IMPLEMENTING INTER-PERIOD CORRELATIONS INTO SCEC BROADBAND PLATFORM SIMULATIONS

REPORT – SCEC Project ID 17138

Introduction

The inter-period correlation of epsilon (ρ_ϵ) is an important component of ground motions for capturing the variability of structural response that is needed in seismic fragility and seismic risk studies. Without the appropriate inter-period correlation of ground motions, variability in the structural response may be under-estimated. This leads to structural fragilities which are too steep (under-estimated dispersion parameter β) and propagates through to non-conservative estimates of seismic risk. This is the motivation for calibrating the inter-frequency correlation of the finite fault simulations. In Bayless and Abrahamson (2018), current state of multiple existing ground motion simulation methods was assessed by evaluating the inter-frequency correlations from forward simulations and comparing with the correlation from empirical models. None of the six finite-fault simulation methods tested adequately capture the inter-period correlations over the entire frequency range evaluated, although several of the methods show promise, especially at low frequencies. Therefore, the inter-frequency correlation from numerical simulation methods needs to be calibrated. This research tests methods for calibrating the simulation method EXSIM (Atkinson and Assatourians, 2014).

We have developed a post-processing approach to implement the correlation into the SCEC BBP codes, and have implemented and tested it using EXSIM (fortran and python).This process is described within this report. Future work is required to implement the correlation at the sub-source level.

In a related study, we have evaluated the long-period correlations of the GP method, and tested the sensitivity of the correlations to various source methods. This has been accomplished by testing the GP method, the SONGS method (which uses GP wave propagation but a modified source representation), and with simulations from Arben Pitarka (which use an Irikura-type source along with GP wave propagation). Based on these results we concluded that modifications to the source can have impactful effects on the between-event and within-event components of the correlation. However, we also concluded that the between-site component of the correlation (which cannot be determined for simulations based on a 1-D velocity model) is non-negligible in the data, and so modifications made to the source alone cannot get correlation of the simulations up to the levels of the data. Modifications to the GP method should be a subject of future research.

<u>Background on SMSIM and EXSIM</u>

The point-source stochastic method for simulating ground motions, based on the pioneering work of Brune (1970), Hanks and McGuire (1981) and Boore (1983), among others, has been developed and refined over several decades. David Boore formalized the method, extended it to the simulation of acceleration time series, and developed a computer code named SMSIM for the implementation (Boore, 1983; Boore, 2003; Boore, 2005). EXSIM is the finite-fault extension of SMSIM. Like SMSIM, EXSIM is also an open-source simulation algorithm for generating time series of ground motion for earthquakes. EXSIM divides a finite-fault rupture into sub-sources with each sub-source modeled as a point source using the point-source stochastic method. The acceleration time series resulting from each sub-source is summed in the time domain after applying appropriate time delays for propagation of the rupture front (Atkinson and Assatourians, 2014). The version of EXSIM used here is described in Atkinson and Assatourians (2014) and is implemented as a FORTRAN code in the Broadband Platform (BBP, Maechling et al., 2015) v17.3.

Implementing the Correlation

SMSIM

SMSIM uses filtered white noise in the time-domain, resulting in ϵ with no correlation between frequencies. To generate a simulated time series with realistic inter-period correlation of the epsilon, the SMSIM procedure can be modified as described below.

First, a symmetric, positive-definite covariance matrix (Σ) for the inter-frequency $\rho_{\epsilon,total}$ of FAS is needed. This matrix is factorized using the Cholesky decomposition $\Sigma = LL^T$, where L is a lower triangular matrix (Seydel, 2012). Then the zero-mean correlated random variables Y can be calculated as $\mathrm{Y} = \mathrm{LZ}$, where Z are independent random variables drawn from a standard normal distribution. The random variables Y are normally distributed with zero mean and covariance matrix Σ. In the SMSIM procedure, ϵ values usually obtained from time-domain noise are replaced with correlated random numbers sampled in this fashion. The sample ϵ is scaled by a standard deviation equal to 0.65 (ln units). The value of 0.65 is consistent with the standard deviation of the FAS that results from the SMSIM procedure, which is not sensitive to the time-domain variance of input white noice. The scaled ϵ are converted to normalized FAS by $\exp(\epsilon * \sigma)$. The correlated ϵ are standard-normally distributed in natural logarithm space, so the normalized FAS are log-normally distributed. For a log-normally distributed variable X, the first moment (mean) is given by Equation 1 (Kenney and Keeping, 1951):

$$E[X] = e^{\mu_\epsilon + \frac{1}{2}\sigma_\epsilon^2} \quad (1)$$

where μ_ϵ and σ_ϵ are the mean and standard deviation of the natural logarithm X. In this application, $\mu_\epsilon = 0$. The normalized FAS need to have unit mean so that implementing the correlation does not change the mean amplitude of the simulations (over a suite of realizations) with respect to the unmodified simulation method. Therefore, to get normalized FAS with mean equal to one, these must be scaled by the adjustment factor given in Equation 2.

$$SF = \frac{1}{e^{\frac{1}{2}\sigma_\epsilon^2}} \quad (2)$$

With the imposed value of $\sigma_\epsilon = 0.65$, the adjustment factor $SF = 0.8096$. Finally, the normalized and adjusted FAS are scaled by the Fourier amplitude spectrum of the considered scenario, and the SMSIM recipe is continued to generate time series with realistic ρ_ϵ: $\mathrm{SMSIM_{corr}}$.

This $\mathrm{SMSIM_{corr}}$ procedure is similar to the method described in Stafford (2017). Individual realizations of correlated ϵ may be positive or negative for frequency bands, but as the sample size is increased, the sampled ϵ have the intended standard-normal parameter values. Therefore, with a sufficient sample size, and with the adjustment factor given by Equation 2, the median FAS or spectral acceleration (PSA) of a simulated scenario should be the same as from the unmodified SMSIM procedure. In Figure 1, example smoothed FAS spectra from the SMSIM and $\mathrm{SMSIM_{corr}}$ procedures are shown for one sample of ϵ.

In this implementation, the model for the inter-frequency ρ_ϵ of FAS developed in Bayless and Abrahamson (2018) is used to generate correlated ϵ. The model usable frequency range is $f =$ 0.1-24 Hz and requires extrapolation for frequencies outside this range. The extrapolation is performed by using the values for model coefficients $\mathrm{A}, \mathrm{B}, C$, and D at either $f = 0.1$ or $f = 24$ Hz, for extrapolating to lower and higher frequencies, respectively. The extrapolation is performed to generate correlated ϵ over the frequency range 0.01-100 Hz in the SMSIM and EXSIM (described below) implementation procedures. This extrapolation may introduce a bias in the correlations at very low and high frequencies, as discussed further below.

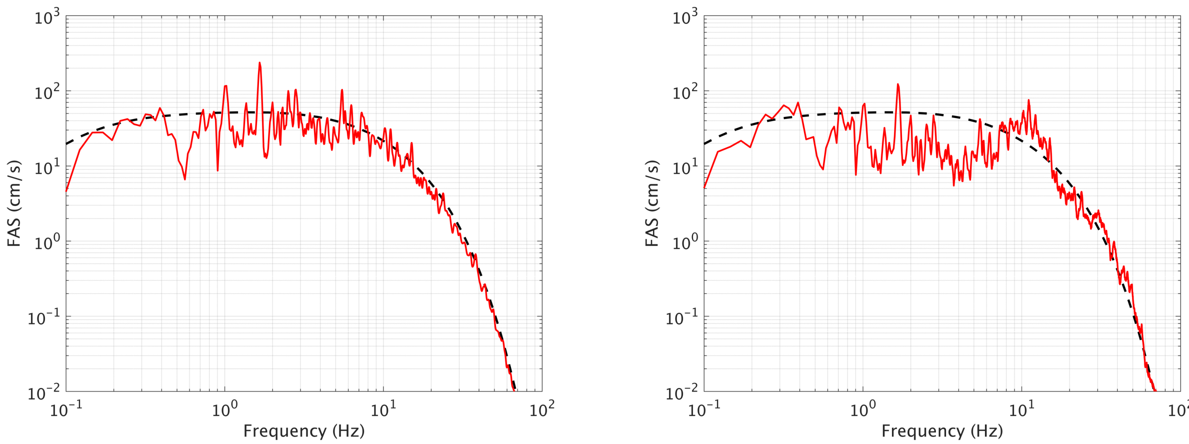

Figure 1. Left: An example smoothed FAS spectrum from the unmodified SMSIM procedure (low ρ_ϵ). Right: An example smoothed FAS spectrum from the SMSIM$_{corr}$ procedure. In both panels the point source scenario spectrum is given by the dashed line.

<u>EXSIM: Sub-sources</u>

As described previously, EXSIM is the finite-fault extension of SMSIM. Ideally, implementing the inter-frequency correlation should involve following the SMSIM$_{corr}$ procedure for all the sub-sources, and the resulting finite-fault time series should also have the appropriate ρ_ϵ. In Bayless and Abrahamson (2018), it is shown that the total ρ_ϵ does not have a strong dependence on magnitude, so the small events (the sub-sources) effectively have the same ρ_ϵ as the larger scenario. Thus, the summation of multiple sub-sources in the time domain should be equivalent to the summation of the sub-source Fourier amplitude spectra due to the linearity of the Fourier transform. This concept is tested in this section.

Each of the sub-source implementations are tested using 300 realizations of the same earthquake scenario for a single site. The scenario is the Northridge earthquake using the source as defined in Goulet et al. (2014) and the site is Sylmar Converter Station East (SCSE), which is located approximately 5km from the rupture plane. The sub-source implementation is tested using three approaches: Method 0, Method 1 and Method 2; these are described below.

Method 0

Method 0 is the unmodified version of EXSIM. Using Method 0, the smoothed FAS of the 300 simulated acceleration time series of the Northridge-SCSE scenario are calculated and ss expected, there is effectively zero inter-period ρ_ϵ using unmodified EXSIM.

Method 1

For Method 1, the code is modified by following the $SMSIM_{corr}$ procedure with a different sample of correlated ϵ within each sub-source of the finite rupture. This means that for a given rupture, the different sub-sources will have correlated Fourier spectra with local peaks and troughs over different frequency ranges due to the random sampling of correlated ϵ.

The smoothed FAS of the 300 simulated acceleration time series of the Northridge-SCSE scenario are calculated The ρ_ϵ calculated from these simulations are much weaker than the ρ_ϵ prescribed to each sub-source, meaning there is significant destructive interference of the correlation between sub-sources. This result led to the second method for sub-source implementation, Method 2.

Method 2

Method 2 is tested with the objective of avoiding the destructive interference of the correlation between sub-sources observed using Method 1. For Method 2, the $SMSIM_{corr}$ procedure is followed with the same sample of correlated ϵ within each sub-source of the finite rupture. In this case, for a given rupture realization, the different sub-sources will have correlated Fourier spectra with peaks and troughs in the same frequency ranges. Between alternate rupture realizations, the correlated ϵ samples vary. The simulation is repeated for the same Northridge-SCSE scenario 300 times and the smoothed FAS of the resulting acceleration time series are calculated. The ρ_ϵ cross-sections from Method 2 are stronger than from Method 1 but are still weaker than the ρ_ϵ prescribed to each sub-source. This means there is still destructive interference of the correlation between sub-sources. Since each sub-source is prescribed the same correlated ϵ values for a given realization, this was not the expected result.

The total ρ_ϵ does not have a strong dependence on magnitude (Bayless and Abrahamson 2018), so the small events (the sub-sources) effectively have the same ρ_ϵ as the larger scenario. Thus, the summation of multiple sub-sources in the time domain was expected to be equivalent to the summation of the sub-source Fourier amplitude spectra due to the linearity of the Fourier transform. Upon further inspection, this is not the case because the real and imaginary parts of the Fourier transform are individually linear, and so they possess the additive property of linearity. The additive property of the linearity of the Fourier transform applies to the real and imaginary parts, but does not necessarily hold for the Fourier amplitudes (magnitudes). This is the effect of differences in phase angles on the finite-fault Fourier amplitude spectrum, which is causing the reduction in correlation between frequencies This effect should be studied further in the future with methods for generating partially correlated phase angles so that the correlation can be implemented at the sub-source level.

EXSIM: Post-Processing

Rather than incorporating ρ_ϵ into each EXSIM sub-source, another approach is to implement the ρ_ϵ as a post-processing step. For a given earthquake scenario and site, the post-processing method steps are:

- Run the unmodified EXSIM algorithm n times.
- Calculate the geometric mean FAS of the n simulated time series.
- Using the mean spectrum as the target (in place of the point source spectrum), shape the sample of correlated ϵ to the target spectrum.

- Perform the inverse Fourier transform using the phase angles from the tapered time domain noise from any of the i^{th} simulation realizations.

This method is tested using the same earthquake and site (Northridge-SCSE) and with $n = 300$. The resulting FAS and the ρ_ϵ are shown in Figure 2. In this case, the ρ_ϵ match the prescribed model well, and with increasing n, the match should also improve.

The post-processing method allows for full calibration of the correlation of the simulations, which has use in practice in the short-term (e.g. structural risk applications), but it is not a desirable approach. The ρ_ϵ observed in the data is an important property of ground motions, and there is some physical reason for the existence of the correlation. The theoretical cause is currently not well understood (although the relative contributions of GMM components to the total correlation have been identified in Chapter 3), but the correlation must be introduced in some combination of the earthquake source, the travel path, and the local site response. Therefore, the preferable approach is to incorporate the correlation into seismological models (EXSIM, as well as others) through these foundational elements so that the models most closely represent the earthquake process. When the post-processing method is applied, the physical process built into the finite-fault simulation is ignored.

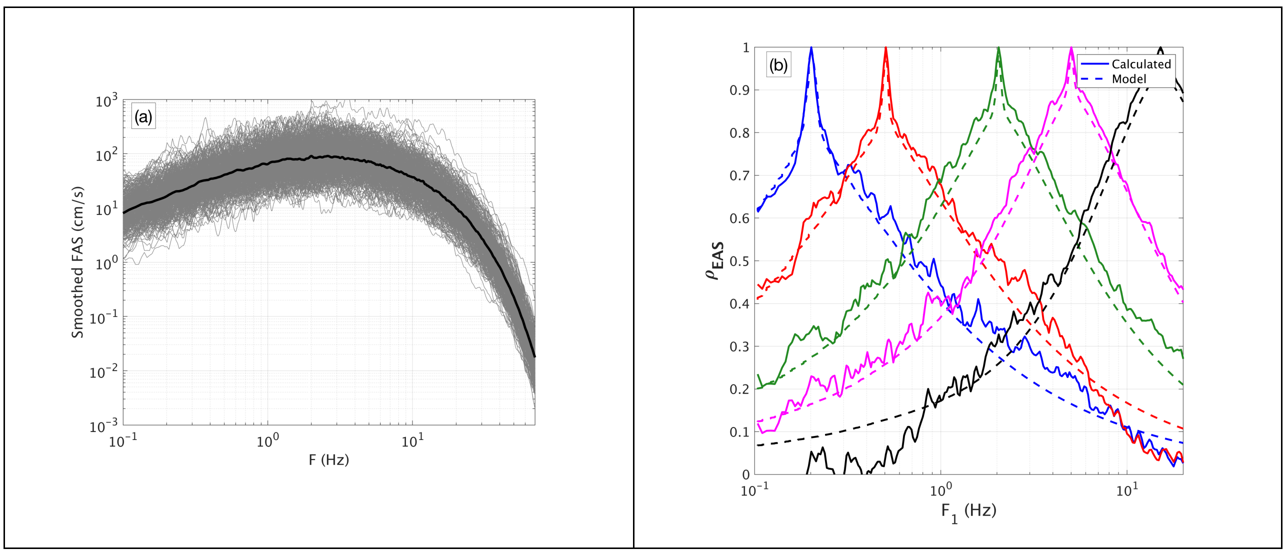

Figure 2. (a) The smoothed FAS of 300 realizations of the Northridge-SCSE scenario created using EXSIM with the post-processing ρ_ϵ procedure, with the geometric mean of these spectra (heavy line). (b) the ρ_ϵ at five conditioning frequencies calculated from these simulations (solid) along with the ρ_ϵ model from Chapter 3 used to generate correlated the ϵ values (dashed).

SCEC Broadband Platform Implementation

Due to the unresolved shortcomings of the sub-source method, the post-processing method is tested on the SCEC Broadband Platform v17.3. Simulations of seven of the Dreger et al. (2014) validation events are analyzed: Alum Rock, Chino Hills, Landers, Loma Prieta, North Palm Springs, Northridge, and Whittier Narrows. The simulations on the BBP use 50 realizations (each with a different random number generation seed) of each event and each earthquake simulates ground motions for approximately 45 sites.

The response spectra goodness-of-fit (GOF) is a summary of the logarithmic residuals of the simulated response spectra relative to the recorded ground motions. The GOF is calculated at each spectral period. The GOF plot is the primary evaluation tool used in the Goulet et al., (2015) validations of the SCEC BBP simulations. Figure 3 shows a GOF plot for simulations of the Loma Prieta earthquake. This plot is created using the combination of all 50 source realizations and all recording stations, where the solid black line is the mean GOF for all stations (with the average of all source realizations representing the PSA at a station). The pink band is the 90% confidence interval for the mean, and the purple band is the standard deviation centered around the mean (Goulet et al., 2014). Figure 4 shows the same summary plot for the Northridge earthquake simulations. In both figures, the top panel shows the GOF summary for unmodified EXSIM, and the bottom panel shows the GOF summary for EXSIM with the post-processing ρ_ϵ procedure.

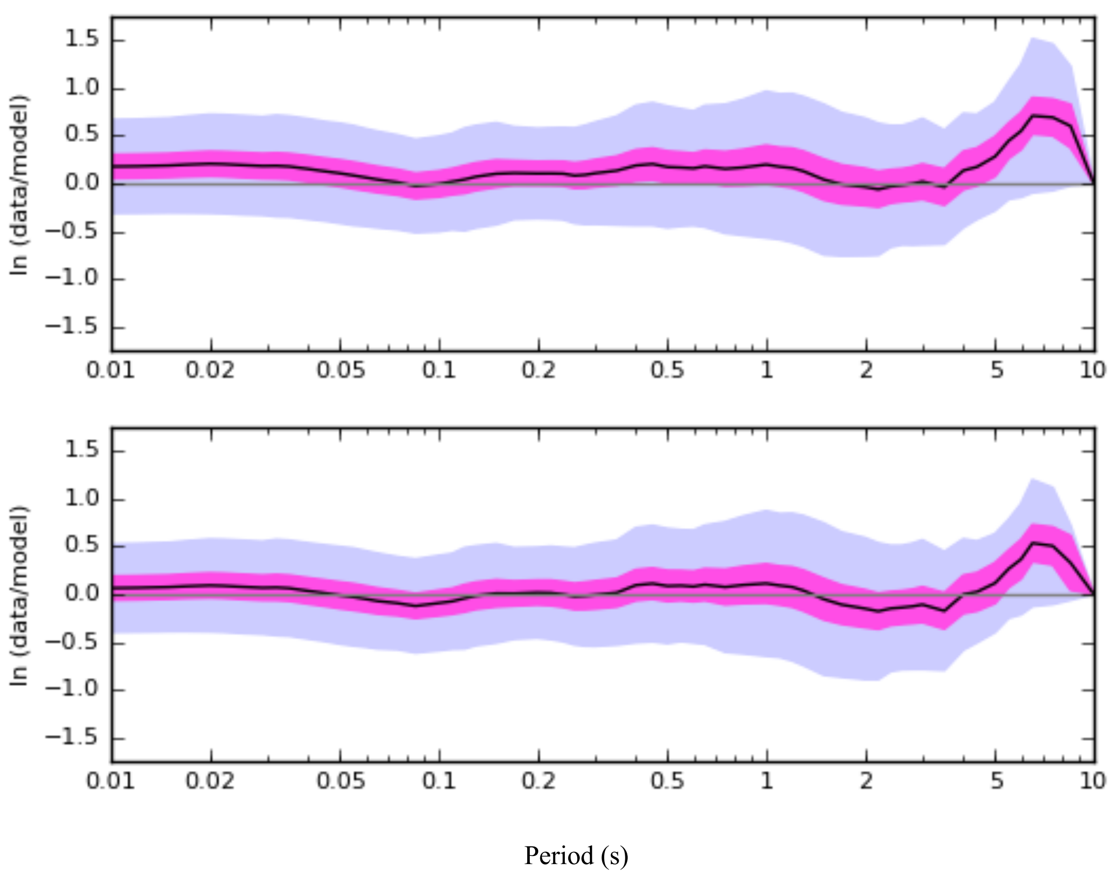

Figure 3. The goodness-of-fit (GOF) for response spectra, simulations of the Loma Prieta earthquake. Top: Unmodified EXSIM, Bottom: EXSIM with the post-processing ρ_ϵ procedure.

As shown in Figure 3 and Figure 4, the post-processing ρ_ϵ procedure does not introduce a significant bias in the mean residual. Minor shifts in the mean residuals are due to the random sampling of correlated ϵ over the 50 source realizations. The difference in the mean residual seen for the two earthquakes in Figure 3 and Figure 4 is similar because the same samples of ϵ were used in the 50 realizations of both earthquakes (a result of the same set of random number generator seeds). If different ϵ sets were used between earthquakes, then the two simulations would have different mean bias effects. Additionally, with more realizations, the mean bias will approach the unmodified EXSIM, so that validations for the median PSA do not need to be repeated.

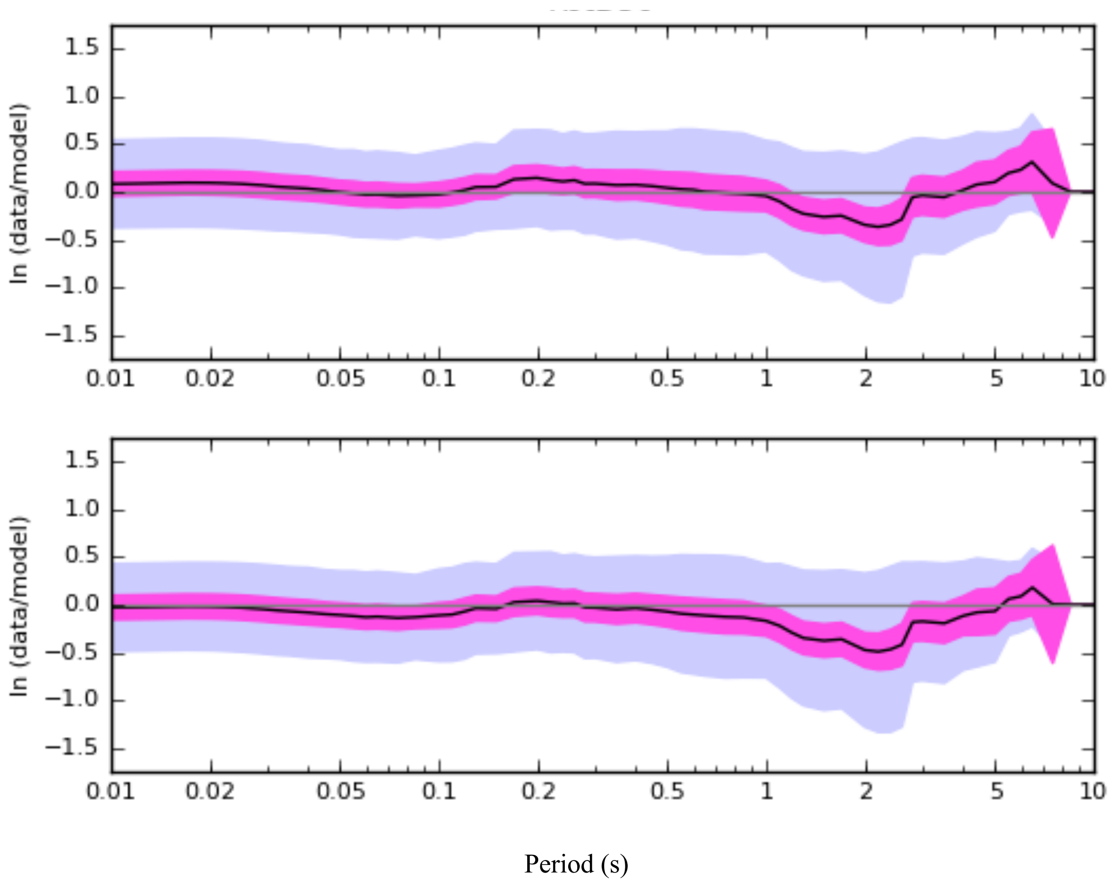

Figure 4. The goodness-of-fit (GOF) for response spectra, simulations of the Northridge earthquake. Top: Unmodified EXSIM, Bottom: EXSIM with the post-processing ρ_ϵ procedure.

Using the simulations, residuals are also calculated from the NGA-West2 GMMs for PSA. These models are smooth, so they produce residuals appropriate for back-calculating ρ_ϵ from the simulated time series. Using the seven validations events, with 50 source realizations of each event, and with approximately 45 stations per event, a database of 13,400 PSA residuals is developed. Figure 5b summarizes the within-event ρ_ϵ of these PSA residuals; panel (a) shows the within-event ρ_ϵ for the unmodified version of EXSIM. This figure indicates that the post-processing implementation of ρ_ϵ, applied to the FAS, performs quite well. The ρ_ϵ for PSA is significantly improved relative to the unmodified EXSIM version. These correlations approximately match the Baker and Jayaram (2008) empirical model, except for the very long periods where the correlation is higher than the model. At long periods, the correlations may be biased due to the extrapolation of the FAS ρ_ϵ model to frequencies as low as $f = 0.01$ Hz. This effect should be studied further.

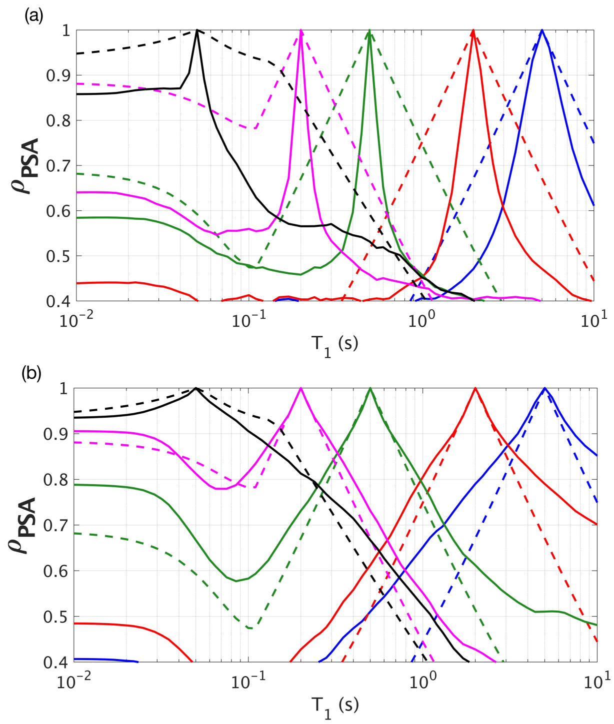

Figure 5. Comparison of within-event ρ_ϵ of PSA, (a) for unmodified EXSIM, and (b) for EXSIM with the post-processing ρ_ϵ procedure. Both plots show cross-sections of ρ_ϵ versus period at conditioning periods 0.05, 0.2, 0.5, 2, and 5 sec calculated from the simulations (solid lines), compared with the Baker and Jayaram (2008) model (dashed lines).

Conclusions and Recommendations for Future Work

The correlation incorporated into the Fourier amplitude spectra of each EXSIM sub-source results in lower than desired ρ_ϵ, meaning there is destructive interference of the correlation between sub-sources. EXSIM has been calibrated using no correlation between frequencies, and in order to incorporate the correlation on a sub-source level, these calibrations will need to change.

The ρ_ϵ can be calibrated as a post-processing modification, which is not preferable, but achieves the short-term goal of being be able to prescribe the total correlation of the full waveform seen in the data for other applications, e.g. structural risk. This method has been implemented and tested on the SCEC Broadband Platform. In the long term, preferable approach is to incorporate the correlation into seismological models (EXSIM, as well as others) through the foundational elements (source, path, site) so that the models most closely represent the earthquake process.

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