Progress Report, 2011–2012
Southern California Earthquake Center

Advanced Numerical Techniques for Dynamic Rupture Simulations

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SCEC3 Summary:

Our group has developed several novel numerical methods for dynamic rupture simulations. There have been two main thrusts, each targeted at specific outstanding scientific questions. Motivated by field and laboratory studies of fault roughness, we sought to develop a code capable of handling geometrically complex domains. Since one of the target research areas was the origin of incoherent high frequency ground motion, we needed a method free from the spurious numerical oscillations that plague many conventional codes. We also need high-order accuracy, both in the interior and also near faults. To accomplish this, we developed a method based on summation-by-parts finite difference operators with weak enforcement of boundary conditions and fault friction laws (both slip-weakening and rate-and-state). The method, including boundary treatment, is provably stable and accurate, as confirmed with rigorous convergence tests. We have also implemented Drucker–Prager plasticity to account for inelastic off-fault deformation. The code was parallelized and strong scaling tests show ideal scaling to 4096 cores (the most we have tested with thus far). This code was used in our published studies of high frequency ground motion from rough faults. We have also used it to study subduction zone earthquakes, a testament to its ability to handle extremely complex geometries.

A second thrust of our work has been on an adaptive mesh refinement (AMR) code for rupture dynamics. Here the motivating problem concerns the earthquake energy balance, in particular which physical processes explain the increase of fracture energy with propagation distance required by self-similarity. The AMR approach adaptively refines and coarsens the mesh to track sharp gradients in the velocity and stress fields, such as wavefronts and rupture fronts. The method is based on a low-order finite volume method, with high accuracy arising in this case from a dense mesh around the features of interest. The implementation is built on the CHOMBO AMR library from Lawrence Livermore National Laboratory. We have implemented slip-weakening and rate-and-state friction into our code, as well as continuum plasticity. The code is written in a dimension-independent manner, permitting both 2D and 3D simulations. We are currently adding thermal pressurization. After that is done, we will be able to study how much dissipation occurs on the fault (through thermal pressurization and frictional sliding) versus off the fault (in inelastic deformation).

Recent reports related to theme of this project:

Kozdon, J. E. and E. M. Dunham (2011), Adaptive Mesh Refinement for Earthquake Rupture Simulations, presented at SIAM Conference on Mathematical and Computational Issues in the Geosciences, Long Beach, Calif., 21-24 Mar

O'Reilly, O., J. E. Kozdon, E. M. Dunham, and J. Nordstöm (2011), Coupled High-Order Finite Difference and Unstructured Finite Volume Methods for Earthquake Rupture Dynamics in Complex Geometries, presented at SIAM Conference on Mathematical and Computational Issues in the Geosciences, Long Beach, Calif., 21-24 Mar

O'Reilly, O., J. Nordstöm (2011), Coupled High-Order Finite Difference and Unstructured Finite Volume Methods for Earthquake Rupture Dynamics in Complex Geometries. *Student thesis* urn:nbn:se:uu:diva-155471

Kozdon, J. E. and E. M. Dunham (2011), Tetemoko: An Adaptive Mesh Refinement Framework for Dynamic Rupture Simulation, in *SCEC Annual Meeting Proceedings and Abstracts*, vol. XXI, p. 185, Southern California Earthquake Center.

Kozdon, J. E., E. M. Dunham, and J. Nordstöm (2011), Simulation of Dynamic Earthquake Ruptures in Complex Geometries Using High-Order Finite Difference Methods. *Journal of Scientific Computing*, submitted: 15 September 2011.

Kozdon, J. E. and E. M. Dunham (2011), Rupture to the Trench in Dynamic Models of the Tohoku-Oki Earthquake, Abstract U51B-0041 presented at 2011 Fall Meeting, AGU, San Francisco, Calif., 5-9 Dec

Dunham, E. M. and J. E. Kozdon (2011), Rupture Dynamics of Subduction Megathrust Earthquakes, Abstract T31E-05 presented at 2011 Fall Meeting, AGU, San Francisco, Calif., 5-9 Dec

Technical Report:

This project involved the continued development of advanced numerical methods for dynamic rupture simulations. This was done in two ways. First the finite difference method our group has developed was extended to allow for coupling with an unstructured finite volume method. This coupling will allow us to simulate complex geometries, specifically fault networks involving multiple branches. Second, we extended our previously developed adaptive mesh refinement code to incorporate rate-and-state friction and off-fault plasticity.

Finite Difference and Finite Volume Methods for Complex Geometries:

In past years our group has developed an efficient, accurate, and stable finite difference method for dynamic rupture simulations. This method uses block-structured meshes and coordinate transforms to handle complex geometries. Using this method we are able to simulate many fault systems that arise in nature as well as heterogeneous material properties and complex free surface topography. This method is highly parallel and efficient, showing almost ideal scaling on up to 4096 cores. However, some geometries are more naturally represented using unstructured grids. One of the main drawbacks of methods based on unstructured meshes is the increased computational cost due to inefficient cache usage. This motivated us to consider a coupled approach, where the unstructured mesh is localized to a small region around the complex geometry. Under this support we have developed a stable and accurate procedure for incorporating rate-and-state boundary conditions into an unstructured finite volume method. We have also developed a methodology for coupling our finite difference method with the unstructured finite volume method.

The finite volume method we have developed is a 2D node-centered method for triangular meshes, with the unknown fields stored at the grid vertices instead of cell centers. To incorporate fault interfaces, we colocate nodes on both sides of the interface, each with an independent set of stresses and velocities. As in our finite difference method, we enforce the fault conditions weakly by penalizing the discretization for not satisfying the friction law. The method is provably stable. It is well-suited for fault branches because we do not have to arbitrarily specify a through-going fault segment as appears to be required by the popular traction-at-split-nodes method.

A similar penalty technique is used to couple the finite volume method with the finite difference method across an interface of colocated grid points; see Figure 1(a). While the coupled method is formally second order accurate, the overall error levels are dramatically reduced compared to a finite volume only approach because of the superior dispersion properties of the finite difference method.

To verify the accuracy and stability of the method we have conducted several numerical tests. By adding a forcing function in the interface conditions and imposing source terms to the interior we are able to construct an analytic solution to the problem, a technique known as the method of manufactured solutions. We cover $\sim 10\%$ of the domain with an unstructured mesh and discretize the remainder of the domain using our high-order finite difference method. We also run the same problem using a fully unstructured mesh. The numerical error and convergence rates are shown in Figure 2.

To test our implementation of rate-and-state friction and demonstrate our ability to handle non-Cartesian geometries we have set up a test involving antiplane strain and a circular frictional fault. Around the fault an unstructured mesh is used, and away from the fault we transition to our high-order finite difference method; see Figure 1(a). Rupture is nucleated at the top of the fault and propagates downward. There are minimal spurious reflections off of the transition between the two meshes.

In the near future we will implement this coupled method into our production-level parallel code, enabling us to consider problems with very complex networks of faults involving multiple branches and segments.

Adaptive Mesh Refinement:

It is an outstanding issue in rupture dynamics to explain the physical basis of the linear increase of fracture energy with rupture length required by self-similarity. Two theories have been suggested in the literature for this: thermal pressurization (Rice, 2006) and plasticity (Andrews, 2005). To date, it has not been possible to confirm either of these mechanisms in simulations due to the resolution constraints of traditional codes based on fixed computational grids. Thus, we have been developing an adaptive mesh refinement (AMR) framework that will allow us to model ruptures which propagate for tens or hundreds of kilometers using laboratory-measured friction parameters. The reason that AMR allows us to accomplish this is that mesh resolution is dynamically placed as required to ensure an accurate solution.

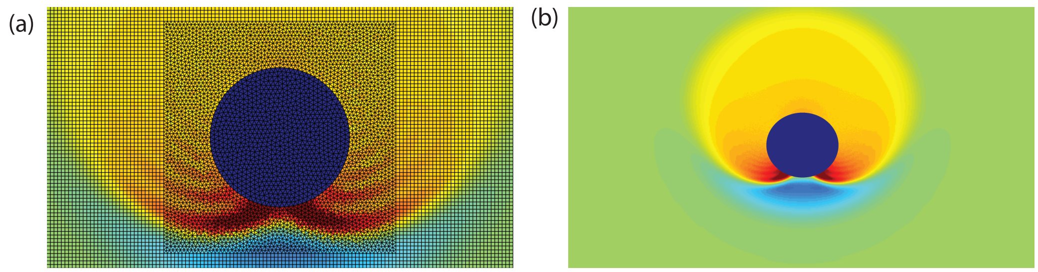
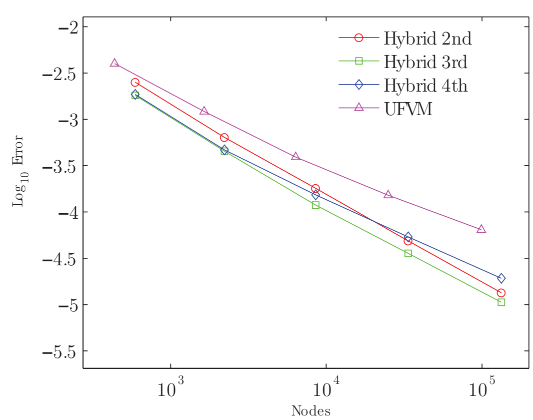


Figure 1: (a) Mesh and particle velocity field for a low resolution simulation of a circular fault. The unstructured mesh is only used to handle the complex geometry, and away from this we use a structured mesh. The methods are coupled only at the interface, and thus no nesting/overlapping strategy is required. (b) The full domain for this problem with a factor-of-two refinement compared with panel (a).



	Convergence rate		
Nodes	○	□	◇
591			
2211	2.09	2.10	2.10
8544	1.87	1.99	1.65
33582	1.91	1.75	1.53
133146	1.87	1.77	1.49

Nodes	△
435	
1636	1.78
6339	1.66
24949	1.39
98985	1.25

Figure 2: Convergence rate and error for the coupled high-order finite difference and finite volume method and the uncoupled finite volume method. For the coupled method, the unstructured mesh covers $\sim 10\%$ of the whole domain. For the uncoupled method, the whole domain is covered with an unstructured mesh.

Additionally, the AMR framework that we have chosen for our code is such that each grid resolution can be advanced independently, thus large ($\sim$10 m) grid cells need not be advanced with the more restrictive time step constraints imposed by small cells ($\sim$1 cm).

We have thus far incorporated both strongly rate-weakening fault friction (in a rate-and-state framework) and off-fault plasticity in our AMR code. We have verified our implementation by comparing with the results of Dunham et al. (2011) as shown in Figure 3. The zone of plastic deformation compares quite well with that seen in Dunham et al. (2011), and the power of AMR is evidenced by the significantly higher resolution.

To demonstrate that we will soon have the ability to study the earthquake energy balance, we show a second test in Figure 4. Laboratory-measured friction parameters were used to propagate a rupture 30 m with off-fault plasticity. This test is motivated by Noda et al. (2009), though we have included plasticity and not thermal pressurization in our model. The breakdown of linear scaling of the region of plastic deformation is a computational artifact due to reflections from the outer absorbing boundary, which (unfortunately) are too accurately accounted for by the adaptive placement of the meshes. We are currently exploring strategies to reduce or eliminate these artificial reflections, such as adding a perfectly matching layer or some other more effective absorbing boundary condition. Within the next year it will be possible to propagate a rupture > 1 km using laboratory-measured friction parameters. This means that once we add thermal pressurization (which will be done using the computationally efficient strategy developed by Noda and Lapusta (2010)), we will be able to computationally explore the self-similarity of earthquakes for the first time using laboratory-measured parameters.

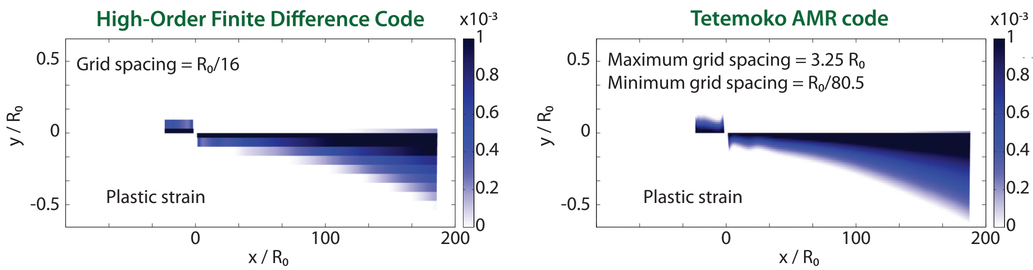


Figure 3: Comparison of our AMR code and our high-order finite difference code, with strongly rate-weakening fault friction and Drucker–Prager off-fault viscoplasticity. R_0 is the characteristic size of the rupture front for quasi-static crack growth. Plasticity occurs in a wedge-shaped region opening at a very small angle with respect to the fault.

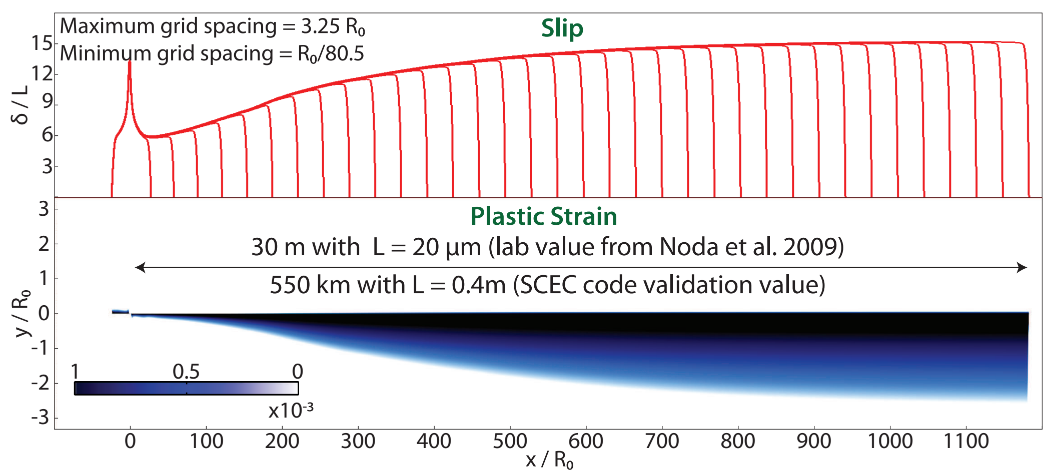


Figure 4: Example of AMR simulation with strongly rate-weakening friction and off-fault plasticity. Artificial wave reflections from the exterior boundaries prevent an accurate solution at later times, and destroy the expected linear increase of slip and off-fault extent of plasticity with propagation distance. We are currently exploring effective ways of minimizing this.