2013 Annual SCEC report

Quantifying variability of seismic source spectra derived from cohesive-zone models of earthquake rupture (PI: Kaneko)

Summary:

Earthquake stress drops are often estimated from far-field body-wave spectra using measurements of seismic moment, corner frequency, and a specific theoretical model of rupture behavior. The most widely-used model is from *Madariaga* [1976], who performed finite-difference calculations for a singular crack radially expanding at a constant speed and showed that $\bar{f}_c = k\beta/a$, where $\bar{f}_c$ is spherically averaged corner frequency, β is the shear-wave speed, a is the radius of the circular source, and $k = 0.32$ and 0.21 for P and S waves, respectively, assuming the rupture speed $V_r = 0.9\beta$. Since stress in the Madariaga model is singular at the rupture front, the finite mesh size and smoothing procedures may have affected the resulting corner frequencies. In this work, we have investigated the behaviour of source spectra derived from dynamic models of a radially expanding rupture on a circular fault with a cohesive zone that prevents a stress singularity at the rupture front. We have found that in the small-scale yielding limit where the cohesive-zone size becomes much smaller than the source dimension, P- and S-wave corner frequencies of far-field body-wave spectra are systematically larger than those predicted by *Madariaga* [1976]. In particular, the model with rupture speed $V_r = 0.9\beta$ shows that $k = 0.38$ for P waves and $k = 0.26$ for S waves, which are 19 and 24 percent larger, respectively, than those of *Madariaga* [1976]. Thus for these ruptures, the application of the Madariaga model overestimates stress drops by a factor of 1.7. In addition, the large dependence of corner frequency on take-off angle relative to the source suggests that measurements from a small number of seismic stations are unlikely to produce unbiased estimates of spherically averaged corner frequency.

Publications and abstracts resulted from this project:

Kaneko, Y. and P. M. Shearer, Seismic source spectra and estimated stress drop derived from cohesive-zone models of circular subshear rupture, *Geophys. J. Int.*, doi:10.1093/gji/ggu030, 2014.

Kaneko, Y. and P. M. Shearer, Variability of seismic source spectra derived from cohesive-zone models of circular rupture. abstract S11A-2282 IN: AGU Fall Meeting 2013, 9-13 December, San Francisco California : abstracts. American Geophysical Union.

Kaneko, Y. and P. M. Shearer, Variability of seismic source spectra derived from cohesive-zone models of circular rupture. p. 45 IN: Ben-Zion, Y.; Rovelli, A. (Workshop directors) 40th Workshop of the International School of Geophysics : properties and processes of crustal fault zones, Erice, Sicily (IT), 18-24 May 2013. INGV.

Technical Report:

Accurate determination of stress drops from body-wave observations is difficult not only because of uncertainties in observations and corrections for path effects, but also due to the fact that the methods for estimating stress drop are derived from a number of assumptions about the dynamics of the source. The most widely-used model is from *Madariaga* [1976], who performed finite-difference calculations for a singular crack radially expanding at a constant speed. However, since stress in the Madariaga model is singular at the rupture front, the finite mesh size and smoothing procedures may have affected the resulting corner frequencies. To address this issue, we have constructed a dynamic model of expanding rupture on a circular fault, similar to that of *Madariaga* [1976], but with a cohesive zone that prevents a stress singularity at the rupture front. Our goal is to study the simplest class of rupture models that are physically realizable, i.e., have no stress singularities, and that are generated by simulations with a proper numerical resolution. We have simulated scenarios of dynamic rupture and analyzed the behavior of farfield body-wave spectra, the relation between the corner frequency and the source radius, and the dependence of corner frequencies on rupture speeds.

The goals of the project described in the 2013 proposal were:

- To determine the relationship between source radius and corner frequencies in cohesive-zone models of earthquake rupture
- To quantify uncertainties associated with data sampling over the focal sphere
- To understand the relationship between fracture energy and corner frequencies

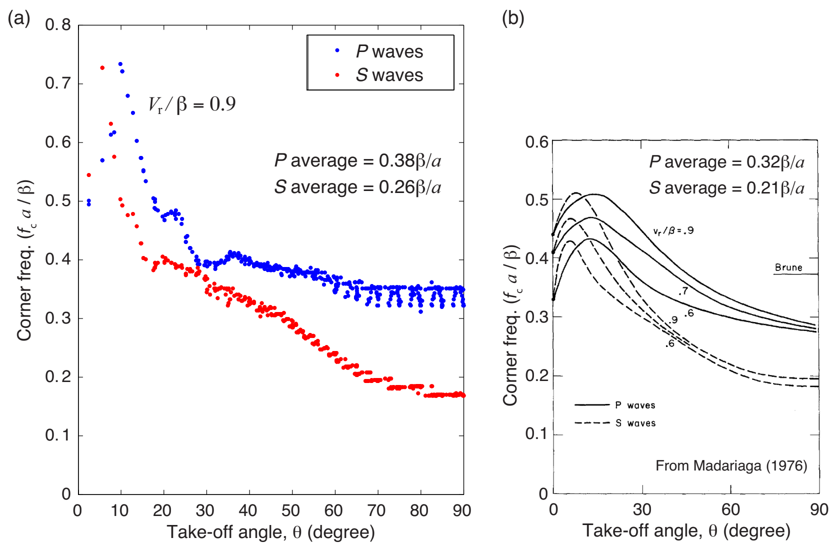

Figure 1: Variations of P and S corner frequencies over the focal sphere. (a) Corner frequencies of displacement spectra at every 5 degrees on the focal sphere are plotted as a function of the take-off angle θ for a representative source model. The average values over the focal sphere for the rupture speed $V_{\mathrm{r}}/\beta = 0.9$ are indicated. (b) The figure adapted from Figure 10 of *Madariaga* [1976].

We have accomplished all the goals discussed above and published the results in Geophysical Journal International [*Kaneko and Shearer*, 2014]. In the following, we summarize our main findings.

The corner frequencies of displacement spectra at take-off angles sampled every 5 degrees over the focal sphere are shown in Figure 1a. Generally, the corner frequency of the P wave is larger than that of the S wave because of the shorter duration of the P displacement pulses. For $\theta \gtrsim 30^{\mathrm{o}}$, the variation is gradual and both the P and S corner frequencies decrease with θ. Although the pattern of the corner-frequency variation is similar to that in the Madariaga model (Figure 1b), the amplitude of the variation is much larger in the model considered in this study (Figure 1a). Since the corner frequency is a function of the take-off angle, we compute the average $\bar{f}_{\mathrm{c}}$ over the focal sphere. The spherical averages of the P and S corner frequencies for the scenario shown in Figure 1a are $0.38\beta/a$ and $0.26\beta/a$, respectively, which are larger than those in *Madariaga* [1976]. The ratio of the P to S corner frequencies is 1.5, consistent with that of the Madariaga model. It is generally agreed that P corner frequency is larger than the S corner frequency for many observed earthquakes [e.g., *Molnar et al.*, 1973; *Abercrombie*, 1995].

The source model presented above assumes the rate of frictional weakening $A'_{\mathrm{w}} = 42$. Increasing A'_{w} generally results in smaller fracture energy G', but G' eventually becomes independent of A'_{w} for models with a cohesive-zone size much smaller than the source dimension. We call this case the 'small-scale yielding limit'. In this limit, the spherical average of corner frequencies does not depend on the rate of frictional weakening (Figure 2a), and the source-time histories of the models with $A'_{\mathrm{w}} = 84$ and $A'_{\mathrm{w}} = 168$ become identical.

The apparent increase in the corner frequencies with the fracture energy shown in Figure 2a comes from the way the rupture front advances in the model, where the rupture speed V_{r} is defined at the intersection of the weakening rate A_{w} and $\tau = \tau_{\mathrm{d}}$ and not at the actual rupture front. Because of this formulation, the actual rupture front is ahead of the location $V_{\mathrm{r}}t$ and encounters the edge of the circular fault sooner than the time a/V_{r}. As a result, the eventual source duration becomes shorter in the model with a smaller frictional-weakening rate A'_{w}, or larger fracture energy G' (Figure 2a). This is why the models with larger fracture energy lead to larger corner frequencies (Figure 2a).

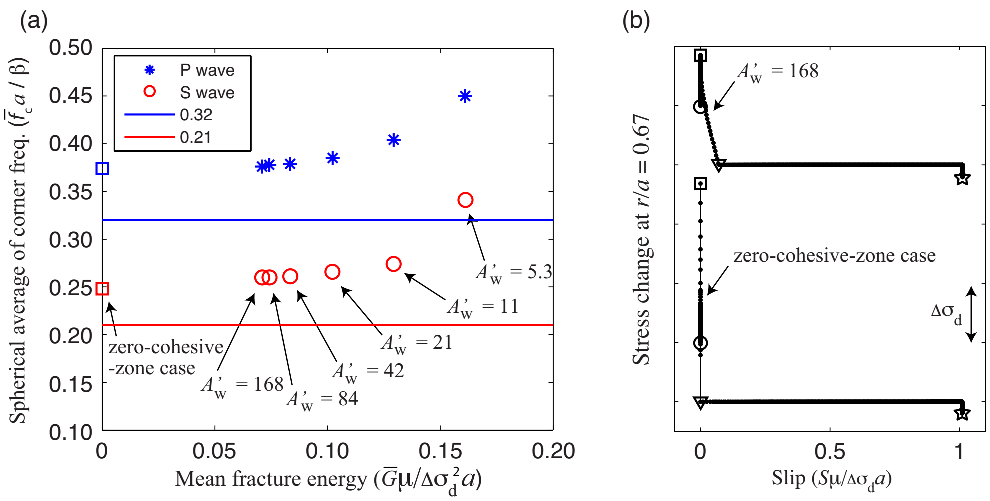

Figure 2: (a) Spherical average of corner frequencies for models with different weakening rates A'_{w}. As A'_{w} becomes larger, the mean of fracture energy $\overline{G}$ over the circular source becomes smaller but eventually becomes independent of A'_{w}. In this small-scale yielding limit, the averages of corner frequencies differ from those obtained by *Madariaga* [1976] by 19 percent for P waves and 24 percent for S waves. (b) Simulated slip-weakening curves for the cases with $A'_{\mathrm{w}} = 168$ and the zero-cohesive-zone case.

Since corner frequencies for the models with small A'_{w} depend on assumptions for the source, we focus our analysis on the small-scale yielding limit. In this limit, the spherical averages of P and S corner frequencies $\bar{f}_{\mathrm{c}}^{\mathrm{p}}$ and $\bar{f}_{\mathrm{c}}^{\mathrm{s}}$ are found as

$$\bar{f}_{\mathrm{c}}^{\mathrm{p}} = k^{\mathrm{p}}\frac{\beta}{a} = 0.38\frac{\beta}{a} \quad (1)$$

$$\bar{f}_{\mathrm{c}}^{\mathrm{s}} = k^{\mathrm{s}}\frac{\beta}{a} = 0.26\frac{\beta}{a} \quad (2)$$

for $V_{\mathrm{r}}/\beta = 0.9$ (Figure 2a). Surprisingly, the corner frequencies are larger than those in the Madariaga model by 19 percent for P waves and 24 percent for S waves, respectively. The values of k are smaller than those of *Sato and Hirasawa* [1973] with $k^{\mathrm{p}} = 0.42$ and $k^{\mathrm{s}} = 0.29$. This is expected because the source duration of the cohesive-zone model is longer, due to the spontaneous healing of slip, than that of *Sato and Hirasawa* [1973].

The results presented in this study have important implications for estimated source parameters of earthquakes. While relative stress-drop variations found in previous observational work are not affected by the difference in the assumed value of k, the absolute level of a stress drop based on the model with the small-scale yielding limit is systematically smaller by a factor of 1.7 than that based on the Madariaga model. For example, with the revised values of k, stress drop estimates reported in *Allmann and Shearer* [2009] range from 0.18 to 30 MPa with the median value of 2.4 MPa. This implies that less accumulated tectonic stress is released by those earthquakes than the previous interpretation. In addition, unusually high-stress-drop earthquakes ($\gtrsim$100 MPa) reported in observational studies may not be actually so large and thus more comparable to the tectonic overburden pressure. Stress drop is also used to infer other source parameters, such as seismological fracture energy. *Abercrombie and Rice* [2005] found that the proxy for fracture energy (denoted as G' in their work) increases with the slip S, from 10^3 J/m^2 at $S = 1$ mm to 10^6-10^7 J/m^2 at $S =$ 1 m. Since $G' = (\Delta\sigma - 2\mu E_{\rm r}/M_{\rm o})(S/2)$ and depends on $\Delta\sigma$, which was estimated based on the model of *Madariaga* [1976], G' would be smaller with $\Delta\sigma$ based on the model with the small-scale yielding limit.

Stress drop observations typically exhibit large scatter, even in relatively compact regions [e.g., *Shearer et al.*, 2006]. A key question is how much of this scatter is real (i.e., true differences in earthquake stress drops) and how much may simply reflect observational uncertainties, such as inaccurate corrections for attenuation. An underappreciated aspect of the Madariaga model, confirmed by our own work, is the large dependence of $f_{\rm c}$ on take-off angle relative to the source (Figure 1a). Thus, measurements from a small number of seismic stations are unlikely to produce unbiased estimates of spherically averaged $f_{\rm c}$, even if we assume attenuation corrections are perfect. A factor of two difference in $\bar{f}_{\rm c}$ will produce a factor of eight difference in stress drop. As an example, synthetic tests of random take-off angles show large scatter in stress drop estimates from single station measurements, which is reduced as more stations are averaged (see Figure 3). Histograms of $\log_{10}\Delta\sigma$ estimates in Figure 3 are approximately log-normal distributed, and standard errors in $\log_{10}$ stress drops are 0.18 and 0.41 for single station P- and S-wave estimates, respectively, which are reduced to 0.09 and 0.20 for 5-station estimates and 0.07 and 0.14 for 10-station estimates. Note that, for example, ± 0.41 in $\log_{10}\Delta\sigma$ gives standard error bars that span from 0.39 to 2.6 times the true stress drop, that is, there is a 68% chance the true stress drop is within that range. These should be taken as lower bounds on the errors because it is unlikely the true station distribution will mimic random take-off angles.

Intellectual Merit:

One of the SCEC science objectives is "to develop physics-based models of the nucleation, propagation, and arrest of dynamic earthquake rupture" that will "contribute to our understanding of earthquakes in Southern California fault system." Our research has established the relationship between corner frequencies and the source radius for dynamic models of a circular fault. As stress drop is often estimated in a way that relies on the validity of a specific theoretical model of rupture dynamics, our results have led to more accurate estimates of stress drops of earthquakes in Southern California and other regions. The results have also impacted interpretations of other source parameters used to infer earthquake mechanics.

Broader Impacts:

Characterization of earthquake source parameters is important for understanding the physics of source processes and seismic hazard. Static stress drop is one of the key earthquake source parameters, which provides hints on earthquake source scaling and insights into tectonic environments in which earthquakes occur. Stress drop is also used as a primary input parameter for ground

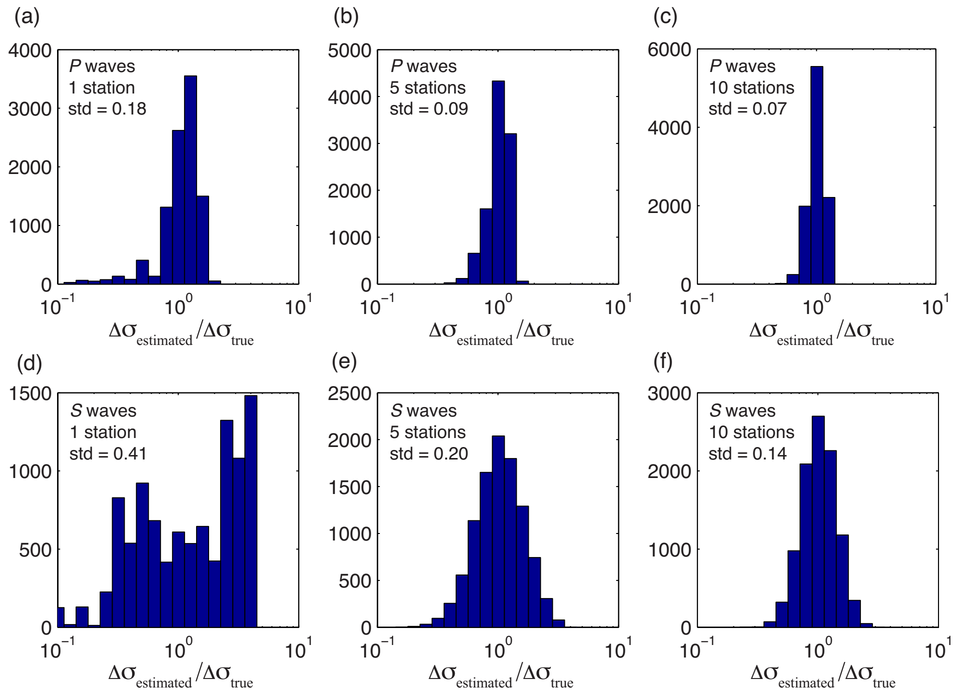

Figure 3: Histograms of normalized stress drops for different numbers of random stations based on the corner frequencies shown in Figure 1a. The horizontal axis shows estimated stress drop $\Delta\sigma_{\mathrm{estimated}}$ normalized by the 'true' stress drop $\Delta\sigma_{\mathrm{true}}$ obtained from perfect station coverage. The standard deviation (std) in $\log_{10}\Delta\sigma_{\mathrm{estimated}}$ and the number of stations are indicated in each panel.

motion simulations with stochastic techniques for quantification of seismic hazards. Hence our project has linked advances in earthquake seismology, crustal deformation, and earthquake hazard. Results from our research have been disseminated through conference presentations, seminars, and publications in peer-reviewed literature.

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