

SCEC 2022 Report:

Validation of Broadband Ground Motion from Dynamic Rupture Simulations: towards better characterizing seismic hazard for engineering applications

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Project Objectives and Overview of Accomplishments

In regions of limited empirical data, such as at near-source distances, earthquake simulations provide an approach to better constraining ground motion amplitudes and trends, enabling improved accuracy of ground motion predictions for seismic hazard analyses. However, most current methods for simulating earthquake ground motions ignore important features of the earthquake rupture and typically employ stochastic approaches at high-frequencies ($> \sim 1$ Hz). Here, we aim to improve methods for simulating earthquake ground motions for seismic hazard applications by employing a group modeling effort that incorporates features of the earthquake fault and rupture (e.g., through complex fault geometry, stress heterogeneity, etc...) that have been demonstrated from both observations and numerical simulations to affect resulting ground motions. Our investigation analyzes how earthquake rupture characteristics influence ground motion behavior and compares these synthetically generated ground motions with ground motion models (GMMs) to evaluate their similarities and differences. We work towards determining conditions for acceptance of broadband synthetic ground motions, towards the goal of ultimately providing confidence in supplementing empirical relations with simulation-derived information. This work will benefit many down-stream applications of ground motion models, including studies of probabilistic seismic hazard analysis, as well as for purposes of structural response and calculating engineering demand parameters.

We have created a database of dynamic rupture simulations (a fully deterministic, physics-based approach) of strike-slip earthquake mechanisms at frequencies computationally accurate up to ~ 3 Hz (varying somewhat depending on initial model conditions). The resulting broadband ground motions are compared with empirical trends predicted by leading ground motion models (GMMs). We focused on the magnitude range of $M_w \sim 5$ up to $M_w 7$ at distances up to 20 km from the source, comparing median spectral accelerations across a range of periods. Additionally, we analyzed the synthetic ground motion variability (which can be isolated in terms of both intra- and inter-event) as a function of both distance and period, for purposes of advancing seismic hazard characterization.

We also initiated a method to unify processing of synthetic ground motions output from multiple codes, in multiple formats, into a structured representation that can be processed uniformly by preexisting software. This will enable a more straightforward approach to aggregating various formats of synthetic time series outputs for comparison with empirical relations. In addition, this will provide the ability for the community to more easily process output data and upload their results to a public repository for a database accessible to scientists and engineers alike.

Methodology: Research Approach

Earthquake simulations can be utilized in areas of infrequent seismicity, or where geologic structures (e.g., sedimentary basins) complicate seismic wave propagation, to help constrain ground motion amplitudes and trends. Simulations allow generation of synthetic ground motion of both historical

and hypothetical events and analysis of the resulting ground motion using a user-defined station distribution. In this project, we generated earthquake sources that produce synthetic ground motion across a wide frequency bandwidth for magnitudes and distances relevant to engineering applications, for which empirical datasets are still poorly populated. We worked towards validating ground motions produced by dynamic rupture models against the empirical trends of GMMs. We simulated suites of events in the magnitude range of $M_w \sim 5$ up to $M_w \sim 7$ along strike-slip earthquakes and generated synthetic ground motion up to 20 km from the fault. Within each magnitude bin, we included multiple variations of initial stress conditions and hypocenter locations, so as to sample a range of earthquake rupture conditions. We initially keep the medium simple—using a 1D layered model (characteristic of a California hard-rock site, with a V_{s30} of 760 m/s) and analyze the variability of the synthetic ground motions (arguably a proxy for single-station standard deviation). This project leverages each PI's expertise and the computational infrastructure developed in past years; each of the codes used here have performed well in the SCEC/USGS Dynamic Rupture Code Verification Project (Harris et al., 2018).

There are several approaches to specify random distributions of fault stress, frictional properties, and/or fault roughness input into the simulations to produce variations in magnitude and distribution of ruptures (and that match observation data of spectral energy across a range of frequencies). We pursued two main routes of rupture variation: (1) imposing stochastic conditions along a planar fault with heterogeneous stress or friction conditions and (2) generating fractally-rough faults, where homogenous background stress conditions naturally introduce initial heterogeneous stress along the fault. Both techniques have been used successfully in previous work to generate ground motion that compares favorably with GMMs.

Results: 2021 SCEC Summary of Work

This year we continued a recently formed SCEC-funded project that focuses on a collaborative approach to validation of ground motions produced from dynamic rupture simulations. We built a synthetic database of ground motion amplitudes from a diverse range of initial conditions and modelling techniques. Each modeler generates earthquake sources that produce synthetic ground motion across a wide frequency bandwidth for magnitudes and distances relevant to engineering applications, for which empirical datasets are still poorly populated. We validate ground motions produced by dynamic rupture models against the empirical trends of GMMs. We simulated suites of events in the magnitude range of $M_w \sim 5$ up to $M_w \sim 7$ along strike-slip earthquakes and generated synthetic ground motion up to 20 km from the fault. Within each magnitude bin, we included multiple variations of initial stress conditions and hypocenter locations, to sample a range of earthquake rupture conditions.

Our group is a coordinated validation effort to model ground motions from dynamic ruptures, encompassing a wide level of well-qualified individuals across all stages of career and background. Each modeler uses their preferred code and dynamic rupture method, creating a diverse distribution of rupture behavior. This year several newcomers joined our group, helping to increase the total number of simulations available for validation and better sample the range of rupture conditions and resulting ground motion behavior. Figure 1 gives an example of a comparison from each different modeler, plotting the arrival time, max slip-rate, and final slip of one example simulation. We compare our synthetically generated ground motion with four current state-of-the-art GMM models (Boore et al., 2014, Abrahamson et al., 2014, Campbell & Bozorgnia, 2014, and Chiou & Youngs, 2014). Figures 2 and 3 show that the overall level of ground motion compares well with GMM's

predictions, plotting our synthetic results versus empirical trends over both period and distance. We find that the intra-event variability is highly dependent on hypocenter location, resulting from azimuthal changes in ground motion amplification. Each modeler also keeps track of fault displacement at the surface, that is used to further constrain our ground motion models is used by other coordinated studies within our group to supplement a paucity of empirical measurements.

We presented our progress at the annual 2021 SCEC and SSA conferences and have an accepted paper and presentation at the upcoming 12NCEE conference. We have received feedback from the community about the importance of our efforts, and have benefited from their suggestions, such as making our database of ground motion records available to the public for further investigation (in progress).

Individual Model Group Results

Part of the value of working within a coordinated team is to develop different approaches for the prescription of model characteristics that lead to validated ground motions. In that spirit, each co- PI constructed models of the benchmark by using their own preferred technique in specifying initial stochastic stress conditions, fault friction properties or geometrical fault complexity. They each developed a distribution of events by running a suite of dynamic rupture simulations appropriate within the magnitude range Mw 5–7, computationally accurate to frequencies ranging from 0.1 up to ~3 Hz. The following list outlines each PI's technique used to simulate a distribution of ruptures and describes their chosen method and associated results.

Withers constructed models with stochastic fault roughness and ran spontaneous earthquake ruptures along rough faults, using a well-verified code (Waveqlab3D) that accurately simulates the rupture along a fault with geometrical complexity at both short and long wavelengths. A suite of simulations was developed for validation by varying the random seed of the rough-fault geometry; this naturally produces varying magnitude events with variation of the hypocentral location and background stress conditions. Figures 2 and 3 show realizations of ruptures along a rough-fault topography profile, with variations in hypocenter location, and corresponding median spectral acceleration trends (compared to GMMs) with distance and associated intra-event variability. The intra-event variability is seen to be highly dependent on hypocenter location, resulting from azimuthal changes in ground motion amplifications. The overall level of variability is lower, as expected, since we currently exclude site effects and full 3D media heterogeneity in our velocity model. Withers also embarked on the unified processing of each groups synthetics results, to ensure uniformity among plotting routines, analysis of ground motion trends, and methods of validation, both via distance and period dependence.

Ulrich/Gabriel take part in the dynamic rupture validation benchmarks efforts using the open-source Discontinuous Galerkin code Seissol (www.seissol.org). They consider rough faults loaded by a uniform regional stress regime and governed by linear slip weakening friction. Their models also include stochastic frictional properties and off-fault plasticity. Band-limited (200m-40km) fractal rough faults are discretized using an unstructured tetrahedral mesh with 50m edge-length elements across the fault and high-order basis functions of degree 4. The statically adaptive mesh adheres to the velocity model. SeisSol's high-order space-time accuracy allows resolving 4-5 Hz broadband ground motion at feasible computational requirements (~ 20k CPU/h per forward simulation). Using a relatively short critical slip weakening distance of 0.15m, combined with stochastic variations of the dynamic friction coefficient (median, 0.2, one sigma [0.17, 0.23], two sigma [0.09,0.29]) translates in fluctuating peak slip rates in the range 5-15 m/s. Their results show very good agreement

with GMMs, illustrated by spectral accelerations from 0.25 to 3s, including the high-frequency content. Intra-event standard deviations at 10-20 km fault distances are in the range of the values predicted by the GMPE, while they are a factor of 2 lower closer to the fault. Further simulations, e.g. incorporating segmented faults, may allow deciphering if ground motions are really less variable in the very near-fault region.

Wang/Goulet used a dynamic rupture method aimed at validating simulated strike-slip fault displacements while producing observationally consistent near-fault ground motions. We simulated a broad magnitude range of events, from M5 to M8 that covers all the occurred surface rupture events. The rupture geometry we adopted in our dynamic rupture follows the scaling relationships of the rupture length and width for the interplate strike-slip events in Leonard (2014). The fault size is guided by the empirical relationships, but the resulting geometries do not strictly follow theirs, as the simulated rupture spontaneously arrests due to the nonuniform stress field. To mimic a realistic irregular coupling between neighboring tectonic stress and fault plane, we follow Andrews and Ma (2016) to exert a self-similar heterogeneous shear stress with horizontal and vertical long-wavelength components controlled by the selected empirical relationship (Leonard, 2014). We choose similar frictional and velocity profiles, as in Andrews and Ma (2016), and apply plasticity near the free surface for accommodating the observationally consistent on- and off-fault partition of fault displacement (Wang and Goulet, 2021). The simulated fault displacements produced from our dynamic rupture method were validated against observations in both the 1992 Landers earthquake and an aggregate empirical fault displacement dataset developed by the Fault Displacement Hazard Initiative (FDHI). Specially, we export three scenarios that have magnitudes of \sim M7 and found that ground motions at periods of 1 s and 3 s within 20km to fault have agreement with GMPEs.

Oral/Ampuero/Asimaki: This group models stress heterogeneity induced by regional past seismicity. Since last year, they increased the resolution of their models to 3 Hz and incorporated a distance-dependent fracture energy for a smooth arrest in natural low-stress areas. They calibrated their models in parallel with the GMPEs at three periods (0.5, 1, 3 seconds) and earthquake scaling laws. By changing the stochastic distribution of past seismicity, they generated 3 different cases of events ranging in magnitude: 7-7.1. Through their analyses on these models, they found that past seismicity induced stress heterogeneity can bring local supershear ruptures, particularly close to recent past events, and a notable variability of ground motion in terms of peak amplitude and polarization. They also found that the smooth arrest helps avoid artificial high frequency radiation that is expected in conventional models with pre-set fault borders. Yet, the source spectrum above the corner frequency, and consequent ground motion, is amplified because of medium stratigraphy. They are in preparation of a paper that details this progress.

Dunyu/Duan: This group simulates two sets of dynamic ruptures governed by the rate- and state-friction (RSF) with strong rate weakening and assume self-similar fault roughness and elastoplastic rock rheology (Shi and Day, 2013). Each dynamic rupture set has a randomized fault roughness with three different nucleation locations. The velocity structure adopts the assigned 1-D velocity structure in the benchmark. Initial stresses are depth dependent with over-pressurization. The pore pressure is assumed to be proportional to the lithostatic stress (Ulrich et al., 2019). The seismogenic depth is set at 14-15 km, which is achieved by linearly increasing the parameter a in the RSF below the seismogenic depth to make the fault velocity strengthening. The model domain is 80, 60, and 40 km along the strike, fault normal, and dip directions, respectively. Cell size is 50 m, which produces 1.5 billion hexahedral elements in each model. We use the high performance computer Stampede2 at

TACC at University of Texas at Austin to carry out the numerical simulations. Each model uses 960 CPUs to run 3.5 hours.

Results from the team. Comparing each modelling team's results individually allowed analysis of each group's approach and dependence on choice of initial conditions. It is also useful to combine each group's results and compare trends on the average of each set of simulations to gain further insight into the similarities and differences based on a common set of parametrizations and source constraints. Figures 2 and 3 shows this comparison for each of our 6 modelling groups, analyzing the synthetically generated ground motions with current state-of-the art GMMs (including Boore et al., 2014, Abrahamson et al., 2014, Campbell & Bozorgnia, 2014, and Chiou & Youngs, 2014) as a function of distance for long to short periods. The averaged ground motion from each individual's group was plotted, for ease of visual inspection, in addition to some individual simulations. We found that the overall level of ground motion is fairly consistent among modelers and compares well with GMM's predictions in term of distance decay and period dependence, but that the intra-event variability is highly variable among models. We note that most models are within 1 inter-event median from this example Mw 7.

Summary

We continued a newly formed SCEC-funded project that focused on a collaborative approach to validation of ground motions produced from dynamic rupture simulations. We worked towards improving models of earthquake rupture for applications to seismic hazard. Here, we utilize a dynamic rupture approach to validate synthetically generated ground motion by comparing with GMMs at a range of magnitudes and distances of engineering relevance. Our method uses physics-based simulations to generate deterministic broadband ground motions; the synthetic median and variability of our simulations are compared with leading ground motion models (GMMs). Our goal is to address relevant needs of the community, particularly the end users of simulations such as engineering researchers and engineers. We intend to make our validation publication available to the community, so that individuals from a variety of background disciplines can leverage our results for a variety of purposes.

In the future, we plan to focus our efforts on continuing to expand our database of ruptures across a wider range of predictor variables, including magnitude, frequency, distance, rake, dip, etc... This will enable comparison of trends with predictor variables and enable investigation into the significance of changes in source conditions for near-source ground motion studies. Motivated by recent studies that explore the significance of velocity structure in resulting ground motions, we also intend to include more complex velocity models within our simulations, including additional site conditions and layered velocity structures, with varying V_{s30} , $z_{1.0}$, and $z_{2.5}$ values. Additionally, we will compile our group's results and submit a summary paper to a peer-reviewed journal, to present to the community our recent results. This will inform researchers and engineers of our progress, provide insight into the methods we have found useful in our approach, and pave a path forward for implementation into seismic hazard studies. Finally, we also will begin targeting additional methods of validation beyond spectral acceleration, including comparison with duration metrics, as well as considering other alternative measures.

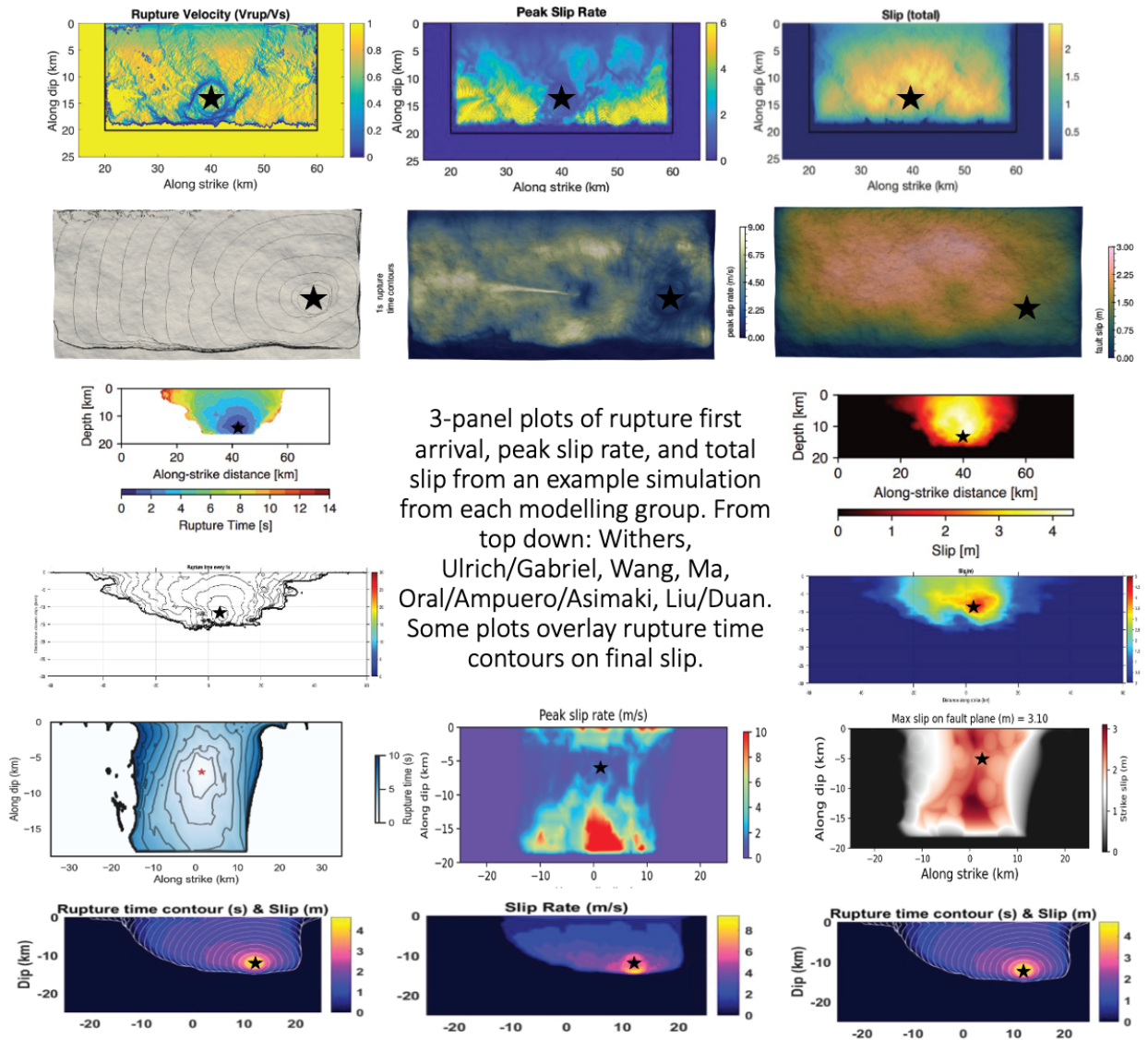


Figure 1. Rupture plots from each modelling group, showing rupture progression, the maximum slip-rate along the fault, and the final slip for one example strike-slip fault at Mw 7.

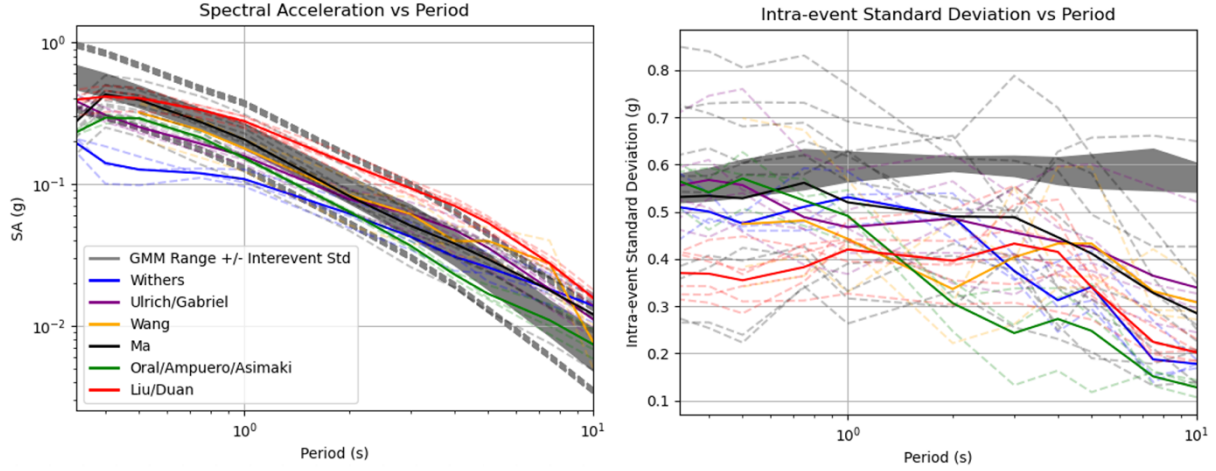


Figure 2. (Left) Ground motion trends versus period compared with empirical models for various different initial conditions, including hypocenter location. (Right) Corresponding intra-event standard deviation for a set of rough-fault simulations for $M_w \sim 7$, where the hypocenter location are varied. We extract ground motion from 4 leading GMM relations using the values of Z1.0, Z2.5, Vs30, Rjb, etc... used in our simulations. Average trends are plotted in bold, while each individual simulation is plotted as a dashed line.

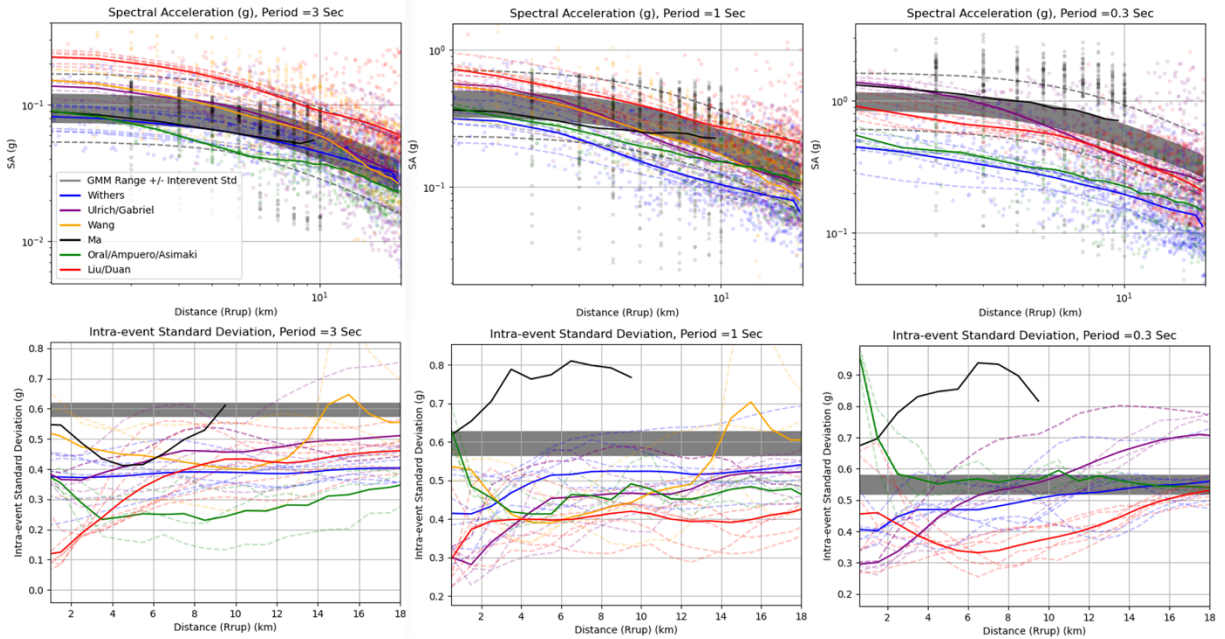


Figure 3. An example ground motion distance decay plotting average SARotD50 and corresponding intra-event standard deviation versus distance at three periods for 6 modeling groups for a set of dynamic rupture simulations. The average magnitude is $M_w \sim 7$, and the hypocenter locations and other initial rupture conditions are varied within each group of simulations. Average trends are plotted in bold, while each individual simulation is plotted as a dashed line. Individual station results are plotted for a random selection of 1000 stations from each modelling group (for clarity the full number of stations, totaling over a million data points per simulation in some cases, could not be plotted).

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