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SCEC Project 17179: Self-similar behavior of rate-strengthening faults

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Motivation

Frequent evidence for stable, aseismic fault slip—in which there is no runaway, earthquake-nucleating instability—includes observations of transient slow slip events, interseismic creep, and postseismic slip. A common presumption is that runaway acceleration of fault slip in these instances is suppressed by the self-limiting effect of a slip-rate-strengthening friction. This may be due to a non-linear viscous rheology, in which fault strength has a power-law dependence on sliding rate, or a non-linear rate and state dependence of friction, in which strength depends logarithmically on the sliding rate, as well as its history. How do such slip-rate-dependent friction laws couple with elastic deformation to determine the spatiotemporal evolution of slip on a fault in response to a driving force? How does an elevated slip rate spread along the fault, at what rate does it decay, and how are these integrated to affect displacement at the surface?

Problem formulation and solutions

We began by examining the consequences of assuming a frictional strength that depends linearly on slip rate. The advantage of starting with such a viscous fault description is that it preserves the mathematical linearity of the problem coupling this strength criterion slip to the linear elastic deformation of the medium. Consequently, classical analytical solution techniques apply and the results have relevance for more general, non-linear dependence on slip rate and state. While key results are summarized below, further discussion and details can be found in Viesca and Dublanchet [2019].

We considered in-plane or anti-plane slip along a planar fault. In these cases, the shear stress on the fault plane τ can be written, for some distribution of slip $\delta(x,t)$, as

$$\tau(x,t) = \tau_b(x,t) + \frac{\mu'}{\pi}\int_{-\infty}^{\infty} \frac{\partial\delta(s,t)\partial s}{s-x}\,ds \tag{1}$$

Where μ' is an effective shear modulus and $\tau_b(x,t)$ is the shear stress resolved on the fault plane in the absence of slip, which can be considered an external forcing. The shear strength of a thin viscous layer of thickness h and viscosity η_f is

$$\tau_s(x,t) = \frac{\eta_f}{h} V(x,t) \tag{2}$$

where $V = \partial\delta/\,\partial t$ is the slip rate, where slip is the displacement of top of the viscous layer relative to that at the bottom. We require that $\tau = \tau_s$ along the fault plane. After non-dimensionalizing, the problem governing the evolution of slip (or similarly, slip rate) reduces to

$$\frac{\partial\delta}{\partial t} = \mathcal{H}\left(\frac{\partial\delta}{\partial x}\right) + \tau_b(x,t) \tag{3}$$

and resembles the diffusion equation

$$\frac{\partial\delta}{\partial t} = \frac{\partial^2\delta}{\partial x^2} + \tau_b(x,t) \tag{4}$$

where a spatial derivative has been replaced by the Hilbert transform $\mathcal{H}[f(x,t)] = \int_{-\infty}^{\infty}\frac{f(s,t)}{s-x}\,ds$.

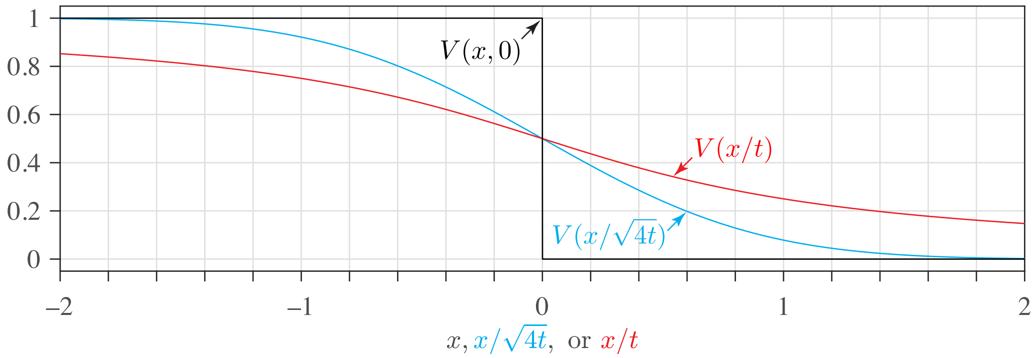

Figure 1. Similarity solutions (red and cyan) for slip rate V for a linear viscous fault having undergone an initial step in stress on $x < 0$, a problem equivalent to having the initial slip rate distribution shown in black. The cyan curve corresponds to the well-known error-function solution satisfying the classical diffusion equation and exhibits characteristic exponential decay. In contrast, the red curve, illustrating the solution (#) satisfying the non-local diffusion equation (#), exhibits power-law decay with a distinct similarity variable, x/t.

We leverage the problem linearity to use its Green's function to write solutions for slip rate in response to a time-varying stress rate $\partial\tau_b / \partial t$ as

$$V(x,t) = \int_{-\infty}^{t} \int_{-\infty}^{\infty} \frac{\partial \tau_b(x,t')}{\partial t'} G(x,t;x',t') dx' dt' \tag{5}$$

Where the Green's function for problem (3) was found to be

$$G(x,t;x',t') = \frac{1}{\pi(t-t')} \frac{1}{1+[(x-x')/(t-t')]^2} \tag{6}$$

Implying that an impulsive loading on the fault leads to a slip rate that spreads in space $\sim t$ and decays with time $\sim t^{-1}$.

As a simple, illustrative example consider the problem determining the spatiotemporal decay of an initially elevation of fault slip rate, in the form $V(x,0) = H(-x)$, where H is the Heaviside step function. The full solution is

$$V(x,t) = \frac{1}{\pi} \text{arccot}(x/t) \tag{7}$$

We may compare this solution to the well-known solution for classical diffusion under the same initial conditions

$$V(x,t) = \frac{1}{2} \text{erfc}(x/\sqrt{4t}) \tag{8}$$

For problems in which fault loading is localized in space in time, we derive asymptotic expansion for slip rate

$$V(x,t) = \frac{a_1}{t} f_1(\eta) + \frac{a_2}{t^2} f_2(\eta) + \dots \tag{9}$$

where the functions f_n of the similarity variable $\eta = x/t$ are known. The coefficients a_n contain information on the spatial distribution of fault loading that triggered the slip-rate transient at early time. Specifically, for a sudden loading on the fault imposed at $t = 0$, the coefficients reflect the moments of the load distribution. In Viesca and Dublanchet [2019], we provide several worked example solutions and their asymptotic expansion. In the following section we focus on the problem of postseismic slip, using the asymptotic expansion for fault slip rate (9) to in turn derive the asymptotic behavior of displacement at the surface of a strike-slip fault.

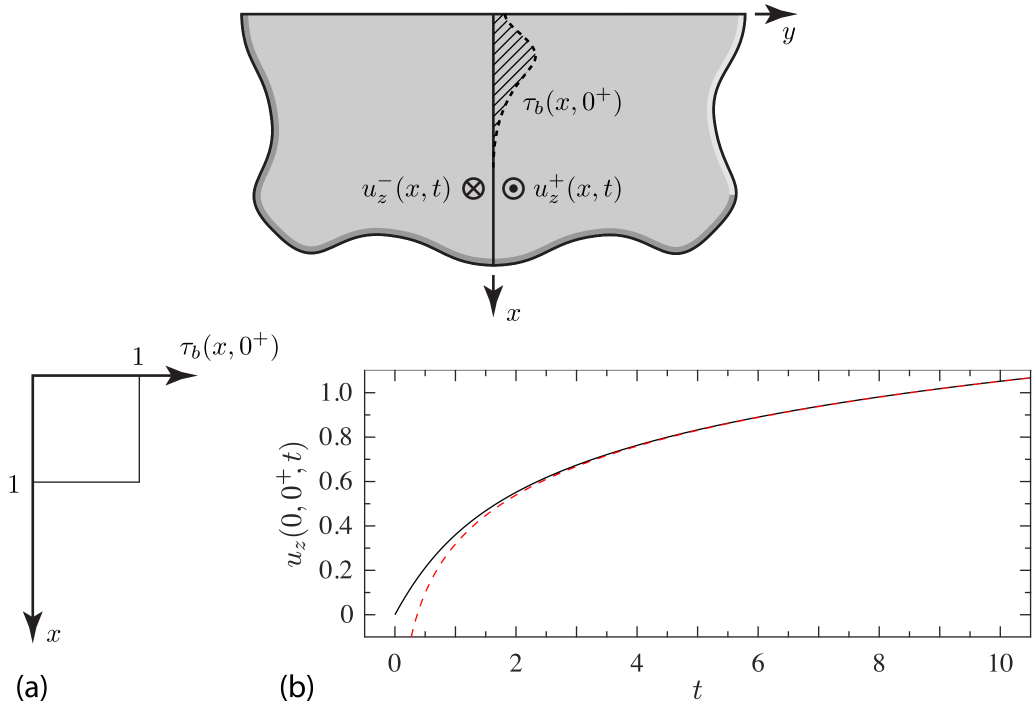

Figure 2. (**top**) A strike-slip fault in the x-z plane intersects the free surface at $x = 0$. The solution to a sudden step in stress τ_b at $t = 0$ along the fault ($x > 0$) is found by method of images. The hatched area is the net force exerted on the fault (per unit distance along z), which determines the leading-order coefficient a_1 of the asymptotic expansion. (**left**) Boxcar step in stress imposed at $t = 0$. (**right**) Evolution of the anti-plane displacement component u_z at the free surface, to one side of the fault. The displacement quickly approaches a logarithmic asymptote (dashed).

Implications for postseismic deformation

The compact, closed-form nature of the solutions allow for general results to be found for displacement at the surface in response to a slip transient on a viscous fault. In Figure 2, we consider the anti-plane slip of a fault subject to an initial step in stress over a finite depth. We derive the full solution for slip along the fault and displacement at the surface. We also derive the long-time asymptotic behavior of each. Displacement at the surface accumulates logarithmically with time.

Previously, logarithmic time-histories of surface displacements were taken as supporting evidence for a slip rate- and state-dependent friction, given that the coupling of such a logarithmic slip rate dependence with a spring-block slider model of afterslip would in turn lead to a logarithmic accumulation of displacement [e.g., Marone et al. 1991; Perfettini and Avouac, 2004; Montesi, 2004; Helmstetter and Shaw, 2009], while other descriptions (linear or power-law viscous) would not. However, we find that this exclusion to be a consequence of the assumption that area of the fault that participates in afterslip the remains fixed, with the fault locked against sliding on all boundaries of this area. Our analysis of the full continuum problem relaxes this assumption and allows slow slip to evolve along the fault. Doing so, we find that any linearizable rate-strengthening description would lead to a logarithmic (or near-logarithmic) accumulation of displacement due to afterslip at long times. While in the example problem considered, the entire fault is allowed to slide in this example for analytical simplicity, enforcing locking conditions over a finite depth and allow for post-seismic creep below, reveal comparably weak time dependence of post-seismic surface displacements.

Relevance for rate-and-state friction and application to models of creeping landslides

Most fault strength descriptions are non-linear. What is the relevance of the results of a linear for these descriptions? First, most non-linear descriptions are linearizable about a finite or zero slip rate, such that the linear description can be used to consider the spatiotemporal evolution of slip in response to small perturbations. Second, for larger perturbations, the asymptotic expansion of our linear results here suggests a route for conducting a non-linear perturbation expansion for non-linear problems. We investigated such a perturbation expansion for a rate- and state-dependent fault friction. We examined the decay of a rate-strengthening fault back to steady creep following a perturbation in stress. We found that the leading order term in the asymptotic expansion (9) applies to a rate-and-sate fault as well. In addition, the subsequent terms in the asymptotic expansion (9) appear in the perturbation expansion, alongside corrections for the non-linearity of the friction.

In parallel to investigating the effects of a rate- and state-dependent friction, we also investigate the quantitative difference of assuming an elastic configuration commonly used to model translational landslide motion: a compliant elastic layer overriding a rigid substrate (Wang et al., 2018). This change entailed replacing the convolution in (1) with another spatial derivative, such that in the case of a linearly viscous basal friction, the problem governing slip and slip rate reduces to the classical diffusion equation (4). We derived the perturbation expansion for the slip rate for a creeping landslide subject to an external loading (e.g., an increase in basal shear stress, or comparably, an increase in basal pore pressure) and whose basal friction is described a rate- state-dependent basal friction. The perturbation expansion describes the long-time behavior of the transient increase in down-slope displacement of the landslide.

References

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Related reports

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