SCEC 2013 Progress Report

SCEC Project 13099: Nucleation of dynamic rupture on faults with heterogeneous rate- and state-dependent friction (aging law)

PI: Robert C. Viesca, Tufts University;
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Summary: We report on results of study into the development of (dynamic rupture-nucleating) slip instabilities on surfaces whose frictional strength is determined by a slip rate- and state-dependent friction law. Specifically, we examine conditions in which frictional parameters vary spatially, as in current models of the seismic cycle. We remarkably find (1) that spatial variations of friction parameters create preferential sites for slip-instability development (and ultimately, earthquake nucleation in such models), and (2) that the manner of that development can be determined a priori if the distribution of the frictional parameters are known. We also highlight (3) results contrasting the influence of heterogeneity in determining the expected nucleation patch size, relative to that expected under homogeneous conditions.

Background: For one-dimensional variation along a fault (e.g., for in- or anti-plane slip between two elastic half-spaces), the expressions for the fault shear stress τ and strength τ_s are

$$\tau(x,t)=\tau_o(x,t)+\mathcal{L}[\delta(x,t)], \quad \tau_s(x,t)=\sigma f(x,t)$$

where τ comprises τ_o, the fault stress in the absence of fault slip (e.g., owed to an external forcing), plus the stress due to a distribution of quasi-static slip δ (determined by the functional $\mathcal{L}$, whose precise form depends mode of slip and elastic configuration); and the strength here is the product of an (assumed) uniform normal stress σ and an evolving friction coefficient f, determined by a slip rate- and state-dependent formulation of friction [e.g., Dieterich, 1979; Ruina, 1983]

$$f(x,t)=f_i+a\ln\left[\frac{V(x,t)}{V_i}\right]+b\ln\left[\frac{V_i\theta(x,t)}{D_c}\right], \quad \frac{\partial\theta}{\partial t}=1-\frac{V(x,t)\theta(x,t)}{D_c}$$

where θ is a state variable whose evolution follows the above aging law and is a characteristic slip distance for state evolution.

The above system of equations gives rise to finite-time instabilities [e.g., Rubin and Ampuero, 2005] in which $V\sim D_c/(t_{nuc}-t)$, where t_{nuc} is the time of instability. We look for solutions where slip rate diverges in the form

$$V(x,t)=\mathcal{W}(x)\frac{D_c}{t_{nuc}-t}$$

where $\mathcal{W}$ is a compact distribution with a domain half-length L. Rubin and Ampuero [2005] did so for the case of in- and anti-plane slip on a fault with uniform material properties (a problem whose sole parameter is the ratio a/b) and found such localized solutions valid for the range $0<a/b\le 0.3781...$. In previous work [e.g., Viesca, 2014] we generalized their results

partly by considering that the state variable may be replaced in the above system of equations by an alternative variable

$$\Phi(x,t) = 1 - \frac{D_c}{V(x,t)\theta(x,t)}$$

which takes on a value of 0 when the system is at steady state and 1 when far from it. The localized acceleration solutions of Rubin and Ampuero [2005] correspond to solutions in which $\Phi = 1$ on the domain of instability. Relaxing that assumption, but still requiring that Φ take on a fixed distribution $\mathcal{P}(x)$, we found that such solutions could be extended to cover the rate-weakening range of $0 < a/b < 1$, retrieving the end-member nucleation length scalings proposed by Rubin and Ampuero [2005].

Project Summary: Here we proposed to examine slip instability development on faults with heterogeneous fault frictional properties, with a focus on the parameters *a*, *b*. In the homogeneous case discussed above, there was a translational symmetry along the fault. This implies that nucleation could occur on one part of the fault as well as another and the location of nucleation was simply determined by initial conditions and the external forcing. Heterogeneity in the frictional parameters breaks this symmetry and creates distinct, preferential sites for the development of the slip instability and, ultimately, the nucleation of a dynamic rupture. These preferential nucleation sites are manifested, for example, in current models of the seismic cycle using depth-dependent frictional properties [e.g., Lapusta and Rice, 2003], in which nucleation consistently occurs within a region in which *a* and *b* are uniform and not within the linear transition zone to rate-strengthening behavior at depth where the system's loading is imposed.

More precisely, we find the location and distributions of diverging slip rate (via $\mathcal{W}$) and state (via $\mathcal{P}$) that are admitted given a prescribed heterogeneity in *a* and *b*. We considered a simple, low-parameter distribution of $a(x)/b$ (Figure 1), which can represent streak- or circular patch-like patch regions of rate-weakening material embedded within a rate-strengthening fault. The preferred nucleation location is about the minimum in $a(x)/b$. By varying the minimum value of $a(x)/b$ as well as the linear rate of variation away from it, the slip instability development changes in character (namely, the nucleation patch size L and the distribution shape $\mathcal{W}$ of the diverging slip rate).

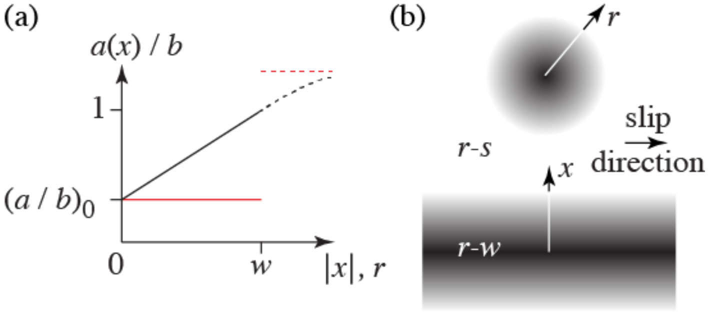

Figure 1. (a) Distribution of the relative rate-weakening parameter *a*/*b* for two model rate-weakening regions: (b) circular and streak-like patches.

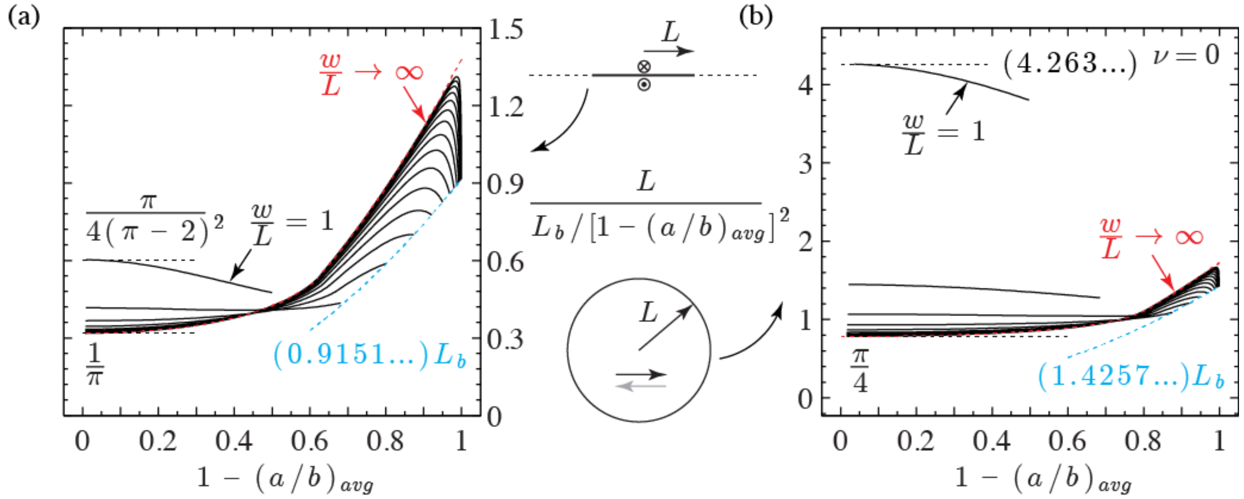

Figure 2. Solution of nucleation patch half-length L under the two-parameter heterogeneous distribution of $a(x)$, and b of Figure 2 under the two elastic configurations of (a) single-mode (in- or anti-plane rupture) and (b) mixed mode rupture. The latter is specific for the case $\nu = 0$ and can be used to approximate cases where $\nu \neq 0$. Axis scalings use a value of a/b averaged over the nucleation patch, $(a/b)_{avg}$. This allows a simple comparison as to how wrong a nucleation patch size estimate could be if it was made assuming a homogeneous value of a/b in the rate-weakening region. This can be done by comparing the vertical distances between a homogeneous solution (a point on the red-dashed line) and a heterogeneous solution vertically above or below it (e.g., a point along a black line). For cases here, the worst estimation is less than a factor of 4. The frictional lengthscale $L_b = \bar{\mu} D_c / \sigma b$ where $\bar{\mu}$ is a shear modulus dependent on the mode of rupture.

We compare these solutions for nucleation patch length L under heterogeneous conditions to those of homogeneous fault properties (red-dashed lines in Figure 2 (a) and (b)). In addition to the solutions under a single mode of slip (e.g., in-plane mode-II, or anti-plane mode-III rupture), we have also found those under mixed-mode slip. The mixed-mode solutions are for the case $\nu = 0$ (ν being the Poisson ratio), in which the problem is axisymmetric. These may be extended to cover more realistic cases where $\nu \neq 0$ by making use of results for energy release rates on curved crack fronts [e.g., Gao, 1988]: the slipping region will be approximately elliptic (elongated in the direction of slip) with a minor axis of length close to L in Figure 2b (red-dashed) and a major axis that is large by a factor of $(1-\nu)^{-1}$. These solutions accurately describe the length and manner of instability development within rate-weakening regions observed in simulations [e.g., Rubin and Ampuero, 2005; Chen and Lapusta, 2009].

For the heterogeneous distributions of Figure 2, we find the two-parameter—$(a/b)_0$, w/L—family of solutions for the single and mixed-mode configurations (Figure 3, black lines). We examine mixed-mode nucleation within the circular rate-weakening patches and single-mode nucleation within the streak-like patches. These solutions allow us to gauge the consequences of homogeneizing heterogeneous frictional properties in seismic cycle models: in addition to missing out on preferred nucleation sites, we could also under/overestimate the critical nucleation patch size. However, here we can anticipate that any mis-estimation would not be severe (by comparing the solid black lines to the red-dashed line): i.e., at most we could expect to be off by less than factor of 4 (if we estimated the patch size assuming the homogeneous results and a single value of a/b, $(a/b)_{avg}$, the average of the heterogeneous a/b over the true, heterogeneous patch size).

In addition, we confirm that the expected preferential nucleation sites are indeed manifested in solutions for the evolution of V and θ under conditions that provoke slip instability. As an example, we take a linear distribution of rate-weakening $a(x)/b$ within an otherwise rate-neutral region (Figure 3, left panel). In Figure 3 (right panel) we examine the evolution of slip rate from an initially homogeneous steady-state distribution in response to a locally peaked external forcing, centered at an intermediate depth (highlighted by open circle). Progressive darkening of slip rate profiles corresponds to later times in the instability development. We find that the expected distribution given by $\mathcal{W}$ (red-dashed line) is asymptotically approached.

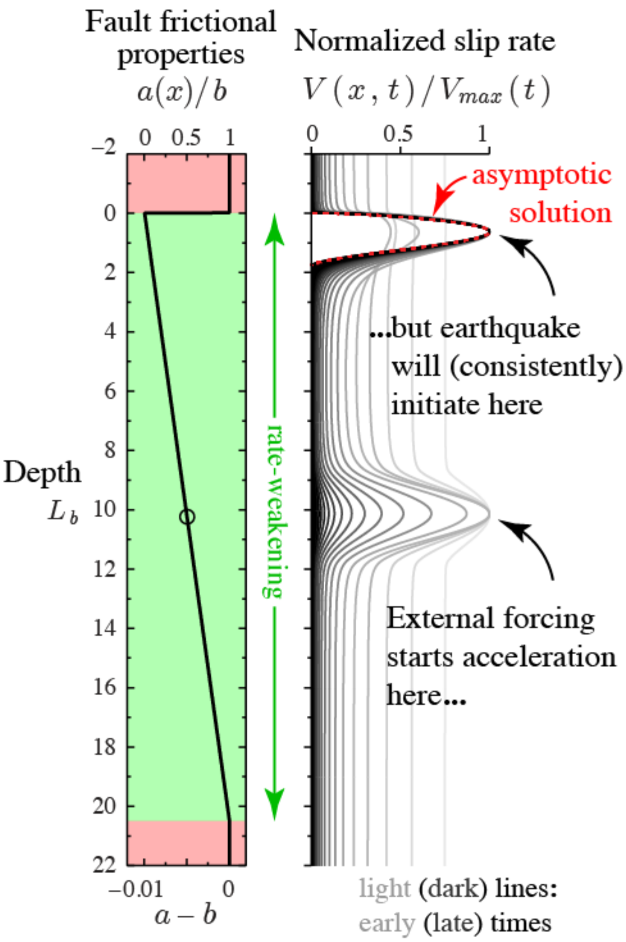

Figure 3. Demonstration that nucleation sites may be determined solely by frictional parameter distributions. (left) Distribution of frictional properties, here given by depth-dependent $a(x)$ and uniform b. (right) Snapshots of normalized slip rate profiles with depth (greyscale; time increasing with progressive darkening) showing that slip instability is attracted to occur with a predictable location and manner (red-dashed) in spite of an initial forcing centered at a different rate-weakening location (open circle).

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