

# 2011 SCEC Grant #11065: Annual Report

*Title of Project*

**Earthquake nucleation mechanisms and damage healing in heterogeneous fault zones, probed with periodic loadings (in models, observations, and experiments).**

*Principle Investigators*

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***Proposal Category:***  
Integration and Theory

***Science Objectives:***

A3,A4,A10

# 1. Can oscillatory stresses be used to predict material failure and large earthquakes?

(Braden A. W. Brinkman, Michael LeBlanc,  
Yehuda Ben-Zion, Jonathan T. Uhl, Karin A. Dahmen)

It has long been speculated that tidal stresses can trigger large earthquakes. However, experimental confirmation remains controversial, and theoretical models have yet to accurately predict the response of faults to external periodic stresses [T.H. Jordan, et al., *Living on an Active Earth: Perspectives on Earthquake Science*, National Academy Press, Washington, D.C., 418 pp., (2003)]. We studied a simple model for potential triggering mechanisms of large earthquakes in response to a slowly increasing loading stress and an added periodic stress with amplitude  $F_0(\omega)$  at frequency  $\omega$ . The model predicts the minimum amplitude  $F_{0\min}(\omega)$  needed to observe stress-correlated triggering of large slip-events in earthquake-fault-zones [Y. Ben-Zion, *Reviews of Geophys.* **46**, RG4006 (2008)], sheared lab-scale rocks [Lockner and Beeler, *J. Geophys. Res.* **104**, 20133 (1999) and 108, ESE 8-1 (2003)], and sheared granular materials [H. M. Savage, C. Marone, *J. Geophys. Res.* **112**, B02301 (2007)]. We predict  $F_{0\min}(\omega) \sim 1/\omega$  for small (non-zero) frequencies and decaying oscillations for  $F_{0\min}(\omega)$  for large frequencies. The predictions explain and unify disparate experimental results. We also argue that correlations between *small* slips and periodic stresses increase as the system approaches a large slip-event/earthquake. These result provide new tools for materials-failure prediction and hazard-prevention studies, applicable to a wide variety of systems [K. A. Dahmen, Y. Ben-Zion, J. T. Uhl, *Phys. Rev. Lett.* **102**, 175501 (2009)].

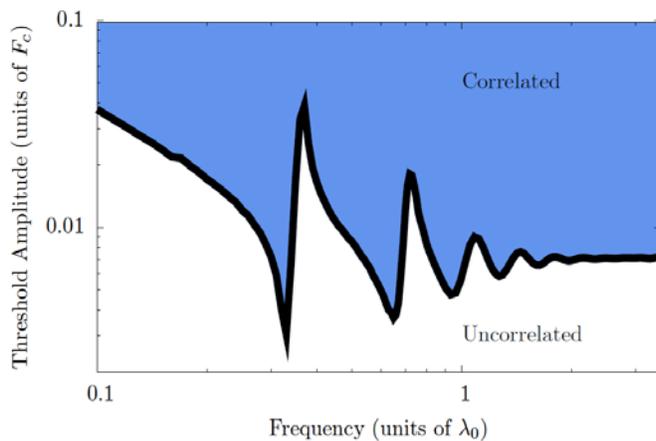


Figure 1: Minimum (“threshold”) amplitude  $F_{0\min}$  required for detecting a correlation between the periodic driving stress and large events as a function of frequency, on a log-log plot. The curve represents the minimum amplitude required to detect a correlation at 99.5% confidence for  $n=500$  recorded events: in the shaded region above the curve we detect significant correlations, while below the curve the correlations do not meet our 99.5% threshold (and are labeled ‘uncorrelated’). As expected, the minimum required amplitude decreases as more events are recorded (see preprint 1). The frequency axis is plotted in units of the small event rate  $\lambda_0$ . The tectonic shear rate was chosen to be  $\Gamma = 0.015\lambda_0$ .

## Preprints on the work supported by this grant:

1. Braden A. W. Brinkman, Michael LeBlanc, Yehuda Ben-Zion, Jonathan T. Uhl, Karin A. Dahmen, “Can oscillatory stresses be used to predict material failure and large earthquakes” preprint 2012, to be submitted to *Nature Physics*.
2. Braden A.W. Brinkman, Yehuda Ben-Zion, Jonathan T. Uhl, Karin A. Dahmen, “Inter-event times in a simple earthquake model and the Weibull Distribution.” Preprint 2012 in preparation.

## Related paper important for future extensions of the work supported by this grant:

3. Karin A. Dahmen, Yehuda Ben-Zion, and Jonathan T. Uhl, “A simple analytic theory for the statistics of avalanches in sheared granular materials”, *Nature Physics*, **7**, 554-557 (2011).

Technical Report:

*Can oscillatory stresses be used to predict material failure and large earthquakes?*

*Braden A. W. Brinkman, Michael LeBlanc, Yehuda Ben-Zion, Jonathan T. Uhl, and Karin A. Dahmen*

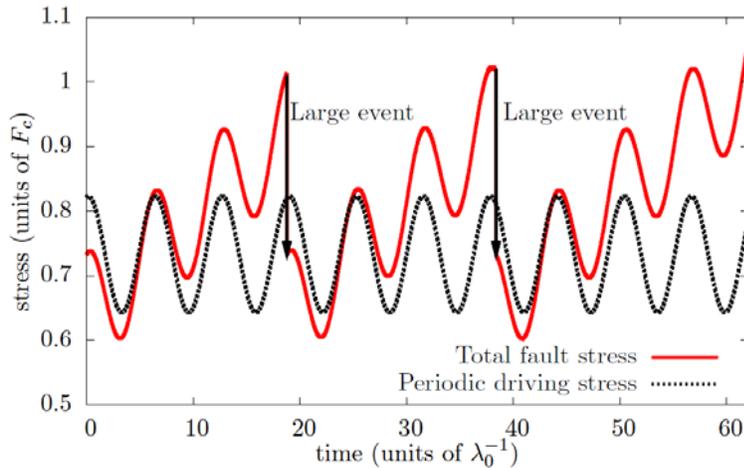
We have studied a probabilistic model of stress-correlated triggering of large mega-thrust earthquakes in the presence of an external sinusoidal driving stress  $F_{0\min}(\omega)$ . The model predicts the minimum amplitude  $F_{0\min}(\omega)$  needed to observe a significant correlation, at a given frequency  $\omega$ , between the stress oscillations and the occurrence of large earthquakes. It also predicts the number of earthquakes necessary to observe such a correlation. Our results can thus be used to predict which periodic stresses (with given amplitude and frequency) are most effective at triggering large earthquakes. After the necessary model parameters are determined from observations, the model results can be used to predict whether seasonal or tidal stresses are more likely to trigger a large event. (Note that the potential for triggering depends on and can be used to infer the state of stress on the fault (the overall average level and degree of heterogeneities).) Comparing the model predictions to data from actual earthquake faults is difficult due to the long times necessary to collect sufficient statistics. Instead, we compare the model to laboratory experiments on systems which exhibit stick-slip behaviour analogous to earthquakes, such as the rock-friction experiments of Beeler and Lockner [Lockner (1999); Beeler (2003)] or the sheared glass bead experiments of Savage and Marone [Savage (2007)]. The behavior predicted by the model qualitatively agrees with the experiments. It also provides new predictions for future experiments at higher (and lower) frequencies than have previously been probed. In particular, the model predicts that the threshold amplitude  $F_{0\min}(\omega)$  is proportional to a power law ( $1/\omega$ ) at low oscillation-frequencies  $\omega$  and exhibits oscillations which asymptotically decay to a constant value at high frequencies. (Here the frequency is compared to the tectonic stress increase rate specified below). Our results primarily concern systems which exhibit stick slip behaviour with recurring large slips. These types of systems often also produce small events which are roughly power law distributed. The much larger slip events (known as “characteristic earthquakes” in fault systems) are over-represented relative to the power law distribution. (This is to be contrasted with other earthquake faults which follow a power law Gutenberg-Richter distribution of sizes on all scales.) For hazard prevention, our primary interest is the effect of periodic stresses, such as tides or seasonal stresses, on these large characteristic events. The model also predicts that small events are more susceptible to periodic stresses close to the occurrence of a large earthquake. This observation can then be used for hazard prediction and prevention studies.

Our simple probabilistic model is based on the following assumptions:

1. Small earthquakes (magnitude  $< 7.0$ ) occur randomly in time according to a Poisson process with constant rate  $\lambda_0$  independent of fault stress  $F$ . (Although in principle the small event rate is expected to depend on stress, we expect the statistics of the large events to be qualitatively the same).
2. Any small event can trigger a large event with a stress-dependent probability,  $P(F)$ . This probability is initially small and rises to 1 quickly near a critical stress  $F_c$ .
3. When a large event occurs, the stress on the fault relaxes to a baseline value,  $f$ , immediately (i.e., the relaxation is much faster than any other timescale in the model).

To model slow tectonic loading with a periodic driving stress, we take the stress  $F$  as a function of time,  $t$ , to be  $F(t) = f + \Gamma t + F_0 (\sin(\omega t + \varphi) - \sin\varphi)$ , where  $f$  is a constant initial baseline stress,  $\Gamma$  is a slow (“tectonic”) shear rate,  $F_0$  is the amplitude of the periodic driving,  $\omega$  is the frequency and  $\varphi$  is the phase. The additional  $\sin\varphi$  term fixes  $F(0) = f$ ; i.e., after a large event the fault stress always relaxes to the baseline value  $f$ . When a large event occurs, the stress relaxes to the baseline value  $f$ , and we reset the time  $t$  to zero. The phase  $\varphi$  is updated such that  $\varphi \rightarrow \omega t_{\text{event}} + \varphi$ , where  $t_{\text{event}}$  is the time that elapsed between the previous large event and the current large event (i.e. the inter-event time), see Fig. 1.

From these assumptions we can compute the inter-event probability density function (pdf) of times between large events. The large event triggering probability  $P(F)$  is calculated using the Ben-Zion and Rice (BZR) earthquake model, described in [Fisher (1997); Ben-Zion (1993), Ben-Zion (1996); Ben-Zion (2003); Ben-Zion (2008)]. Related models [Dahmen (2009); Dahmen (2011)] give the same results for plasticity, rock deformation and sheared granular materials. . Using the triggering probability and the inter-event time pdf we generate a synthetic time series of large earthquake events. We then analyze the correlations between the large events of this time series and the periodic components of the driving stress.



*Figure 1: Stress on earthquake fault compared to the periodic driving stress. We analyze correlations by extracting histograms of the phase of the driving periodic stress when the large earthquakes occur. As seen in the example in the figure, at high frequencies (compared to the slow tectonic loading rate), the large events tend to occur near stress maxima. The large event most likely occurs near the critical stress  $F_c = 1$ . In a finite system the large events may not occur until after the system reached the critical stress, while in an infinite system they occur at or before the critical stress with certainty. For the two large events shown in the sketch, the phase is  $\varphi \approx \pi/2$ .*

Correlations between the timing of the large events and the periodic stress can be observed by analyzing the distribution of phases  $\varphi$  of the oscillatory stress at which the large events occur. The distribution of these phases provides information about how strongly the fault is affected by the periodic stress. For small amplitudes, correlations are weak and consequently the distribution of phases is roughly uniform, while for larger amplitudes, correlations are stronger and obvious sinusoidal variations appear in the distribution of phases, as seen in Fig. 2. Additionally, the model predicts that at *low* frequencies the most likely phase at which a large event occurs is near the maximum of the stress *rate*, i.e.  $\varphi = 0$  (Fig 2, left). At higher frequencies they occur near the maximum stress i.e.  $\varphi = \pi/2$  (Fig. 2, right). Here “low” or “high” frequency is measured relative to the tectonic stress increase rate,  $\Gamma/F_c$ . This prediction agrees with experiments on a laboratory-scale earthquake fault with granular gauge material [Savage (2007)]. Intuitively, one expects that large events are most likely to occur at stress maxima ( $\varphi = \pi/2$  in Fig. 2), as observed at high frequencies. The reason this is not the case at low frequencies is as follows: For  $\omega \ll \Gamma/F_c$  many large events occur during each stress-oscillation cycle. In fact, they occur at every part of the cycle (i.e. at any phase  $\varphi$ ). Furthermore, the number of events observed per unit time is highest when the stress increase per unit time is highest. Consequently at low frequencies most events occur when the

stress rate is a maximum, which is at  $\varphi = 0$  or  $2\pi$  in Fig. 2. In contrast, for  $\omega \gg \Gamma/F_c$  the periodic stress oscillates through many cycles before a large event is triggered. In this case, most large events occur near the stress maxima, i.e. near  $\varphi = \pi/2$  as intuitively predicted and illustrated in Figs. 1 and 2.

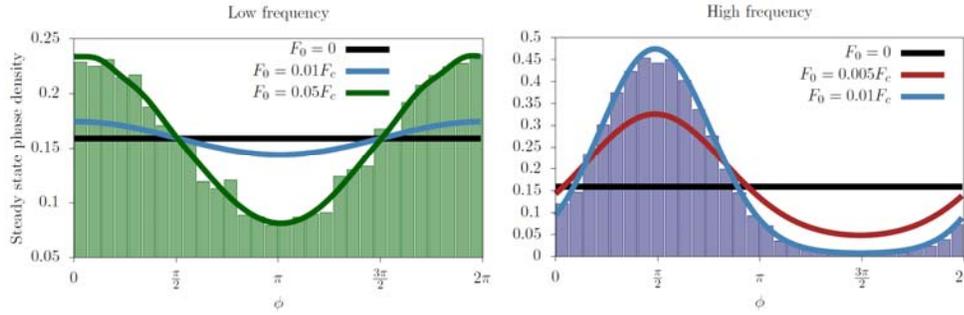


Figure 2: Steady state phase distributions (histograms) at low (left figure) and high (right figure) frequencies  $\omega$  (relative to the tectonic shear rate  $\Gamma/F_c$ ), and for a range of stress amplitudes  $F_0$ . For low amplitudes  $F_0$ , the distributions appear uniform, as expected, since

correlations with the oscillatory driving stress are practically undetectable in this case. As the amplitude  $F_0$  is increased, the phase distributions become more sinusoidal. For low frequencies, the most likely phase is  $\varphi = 0$  (equivalent to  $2\pi$ ) for periodic driving of the form  $F_0 \sin(\omega t + \varphi)$ . At higher frequencies the most likely phase approaches  $\varphi = \pi/2$ . This is consistent with the experimental observation that at low frequencies large events occur near the maxima of the stress increase rate, while at high frequencies they occur near the maxima of the stress itself. Histograms were obtained numerically from runs with 10,000 large events. The solid lines are envelopes of the histograms for different amplitudes  $F_0$ . Note that at low frequencies a much larger amplitude  $F_0$  is required to observe the same amount of correlations with the oscillatory stress (i.e. to observe the same deviation from a uniform phase distribution) than for high frequencies. (The blue curves in both figures correspond to the same amplitude  $F_0$ ).

Following the experimental analyses [Lockner (1999); Beeler (2003); Savage (2007)], we quantify correlations by likening the observed phases to the angles of a drunkard's (random) walk in two dimensions.

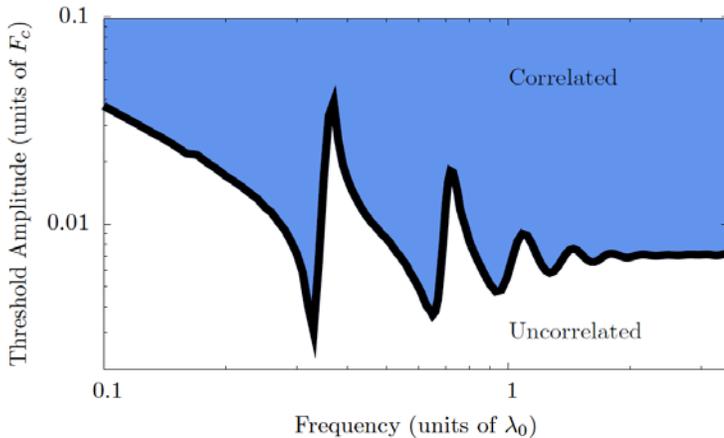


Figure 3: Minimum ("threshold") amplitude  $F_{0min}$  required for detecting a correlation between the periodic driving stress and large events as a function of frequency, on a log-log plot. The curve represents the minimum amplitude required to detect a correlation at 99.5% confidence for  $n=500$  recorded events: in the shaded region above the curve we detect significant correlations, while below the curve the correlations do not meet our 99.5% threshold (and are labeled 'uncorrelated'). As expected, the minimum required amplitude decreases as more events are recorded (see SI). The frequency axis is plotted in units of the small event rate  $\lambda_0$ . The tectonic shear rate was chosen to be  $\Gamma = 0.015\lambda_0$ .

The model results share several features with previous experimental results, in particular the inverse frequency dependence  $F_{0min}(\omega) \sim 1/\omega$  at low frequencies [Savage (2007)] and a minimum in the threshold curve [Lockner (1999); Beeler (2003)]. The  $1/\omega$  decay simply reflects a competition between the slow tectonic loading rate,  $\Gamma$ , and the oscillatory loading rate,  $\omega F_0 \cos(\omega t + \varphi)$ . A correlation will only be observable when the oscillatory loading rate is larger than the tectonic loading, i.e.,  $F_0 \omega \geq \Gamma$ , and hence  $F_0 \sim 1/\omega$  at low frequencies. In addition, the model predicts new features, such as the decaying oscillations of  $F_{0min}(\omega)$  for larger  $\omega$ . These oscillations have not yet been observed in experiments, possibly due to limited frequency ranges in experiments restricting observation to a narrow window of the  $\omega$ - $F_0$  plane. Another reason might be that

the number of events observed per run in experiments may have been too small. The asymptotically decaying oscillations offer a possible resolution to the conflicting high frequency behaviors predicted by other models and discussed in the context of some experimental studies. Experiments which see an increase in threshold amplitude as frequency increases may be seeing evidence of an upward branch of the oscillations of our predicted  $\omega$ - $F_0$  plot [Lockner (1999); Beeler (2003)], while experiments which observe a constant threshold at high frequency may be seeing the asymptotically constant tail of the  $F_0$  versus  $\omega$  curve in Fig. 3 [Savage (2007)]. To test this conjecture, measurements of the small event rate  $\lambda_0$  and other model parameters are needed to identify the range of frequencies for which we expect oscillations in  $F_{0\min}(\omega)$  and to compare the model predictions to experimental results. Furthermore, as shown in the SI the frequency-scale of the oscillation decay decreases with decreasing number  $n$  of observed events, such that oscillations are quickly damped out when only  $n \sim 20$  events are recorded. As a result, experiments which do not measure large numbers of events may not observe strong oscillations in their data.

For large numbers of recorded events, we can easily understand the frequencies at which the minima and maxima in  $F_{0\min}(\omega)$  occur. The frequencies at which  $F_{0\min}(\omega)$  has a local minimum, are determined by the average inter-event time of the large events,  $\langle t \rangle$ , in the absence of periodic driving (i.e. in the “unperturbed” case). In particular, the  $m^{\text{th}}$  minimum occurs at a frequency  $\omega_m = 2\pi m / \langle t \rangle$  within numerical statistical errors, where  $m$  is a nonzero positive integer. The physical reason is as follows: The average time to the next large event is  $\langle t \rangle$ . If the periodic component of the stress is near a maximum at the triggering time (in the unperturbed fault), the amplitude need not be as large to trigger the event slightly early and we have a local minimum in  $F_{0\min}(\omega)$ . If the oscillating stress is near a minimum at this time, however, then a larger amplitude is needed to cause a large event to be triggered early and we have a local maximum in  $F_{0\min}(\omega)$ . Measurements of  $\langle t \rangle$  can be used to predict the frequencies which experiments must probe in order to observe these maxima and minima. Since  $\langle t \rangle \sim 1/\Gamma$ , the model predicts that the minimum frequency increases with background loading rate  $\Gamma$ , i.e.  $\omega_m \sim \Gamma$  in agreement with experiments on rock friction [Lockner (1999); Beeler (2003)].

Our model provides a new framework for understanding the effects of periodic stresses, such as tidal or seasonal stresses, on large events in earthquake faults and on large events in experimental stick-slip systems such as rock-friction or granular shear experiments. Our results suggest an interesting structure in the threshold amplitude  $F_0$  as a function of frequency  $\omega$ : a  $1/\omega$  power law drop-off at low frequencies and decaying oscillations at higher frequencies. We also predict how many events are needed to detect correlations as a function of frequency and amplitude. The results are in agreement with previous experimental observations of the most likely phase and the inverse frequency dependence of the minimum amplitude at low frequencies [Lockner (1999); Beeler (2003)] and the loading rate dependence of the frequency of the minimum in  $F_{0\min}(\omega)$  [Lockner (1999); Beeler (2003)].

**Effect on small events and earthquake/failure forecasts:** So far we have focused on correlations between periodic stresses and large events, since this is most relevant to experiments on lab-sized rocks. But in systems such as earthquake faults, where the small event rate is expected to depend on stress, we can also look for correlations between periodic stresses and the small events. The BZR earthquake model indeed predicts an increase in the small event rate near the critical stress  $F_c$  where the large earthquakes occur. The sharp increase in number of events near the critical stress suggests that correlations between the number of small events and the periodic stress grow stronger closer to the critical

stress. The reason is that small changes in stress will lead to comparatively larger changes in the number of observed events as the stress approaches the critical stress. In characteristic earthquake faults, the increasing strength of small-event correlations with oscillatory stresses is then expected to be an indication of an impending large event. In fact, increased correlations between tidal stresses and small earthquakes were reportedly detected for several years prior to three large mega-thrust earthquakes in the Sumatra region [Tanaka (2010)]. By studying our model with a stress-dependent small event rate we can predict the frequency and amplitude dependence of the strength of stress-correlations with both small and large events. We will also be able to predict how small-event correlations increase prior to large events, providing a potential signal which could be used to forecast large earthquakes. The same approach (i.e., applying oscillatory stresses to a stressed material and measuring correlations of the acoustic emission with the oscillatory stresses) could also be used to forecast large failure events of materials in experiments, non-destructive materials testing, or commercial applications.

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