Brune spectrum:

$$S(f) \sim \frac{M_0}{1+(f/f_c)^n},$$

and calculate the Brune stress drop using the usual relation:

$$\Delta\sigma = \frac{7}{16}\, M_0 \left(\frac{f_c}{k\beta}\right)^3,$$

where β is the shear wave velocity at the source location and k is a numerical constant whose value depends on the assumed source model (Kaneko & Shearer, 2014).

For each of the 14 M5 target events, we select potential EGFs within 5km laterally and vertically and that are 1-2 magnitude units smaller. We then form spectral ratios between the horizontal component S-waves of target and EGF on each broadband or strong-motion station within 100km with adequate signal to noise on all traces. We implement a number of quality control checks to ensure only the most informative data are used in the ultimate inversion, most notably a cross-correlation procedure to ensure the mainshock and EGF event have sufficient mechanism similarity so as not to bias the spectral and directivity inference.

With this database of spectral ratios in hand, we use the PyMC3 probabilistic programming package (Salvatier et al., 2016) to solve for posterior probability distributions of seismic moment, corner frequency, and high-frequency falloff rate for target events and mainshocks alike. In addition, for the target events, which generally have good azimuthal station coverage, we generalize the directivity formulation of Boatwright (2007) to infer the "directivity" ratio *e* of each event. This parameter, sometimes called the "percent unilateral rupture", is a normalized metric with absolute values between 0 and 1, with values near 0 implying bilateral directivity and values near 1 implying unilateral rupture. The underlying assumption is that the presence of rupture directivity will modulate the apparent corner frequency in a systematic fashion with source-receiver azimuth, and thus observations at different stations will allow us to infer the implied directivity.

Bayesian inference requires suitable selections for prior distributions for all parameters of interest. Following careful experimentation and testing and guided by known physical constraints we settled on lognormal distributions for corner frequency, moment, and falloff rate, a Beta distribution for the apparent rupture velocity of the target event, and a uniform distribution for directivity ratio. Where possible, the parameter values are based on observational constraints such as catalog moments and previous measurements of stress drop values and variability. For directivity, moment tensor solutions help constrain the choice of plausible fault planes.

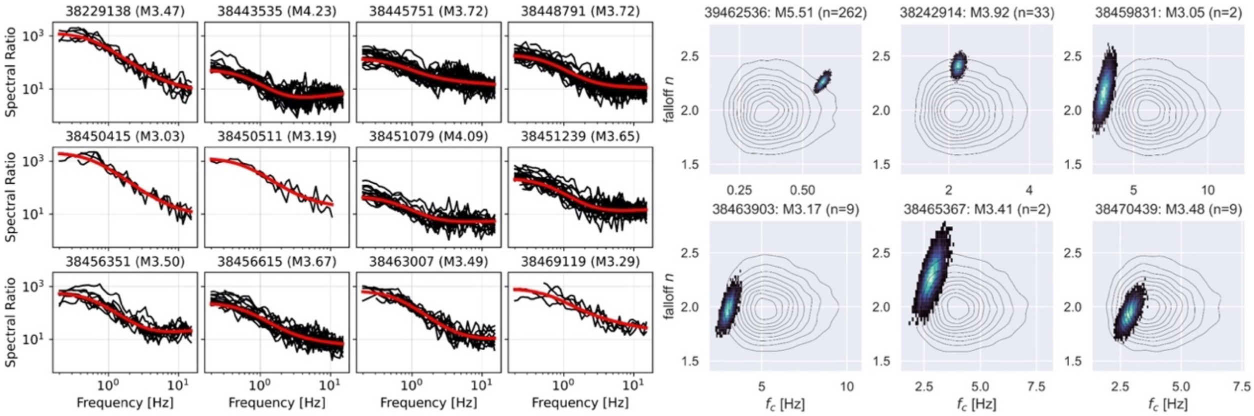

Figure 1: Examples of source parameter estimates. Left panel: examples of spectral ratios of a single target event with 12 distinct EGFs. The posterior prediction (red) matches well the data. Right panel: examples of posterior parameter estimates for a target event (top left) and 5 of its EGFs. There is a clear tradeoff between corner frequency (x-axis) and falloff rate (y-axis). Events with larger numbers of observations have tighter posterior distributions than those that are data limited. Contours from the prior distribution are shown in solid black lines for reference.

III. Key Results

The intriguing initial results of this pilot study showcase the promise and potential of the application of Bayesian inference to the analysis of source spectra. Examination of the posterior distributions of individual events (Figure 1) clearly demonstrate two important concepts with important implications for spectral analysis. First, there is a positive correlation between corner frequency and high-frequency falloff rate. This means that fixing the high-frequency falloff rate, as is commonly done in classical spectral ratio and spectral decomposition analysis will both bias the corner frequency measurement and underestimate its marginal uncertain. Second, well-recorded events have markedly smaller posterior uncertainties than those with few observations. Bayesian inference allows us to track these uncertainties in a quantitative fashion and employ all available EGFs in our inversions in a stable fashion.

One of the most striking observations from this study is the strong evidence for regional variations in earthquake source parameters in southern California (Figure 2). Events in the Salton Trough in the southern part of the study region have relatively low stress drops, while events along the San Jacinto Fault and Transverse ranges have higher stress drop values. The physical reasons for this are not entirely clear. We do observe strong evidence for an increase in stress drop with hypocentral depth but the regional trends appear to be robust independent of this conclusion. Future work with a larger sample size of target events may help elucidate this issue.

Our methodology also allows us to measure directivity parameters for the M5 target events, and the preliminary results are equally compelling. We observe a strong preference for unilateral rupture mode, with 11 out of the 14 events we consider showing clear evidence of directivity in the variations in source spectra with azimuth (Figure 2). All three of the bilateral events occur in the northern part of the study region in the Owen's Valley or in the Ridgecrest rupture zone, though several other Ridgecrest events are likely unilateral. Interestingly, we also observe a correspondence between the inferred rupture directivity and the spatial pattern of aftershocks. For events with clear unilateral rupture modes, aftershocks tend to concentrate in the forward rupture direction where dynamic stresses are likely enhanced. The correspondence is not perfect of course, as both secondary triggering and postseismic stressing undoubtedly play a role in aftershock occurrence. But the clear preference provides both an external form of validation of the method and motivates the importance of the study of directivity effects.

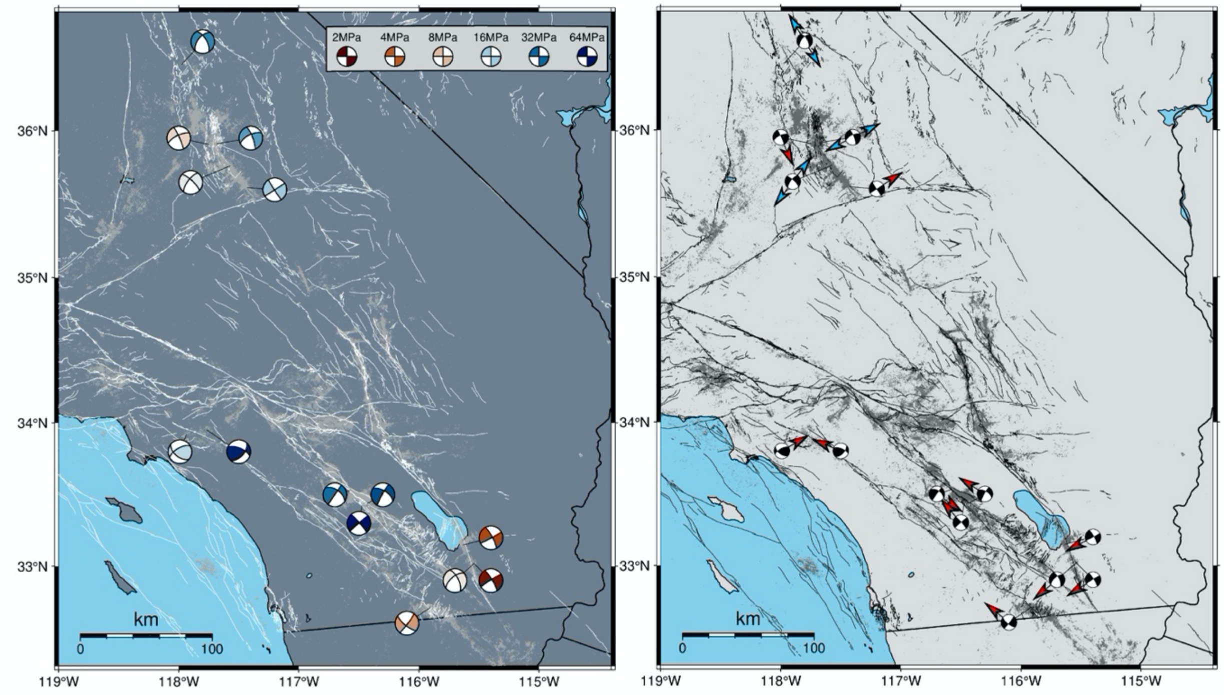

Figure 2: Spatial patterns in earthquake source parameters. Left panel: earthquake stress drop for the fourteen target events, with bluer colors indicating higher values (see key at top right). Right panel: shows inferred directivity for these events, with red arrows denoting unilateral events, and blue bilateral events. There are clear regional variations in stress drop and a predominance of unilateral rupture. Bilateral ruptures are most common in the northern part of the study region.

IV. Outlook and Significance

Support from SCEC for this project has instrumental in the development and preliminary application of a Bayesian inversion framework for the analysis of source spectra and rupture directivity. In our pilot study of 14 M5 target events and their EGFs in Southern California, we observe clear regional and depth-dependent variations in stress drop and rupture directivity. These measurements are at true "cutting edge" of observational seismology and integrate well with other SCEC projects focused on multiscale analyses of source rupture complexity as well as improve approaches to source parameter uncertainty quantification.

Our group aims to build on these initial results in several ways. With the basic inversion framework in place and properly benchmarked, we can now begin to apply the method at scale and across different study regions. The technique, while computationally intensive, is not prohibitive on a mid-scale computing cluster. For large-scale analysis with thousands of events, access to institutional or SCEC-supported supercomputing resources may be necessary. Two follow-on studies are already in progress by the Earthquake Science team at UT Austin. PI Trugman is part of an NSF-FRES effort to understand subduction zone hazards; source spectral analyses will be leveraged to map out along-strike and depth-dependent changes in megathrust frictional regimes. In parallel, Postdoc Nadine Igonin will build on her expertise in induced seismicity to study the source spectral properties of human-triggered sequences in Canada in Texas (manuscript in prep at present writing). These detailed studies take advantage of dense local arrays and will help us better understand the physical processes driving induced earthquake sequences.

Looking forward, we also hope this study will help build collaborations with other researchers in the SCEC community. The basic coding environment is already publicly available on GitHub. We will seek out collaborators with both observational expertise and interest in other study regions, as well as modeling and theoretical expertise to help revise the method to better address different forms of rupture complexity that are beyond the scope of the current modeling framework. In tandem, these collaborations will SCEC research help further advance earthquake science.

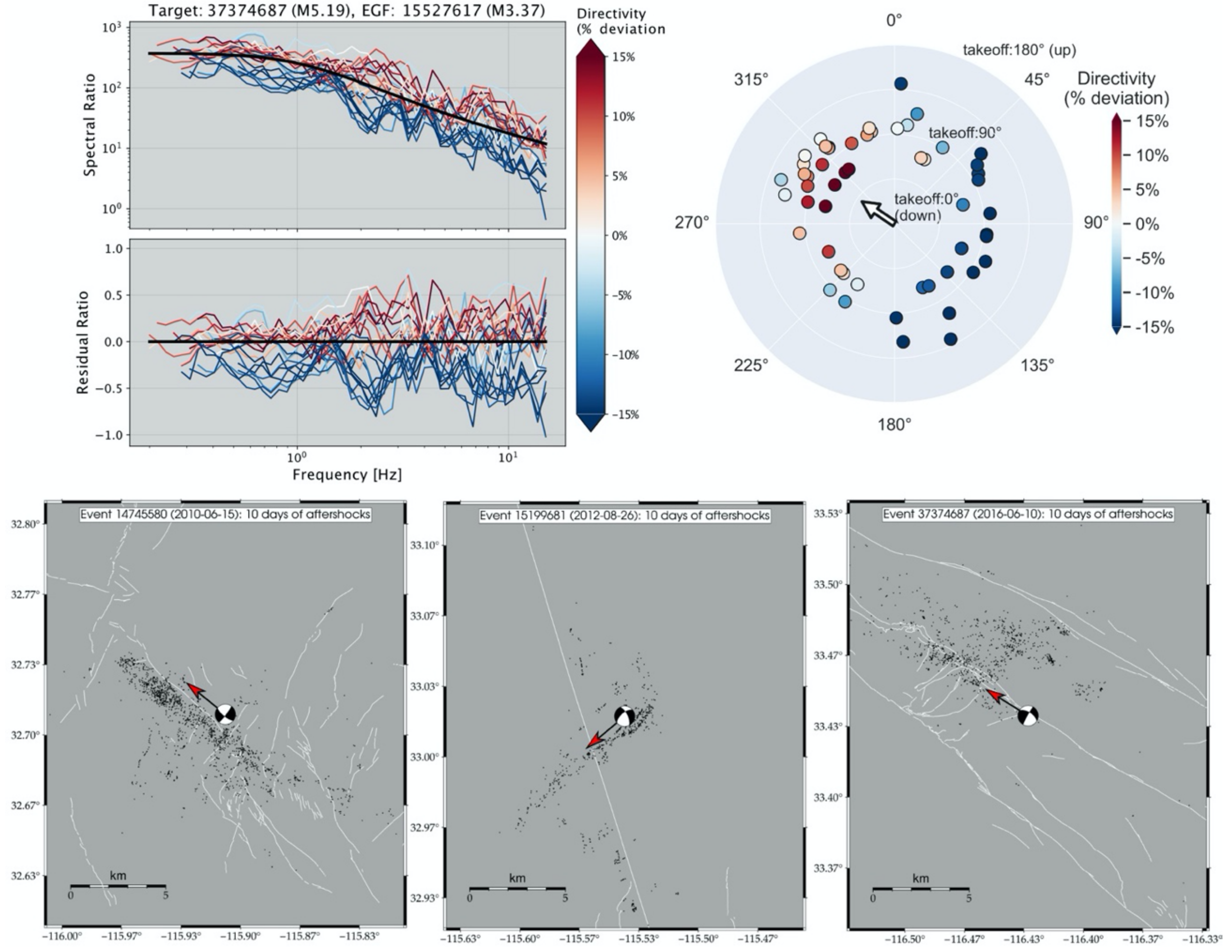

Figure 3. Directivity observations. Top panel: azimuthal variations in source spectra for target event 37374687 (the 2016 Borrego Springs event). Each spectral ratio with EGF 15527617 is color-coded by the model-inferred modulation in corner frequency (%). The clear gradient from blue to red as one traverses the focal sphere indicates strong unilateral directivity. Bottom panel: relation between directivity and the spatial pattern of early aftershocks. For events like these three with clear unilateral directivity, there is a tendency for aftershocks to concentrate in the forward rupture direction.

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