1 Introduction

The aim of this project is to characterize seismicity on geometrically heterogeneous (fractal) faults with pseudo-dynamic simulations of earthquake cycles. We considered a 2-D, plane strain fault loaded at an approximately constant shear stressing rate, and found that seismicity concentrates in the final phase of the cycle. We therefore focused on understanding foreshock sequences, and specifically the interplay of aseismic slip and seismic events leading up to the mainshock (system-size rupture).

2 Numerical model

We ran 2-D plane strain simulations with the quasi-dynamic boundary element code FDRA (Segall and Bradley, 2012). Fault slip is governed by the following equation of motion:

$$\tau_{\text{el}}(x) - \tau_f(x) = \frac{\mu}{2c_s}v(x),$$  \hspace{1cm} (1)

where $\mu$ is the shear modulus, $\tau_f$ the frictional resistance, and $\tau_{\text{el}}$ the shear stress due to remote loading and stress interactions between elements. The stress from each element is computed from dislocation solutions (e.g. Segall, 2010), accounting for variable element orientation. The right hand side is the radiation damping term, which represents stress change due to radiation of plane S-waves (Rice, 1993), with $c_s$ the shear wave speed. Earthquakes are defined as times when the slip velocity anywhere on the fault exceeds the threshold velocity $V_{\text{dyn}} = 2a\sigma c_s/\mu$ (Rubin and Ampuero, 2005).

Frictional resistance evolves according to rate-state friction (Dieterich, 1978):

$$\tau_f(v, \theta) = \sigma \left[ f_0 + a \log \frac{v}{v^*} + b \log \frac{\theta v^*}{d_c} \right],$$  \hspace{1cm} (2)

where, $a$, $b$ and are constitutive parameters; $d_c$ is the state evolution distance; $\sigma$ is the effective normal stress; $v_0$ a reference slip velocity; $f_0$ the steady-state friction coefficient at $v = v^*$, and $\theta$ is a state-variable. We employ the ageing law Ruina (1983) for state evolution:

$$\frac{d\theta}{dt} = 1 - \frac{\theta v}{d_c},$$  \hspace{1cm} (3)

such that steady-state friction at sliding velocity $v$ is

$$f_{\text{ss}}(v) = f_0 + (a - b) \log \frac{v}{v_0},$$  \hspace{1cm} (4)
The fault is loaded by resolving a uniform stressing rate tensor on each element (for more details, see Cattania and Segall (2020)). The fault is oriented at 45° from $\dot{\sigma}_1$, so that the average normal stress does not vary with time. Fault geometry leads to local variations in shear and normal stressing rate, which we find to be minor compared to the slip-induced normal stress perturbations discussed below; we can therefore approximate the loading rate as uniform.

3 Results

3.1 Normal stress variations

The most important effect of roughness is to introduce spatial variations in normal stress caused by slip, consistent with earlier studies (Chester and Chester, 2000; Dunham et al., 2011; Romanet et al., 2020). As shown in Fig. 1, we find that these are well approximated by the following expression:

$$\Delta \sigma(x) = \frac{\mu' S}{2} \int_{k_{\text{min}}}^{k_{\text{max}}} k^2 \hat{y}(k) e^{i(kx + \pi/2)} \, dk,$$

where $S$ is the total slip (assumed uniform), $k_{\text{min}}$ and $k_{\text{max}}$ the minimum and maximum wavenumbers in the fault elevation profile and $\hat{y}(k)$ its Fourier transform. These normal stress perturbations control the fault slip behavior and the coexistence of creep and foreshocks.

3.2 Interseismic and preseismic slip patterns

The interseismic slip behavior on a rough fault is remarkably different from its planar counterpart (Fig. 2): while a planar fault is entirely locked ($V \sim 10^{-14} - 10^{-12} \text{m/s}$) a rough fault develops localized creeping patches ($V \sim 10^{-11} - 10^{-10} \text{m/s}$). These correspond to locations where the normal stress perturbations induced by roughness are negative (tensile), reducing the effective normal stress. By
virtue of the reduced $\sigma$, these regions have low stress drops, and hence start slipping early in the cycle. Since low $\sigma$ promotes stable slip, creeping patches do not accelerate towards unstable (seismic) slip, but instead continue sliding at steady-state velocity. In contrast, regions with higher than average $\sigma$ (asperities) remain locked during most of the cycle, but can nucleate seismic events: foreshocks, and eventually a mainshock (Fig. 3).

### 3.3 Acceleration leading up to the mainshock

The foreshock sequence is characterized by a migratory behavior (Fig. 3): earthquakes tend to occur just outside the rupture area of previous ones, consistent with static stress changes and in agreement with observations (Ellsworth and Bulut, 2018; Yoon et al., 2019). However, abundant aseismic slip also takes place during the sequence, loading asperities and also contributing to subsequent foreshocks. In turn, each subcluster of foreshocks increases stresses in the creeping areas, where slip velocities increase (Fig. 3).

Both foreshocks and creep velocities increase as $1/t$, where $t$ is the time to the mainshock (Fig. 5 in Cattania and Segall (2020)). Based on the mutual stress transfer between asperities and creeping regions, we suggest that the foreshock sequence is controlled by a positive feedback between the two. We derive a simple model based on the following assumptions: 1. seismicity rate varies linearly with creep rate; 2. each foreshock loads creeping areas by a constant stress amount; 3. creep velocity increases by a constant factor as a response to a coseismic stress change (as predicted by rate-state theory). This model reproduces the $1/t$ acceleration observed in the simulations and in observed catalogs (Jones and Molnar, 1979; Ogata et al., 1995), and provides an alternative mechanism to previous explanations appealing to nucleation on a uniform fault (Dieterich, 1992; Rubin and Ampuero, 2005) or direct earthquake triggering (Helmstetter and Sornette, 2003).
Figure 3: Creep acceleration and seismicity leading up to the mainshock. Top: slip velocity on the fault vs. time to the end of the mainshock, with red bars marking the rupture length and triangles marking the nucleation point (mid-point of the region where \( v > V_{\text{dyn}} \) during the first earthquake time step). Note the sudden acceleration in nearby creeping patches and the widening of the fast slipping region with each successive seismic burst. Bottom: subset of the top panel, with events numbered by occurrence time. Small black dots indicate the location of maximum slip velocity at each time step, showing accelerated creep at the edges of each burst, where the subsequent ones initiate. Grey panels show close ups of a few clustered foreshocks.
References


