Validating site response predictions in deterministic physics-based ground motion simulations

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Abstract

Physics-based earthquake simulations are nowadays producing ground motion time-series for engineering design applications. Of particular significance to engineers, however, are simulations of near-field motions and large magnitude events for which observations are scarce. These events are important because they can cause large ground deformations and ground failure (liquefaction, lateral spreading), namely effects that frequently control the risk of infrastructure systems vital to societies, like lifelines and critical facilities. Thus, if ground motion models are to be used in physics-based risk assessment of these systems, they should have capabilities to predict nonlinear site effects and realistic ground deformations. In the first year of the project, we focused on the development of a new constitutive model that can predict realistic simple shear ground deformations over the entire strain range, from small strains to failure. Most geotechnical engineering site response models have been developed using laboratory experiments of cyclic material loading in the low to medium strain range ($\leq 1\%$), without imposed constraints on the shear strength of the material. As a result, when these experimental data are synthesized into stress-strain constitutive relations, the material response at higher strains (~5% to failure) –and therefore the predicted ground deformation– is not reliable. On the other hand, elastic perfectly plastic (EPP) models have been developed with explicit consideration of capturing the material strength, but perform very poorly at lower strains, and overestimate the material hysteretic damping during cyclic loading. Our new hybrid constitute model bridges the gap between EPP models in solid mechanics and cyclic soil models in geotechnical engineering, and is capable of predicting both stiffness degradation and failure. In the second year of the project, we will use this model to "correct" simulated ground motions at sites with known velocity profiles, and examine the extent to which predictions improve by explicitly accounting for site-specific response. Our long term plan is to extend this model to 3D.

1 Background

1.1 The hyperbolic model

Numerous stress-strain models that capture the cyclic behavior of soils have been proposed in the last 50 years. The hyperbolic model, originally proposed by *Kondner and Zelasko* (1963), has been extensively used because it has a simple form with parameters that have clear physical meaning. Its formulation is

$$\tau(\gamma) = \frac{\gamma}{\frac{1}{A} + \frac{\gamma}{B}} \tag{1}$$

where A and B are the two parameters of the model. The hyperbolic model has two properties:

1. $\partial\tau/\partial\gamma = A$ when $\gamma \to 0$, which means that A is the initial slope of the τ-γ curve, also known as the "initial shear modulus" or "maximum shear modulus". Thus $A = G_{\max}$.

2. $\lim_{\gamma\to+\infty} \tau = B$, which means that B is the upper asymptote of the τ-γ curve. We here denote B as τ_{f}, with the subscript "f" indicating "failure".

Hence Equation (1) can be written as

$$\tau(\gamma) = \frac{\gamma}{\frac{1}{G_{\max}} + \frac{\gamma}{\tau_{\mathrm{f}}}} \tag{2}$$

An alternative formulation can be derived by substituting $G_{\max} = \rho \cdot V_S^2$, where ρ is the mass density and V_S is the shear wave velocity; and defining the secant modulus as $G = \tau/\gamma$:

$$\frac{G}{G_{\max}} = \frac{1}{1 + \left(\frac{G_{\max}}{\tau_{\mathrm{f}}}\right)\gamma}$$

The disadvantage of the hyperbolic model is its lack of flexibility. The two parameters have explicit physical meanings, and since one set of $(G_{\max}, \tau_{\mathrm{f}})$ defines only one curve, there is no room for incorporating other factors that affect the stress-strain soil behavior such as plasticity index or overconsolidation ratio. As a result, the hyperbolic model often does not agree with experimental data.

1.2 The modified hyperbolic model

Motivated by the need to develop a more flexible model to simulate cyclic soil behavior, *Matasovic and Vucetic* (1993) proposed the modified hyperbolic model, also known as MKZ. As shown in the following equation, MKZ was based on the original hyperbolic model, with two additional parameters for flexibility:

$$\tau(\gamma) = \frac{G_{\max}\gamma}{1 + \beta\left(\frac{G_{\max}}{\tau_{\mathrm{f}}}\gamma\right)^s} \tag{3}$$

where β and s are the two additional parameters. When $\beta = 1$ and $s = 1$, MKZ reduces to the original hyperbolic model. It is worth noting here that in the popular study by *Darendeli* (2001), the stress-strain model that was used for curve-fitting large amount of resonance column test results was an MKZ model, only with β fixed at 1.0.

Using MKZ model, the slope at the origin correctly predicts the initial shear modulus,

$$\left.\frac{\partial \tau}{\partial \gamma}\right|_{\gamma=0} = G_{\max}$$

However, $\lim_{\gamma \to +\infty} = +\infty$, which means that the stress predicted by this model has no upper bound as the strain increases. Since MKZ is unbounded for large strains, the parameter τ_{f} in Equation (3) has no longer a physical meaning; therefore, with an auxiliary reference strain defined as $\gamma_{\mathrm{ref}} = \tau_{\mathrm{f}}/G_{\max}$, Equation (3) becomes

$$\tau(\gamma) = \frac{G_{\max}\gamma}{1 + \beta\left(\gamma/\gamma_{\mathrm{ref}}\right)^s} \tag{4}$$

Both *Darendeli* (2001) and *Matasovic and Vucetic* (1993) have showed that although the shear stress in MKZ is not bounded, its predictions in the lower strain range ($<$ 3 to 5%) are satisfactory compared to laboratory experiments. Note that 3 to 5% is a typical upper strain limit of the resonance column test, based on which the MKZ, the hyperbolic model and several other soil models for cyclic behavior have been based.

However, for extreme ground motion cases, the shear strain –at least at certain points near the ground surface– can potentially go beyond 5%, and the shear stress in those portions is capped by the shear strength. As stress approaches the soil strength, soils exibit plastic deformation, which means a small increment of stress can induce a large increment of strain. As a result, the neighboring soil layers will experience large deformation as well, which will ultimately lead to larger motions on the ground surface. In other words, if a model without a stress bound is used in the simulation, the surface ground motion is underestimated.

2 A new hybrid hyperbolic model (HH model)

2.0.1 Formulation

Motivated by the drawbacks of leading models in site response analysis described above, we have developed a formulation heretofore coined hybrid hyperbolic model (abbreviated as the HH). The form of this new model is as following

$$\tau(\gamma) = w(\gamma) \cdot \tau_{\mathrm{MKZ}}(\gamma) + [1 - w(\gamma)] \cdot \tau_\mu(\gamma) \tag{5}$$

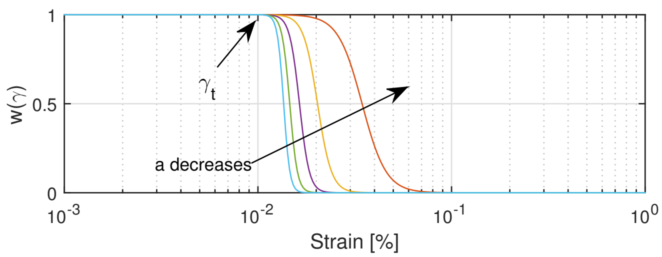


Figure 1: The curve of the transition function $w(\gamma)$

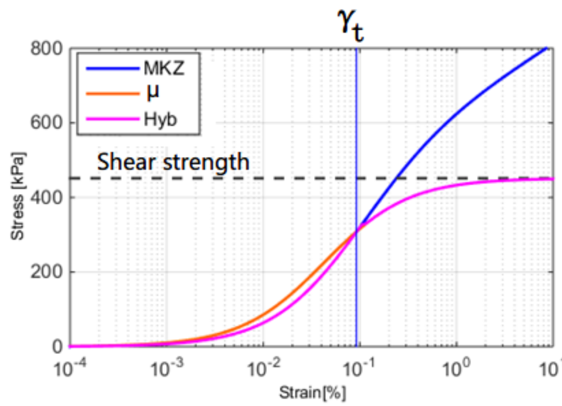


Figure 2: Hybrid stress as determined by τ_{MKZ} and τ_μ

where $\tau_{\mathrm{MKZ}}(\gamma)$ is the stress predicted by the MKZ model, i.e., Equation (4), and $\tau_\mu(\gamma)$ is the stress predicted by an auxiliary stress-strain relation:

$$\tau_\mu(\gamma) = \frac{\gamma \cdot \mu}{\frac{1}{G_{\max}} + \frac{\gamma \cdot \mu}{\tau_{\mathrm{f}}}}$$

where μ is an additional parameter. Note that $\lim_{\gamma \to +\infty} \tau_\mu(\gamma) = \tau_{\mathrm{f}}$. In Equation (5), $w(\gamma)$ is a transitional function which has the following form

$$w(\gamma) = 1 - 1/\left[1 + 10^{-a\left(\log_{10}(\gamma)/\gamma_{\mathrm{t}} - 4.039a^{-1.036}\right)}\right]$$

where a and γ_t are two curve-fitting parameters. The curve of $w(\gamma)$ is shown in Figure 1. We can see from the figure that γ_{t} defines the point at which the value of $w(\gamma)$ starts to deviate from 1.0, and a controls how fast the transition from 1.0 to 0 is. Applying $w(\gamma)$ to τ_{MKZ} and $[1 - w(\gamma)]$ to τ_μ means that the total shear stress exactly follows τ_{MKZ} when $\gamma \leqslant \gamma_{\mathrm{t}}$, and then as $\gamma > \gamma_{\mathrm{t}}$, τ transitions into the value of τ_{f}. The physical meaning of γ_{t} is that beyond that level of strain, $\gamma > \gamma_{\mathrm{t}}$, the MKZ model doesn't capture the target soil behavior; thus, beyond that strain level, we transition towards the asymptote (the shear strength) using $\tau_\mu(\gamma)$. Figure 2 shows an example of hybrid stress as determined by τ_{MKZ} and τ_μ.

2.0.2 Parameters of the HH model

There are 8 parameters in total in the HH model; they are summarized in Table 1. Three, γ_{t},$G_{\max}$, and τ_{f}, carry explicit physical meanings, and need to be determined from laboratory experiments, field data or empirical correlations.

For example, $G_{\max}$ can be determined from in-situ testing of the soil layers $G_{\max} = \rho V_S^2$ where ρ is the mass density of the soil, and V_S is the shear wave velocity of the corresponding soil. V_S can be obtained from tests like CPT (cone penetration test), CHT(cross hole test), suspension logging test, MASW (multi-channel analysis of surface waves); whereas the mass density, ρ, could be empirically estimated from V_S measurements as suggested, for example, by *Burns and Mayne*, 1996.

Parameter	Range	Typical value	Physical meaning
a	> 0	1~50	Steepness of $w(\gamma)$
γ_t	> 0	10^{-4}%~3%	Strain at which the transition from τ_MKZ to τ_μ starts to happen
G_max	> 0	(Depends on soil)	Shear modulus of soil
γ_ref	> 0	(Depends on soil)	Parameter that controls the shape of MKZ
β	> 0	Around 1.0	Parameter that controls the shape of MKZ
s	> 0	Around 1.0	Parameter that controls the shape of MKZ
τ_f	> 0	(Depends on soil)	Shear strength of soil under 1D cyclic loading condition
μ	> 0	Around 1.0	Parameter that controls the shape of τ_μ

Table 1: Summary of the 8 parameters in the HH model

The cyclic shear strength, τ_f, can be obtained from in-situ tests like vane shear test, or laboratory tests, like direct shear test or triaxial tests. Note that since the dynamic motion of the soil modeled by HH is fast and transient, the undrained shear strength, s_u, can be directly used as a measure of the material strength (τ_f here).

As a rule of thumb, the "transition strain", γ_t, can be chosen around 3%, which is the largest strain level measured in resonance column tests. However, in some circumstances, γ_t can be adjusted to smaller values, in order to achieve a smooth stress-strain curve.

The other five parameters of the this model can be determined by curve fitting, through approached such as genetic algorithms.

3 Comparison with Asimaki and coworkers previous model

As a first insight of the performance of the HH model, a test case is performed using both MKZ model and HH model. The subsurface soil profile is a simple 10-meter thick homogeneous layer with V_S = 500 m/s, ρ = 1800 kg/m^3, and damping ratio ξ = 3%. And input waves are prescribed at the base of the layer (i.e., rigid bedrock analogy). The input motion is a sweeping sinusoidal wave with increasing amplitude, as shown in Figure 3. The peak acceleration of this wave is 8.33 m/s^2, equivalent to 0.95g.

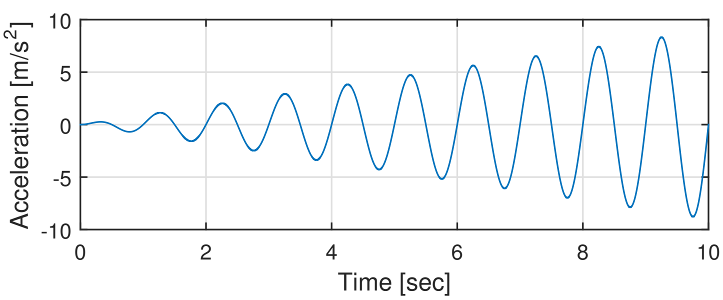


Figure 3: Input signal, a sweeping sinusoidal wave

Figure 4 shows the output motion (on the ground surface) using both MKZ model and HH model, with the input motion overlain. From the figure we can see that when the input motion is small (the first 2 seconds), the output by MKZ and by HH are essentially the same, because the strain induced by such weak input is below γ_t (the transition strain), so $\tau_\mathrm{HH} = \tau_\mathrm{MKZ}$. And as the input motion increases (about 4 seconds), the acceleration by HH starts to "yield" and decreases to smaller than the input, indicating the "sliding" behavior within the soil layer (which is expected). Looking at the result by MKZ, the acceleration is still larger than the input at 4 seconds, and the rate of amplitude increase starts to tone down only at about 8 seconds. This indicates that the soil elements modeled by MKZ are more linear (and thus less nonlinear) than by HH, because MKZ predicts "stiffer" shear modulus even at very high strain range and even when failure occurs (when the shear modulus should be 0). This can be more clearly perceived in Figure 5, where the strain-stress loops of the lowest sub-layer (at 9.5 meters) are shown for both MKZ and HH. We can see

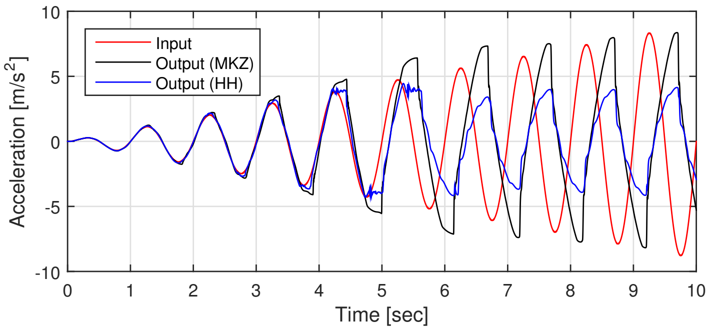
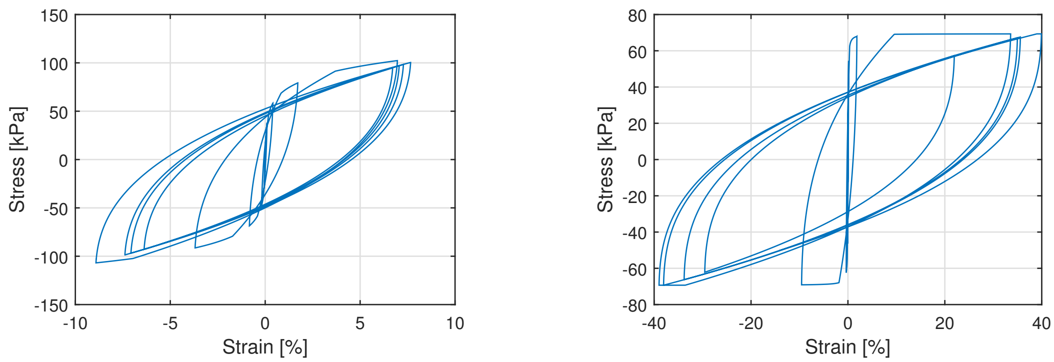


Figure 4: Comparison of output motions by MKZ and HH



(a) MKZ

(b) HH

Figure 5: Stress-strain loops predicted by MKZ and HH

that the stress predicted by HH is capped at the shear strength, 70 kPa, whereas the stress predicted by MKZ exceeds 70 kPa. As a result, the loops predicted by MKZ are much "thinner", than those by HH, thus the maximum strain level in the layer by MKZ is smaller than that by HH. More energy is dissipated when failure occurs in the soil elements, and thus the elements "slide", which results in the surface motion predicted by HH being smaller than MKZ.

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