2012 SCEC Annual Report

Quantizing Southern California Tectonics for Stressing Models

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1. Summary of Research Findings

1) Brendan Meade and graduate student Eileen Evans integrated the Total Variation Denoising solution method into the BLOCKS code. This algorithm is designed to allow for quantized estimation of block motions. The purpose of this approach is to deterministically estimate which particular subsets of the southern California fault network might be most important without having to work to rely on the human or consensus agreement of which faults are most kinematically important *a priori*.

2) Graduate student Eileen Evans assembled a high-density block model based on not only the Community Fault Model (Plesch et al., 2007) but also the USGS Quaternary fault and fold database (http://earthquake.usgs.gov/hazards/qfaults/) for the entire western United States (Figure 1).

3) Evans developed the workflow necessary to run and store the results from estimation with different GPS data subsets and quantify the sensitivity of the TVDN block motion models to the variation in the quantization penalty parameter (Figure 2). These calculations are computationally expensive so substantial planning was required to avoid redundant calculations.

4) Work has been initiated on the calculation of stressing rates calculated from these models (Figure 3).

2.-3. Technical Report and Figures

The goal of developing a Community Stress Model is to provide a framework for comparing different inferences of stress and understanding how similar or different these might be based on data type and interpretation. One of the data types used in these analyses are nominally interseismic GPS velocities. Interseismic GPS velocities can be used to constrain mechanical earthquake cycle models in which stresses can be calculated both at the surface and at all depths. A recent example of this approach is the *Loveless and Meade* (2011) paper "Stress modulation on the San Andreas fault by interseismic fault system interactions". Here, a geodetically constrained three-dimensional block model based on the rectilinear Community Fault Model was used to analytically calculate interseismic stressing rates using analytic Greens functions (*Okada*, 1992). The advantage of this approach is that it provides a mechanically consistent methodology for determining stresses in between observation points. The disadvantage of this approach is that it is predicated on a particular block model geometry that was selected a particular set of researchers.

In our work this year we took an algorithmic approach to moving past the problem of researcher dependent model selection by applying the Total Variation Denoising (TVDN) algorithm (e.g., *Chambolle*, 2004) to block models of southern California and the western United States. This was developed in the context of computer graphics to remove high frequency noise from two-dimensional images by minimizing the total variation between pixels to simplify the final image. Effectively, TVDN serves as an algorithmic Ockham's razor. In the context of the work proposed here, the TVDN algorithm can be used to deterministically select active block model geometries from extremely large subsets of possible models (Figure 1). This approach is primarily useful because it provides deterministic methodology for constraining which parts of the southern California fault system are most actively contributing to interseismic stressing rates, moving some faults closer to failure.

Total variation denoising for block model selection

A long-standing question in the block modeling community is how are block boundaries chosen? To say it another way: Given a large number of blocks, what is the fewest number of distinct rotation vectors (*i.e.*, Euler poles and rotations rates) required to explain the observed geodetic data? In this case we want to solve for a state vector that is quantized (*i.e.*, populated by on a few discrete values). The Total Variation Denoising algorithm (*e.g.*, *Chambolle*, 2004) has been designed to do just this by minimizing the function,

$$J = \|\mathbf{Gm} - \mathbf{d}\|_2 + \lambda \|\nabla \mathbf{m}\|_1 \quad \text{(Total Variation Denoising)}$$

Where $\mathbf{G}$ is a linear operator and the variation in the state vector, $\nabla\mathbf{m}$, can be written with a linear differential operator as $\mathbf{Dm}$. This problem is convex, and a global minimum can be determined using standard quadratic programming techniques. However, it is not immediately obvious that minimizing the above objective function should in any way produce a quantized state vector, $\mathbf{m}_{\text{est}}$. While this result is computationally verifiable for small problems, the theoretical reason for this has only emerged recently in the context of compressed sensing theory where it has been shown that the l_1 regularization is equivalent to the l_0 regularization. The latter formulation $\|\nabla\mathbf{m}\|_0$ is an exact statement of jump minimization in the state vector. However the direct solution to this problem is combinatorial and therefore NP-hard. Remarkably, the l_0 regularization has an "overwhelming probability" (*Candes, Romberg and Tao*, 2005; *Donoho*, 2004) of being equivalent to the l_1 regularization, the latter of which is solvable by non-linear optimization techniques.

Application to southern California and the Western United States

The TVDN algorithm can also be applied to geodetically constrained block model estimation of rotation vectors and fault slip rates that are used to generate stressing rate estimates. All published block models of southern California have substantial researcher dependent choices with regard to which geologic structures are included in a particular block model [*Bennett et al*., 1996; *McCluskey et al*., 2001; *McCaffrey*, 2004; *Becker et al*., 2004; *Meade and Hager* 2005; *Chuang and Johnson*, 2011; *Loveless and Meade*, 2011] and sensitivities of and stress fields to assumptions about fault system geometry are rarely, if ever, discussed. The primary reason that such small subsets of the southern California fault system are included in block models is that as the number of blocks in a model increases, the spread in eigenvalues increases and damping must be introduced for numerical stability. However, TVDN allows for the rotation vectors associated with a very large number of blocks to be estimated through a nonlinear solution method that minimizes the number of different rotation vectors. A practical effect of using this method is that for many small blocks the default behavior would be to rotate similarly to neighboring blocks unless this would result in substantially large residual velocities. In application this means that we can build block models honoring the entirely of the southern California fault system geometry that include hundreds of blocks. We can also easily introduce hypothetical structures (faults) that may not have been mapped and interrogate the geodetic data to determine whether or not activity on hypothesized structures is necessary to explain the geodetic velocity field at a given level of resolution (*e.g.*, mean residual magnitude of 2 or 4 mm/yr).

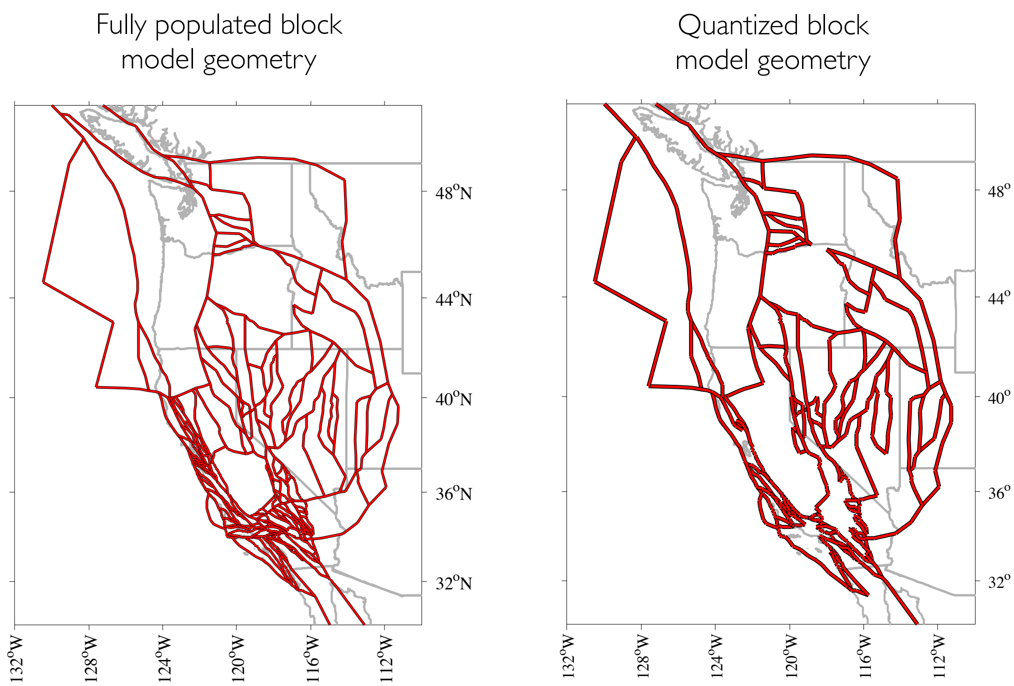

Figure 1. Examples of fully populated block model geometry (left) and a subset of most active blocks determined by quantized state vector estimation. Notice the substantial reduction in the estimate of the number of active boundaries.

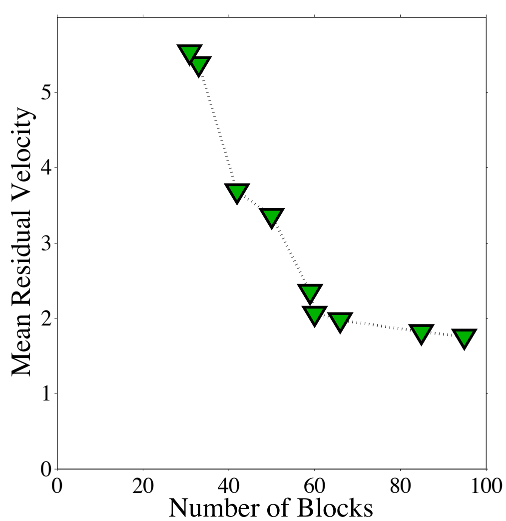

Figure 2. Trade-off between goodness-of-fit and the number of unique block rotation vectors blocks. Fewer unique rotation vectors result in a poorer fit to the data, to a point. Note that the goodness-of-fit starts to saturate near a minimum at ~60 unique Euler poles, a subset of the ~100 blocks in the model.

Stressing rate orientations and magnitudes

The purpose of doing all of this work is to exploit the fact that stressing rates can be calculated analytically with elastic block models. Block rotations obviously do not contribute to the strain/stress fields, but elastic strain accumulation does. Because we have analytic formulas giving the stresses in response to slip, or slip deficits, in a homogenous elastic half space (*e.g.*, *Okada*, 1992). The elastic deformation fields automatically satisfy the stress equilibrium conditions so that we know we are interpreting a mechanically consistent stressing rate field.

This is in contrast to classical stressing rate estimates inferred from the differentiation of GPS velocities where there is no guarantee that the spatial variations in stresses satisfy Newton's second law. An example calculation is shown below and we continue to develop the software to do this calculation at large scale.

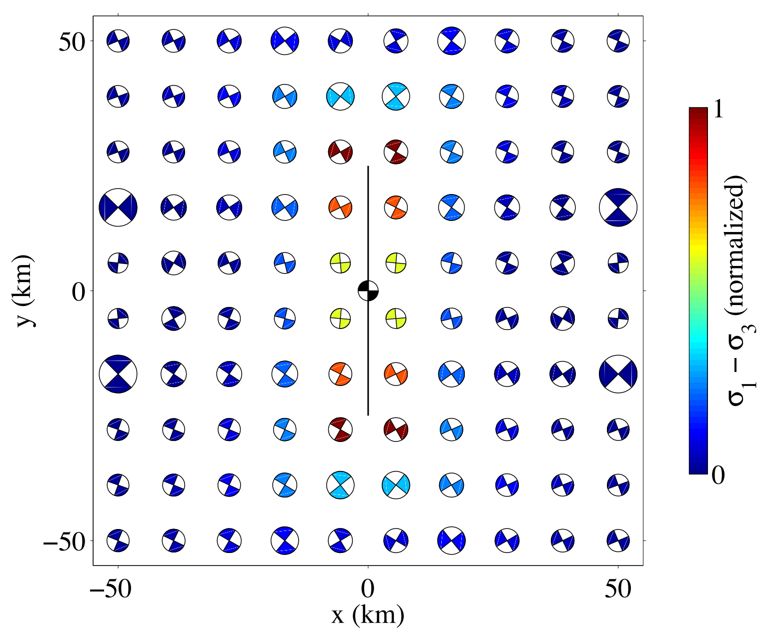

Figure 3. Stress rotations due to elastic deformation following slip on a fine strike slip fault (vertical black line). The size of the focal mechanism is proportional to the base ten log of the normalized differential P-axis orientation and the color indicates the normalized differential stress. Near the center of the slipping fault (vertical black line), the differential stresses are relatively large but the change in stress orientation is small. There is no simple rule for how these relationships very with distance but it is clear that some of the largest angular differences appear at substantial distances away from the fault where differential stresses are small. We are continuing to work on similar calculations for all of southern California based on the quantized block models described above.

4. Outreach Activities

There were no formal outreach activities associated with this work. However it did contribute to the content of a talk that I gave to first year graduate students at Harvard University about applied mathematics in earth science research.

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