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Extracting high-frequency source properties from strong motion data by the Wavelet Transform Modulus Maxima technique

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Abstract

We assessed the applicability of a multiscale signal processing technique, the Wavelet Transform Modulus Maxima (WTMM) method, to characterize high frequency properties of strong motion waveforms, in particular the temporal distribution and strength of singularities (strong phases). We first explored their relation to earthquake source complexity through dynamic rupture simulations on faults with heterogeneous initial stresses. These showed that the timing and exponent of singularities measured by the WTMM method on the radiated wavefield are directly related to the position and exponent of assumed initial stress singularities on the fault plane. This suggests the possibility of inferring properties of fault heterogeneities from the high-frequency properties of the radiated field. We then applied the WTMM analysis to strong motion recordings of three earthquakes, the M5.4, 2008 Chino Hills M5.4, 2004 Parkfield M6.0 and 1999 ChiChi M7.6 earthquakes. We found that strong motion recordings at very high frequencies ($f > 5$ Hz) are represented by a single singularity exponent, $h \approx 2$, with insignificant variability for each earthquake across stations, which suggests a mono-fractal process. These observations cannot be attributed unambiguously to a source effect. The results warrant the analysis of datasets in which attenuation and source effects can be distinguished.

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1 Introduction

The spatial and temporal complexity of the earthquake source is strongly affected by spatial heterogeneities of fault strength and stress, and has a major impact on the amplitude and spatial variability of ground motions in the near-field. Coseismic slip inversions of seismological and geodetic data show notorious spatial heterogeneity. Self-similarity arguments suggest that this complexity extends over a broad range of length scales. However, due to large uncertainties in the crustal structure at fine scales, kinematic source inversions are limited to frequencies lower than 1Hz and are poorly resolved at length scales shorter than several kilometers. This limits their usefulness in constraining quantitative aspects of fault mechanics such as the spatial distribution of strength and initial stress. To retrieve fine scale information from currently available strong motion data requires the definition of appropriate high-frequency seismogram attributes, beyond the wiggle-by-wiggle waveform analysis. For that purpose, the current project [Ampuero, 2008] proposed a novel application of multiscale signal analysis techniques to identify and quantify signatures of spatio-temporal rupture complexity in the high-frequency band of strong motion recordings, specifically the temporal distribution and singularity exponent of strong phases defined as signal singularities. We also proposed to study the relation between these high-frequency attributes and earthquake physics, specifically the spatial distribution and singular character of initial fault stress concentrations.

2 The Wavelet Transform Modulus Maxima

We define strong phases in accelerograms as discrete occurrences of temporal singularities. A signal $g(t)$ with local singular behavior at t_o can be locally expanded as the superposition of a smooth polynomial and a power law singularity:

$$g\left(t\right) \approx P_n\left(t - t_o\right) + O\left(\left(t - t_o\right)^h\right) \tag{1}$$

where h is the local Hölder exponent characterizing the sharpness of the singularity at t_o, and $P_n\left(t - t_o\right)$ is a polynomial of degree $n < h$. Signal processing techniques to detect and analyze isolated singularities or to quantify statistical properties of multiple singularities in a signal are based on the Wavelet Transform Modulus Maxima [Arneodo, 1996]. These multiscale techniques are well developed and have been applied in a variety of fields but have not been applied before to analyze strong motion data and earthquake source processes.

The continuous wavelet transform W of a signal $g(t)$ is defined as

$$W\left(s, \tau\right) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} g\left(t\right) \Psi\left(\frac{t - \tau}{s}\right) dt, \tag{2}$$

where s is the temporal scale, τ is a temporal translation and $\Psi\left(t\right)$ is the analyzing wavelet or mother function. In contrast to the short windowed Fourier Transform, the Wavelet Transform uses the finite support of the analyzing wavelet at different scales to detect localized variations both in time and frequency without loss of resolution.

The Wavelet Transform Modulus Maxima (WTMM) are the local maxima of the modulus of W considered as a function of τ for each scale s. The set of modulus maxima points, M_s, at scale s is

$$\mathrm{M}_s = \left\{\tau_o \quad \text{such that} \quad \frac{\partial |W|}{\partial \tau}\left(s, \tau_o\right) = 0\right\} \tag{3}$$

WTMM lines are curves in the scale-time space (s, τ) that connect WTMM points across scales. The total set of WTMM lines is called the *skeleton* of the signal. Each maxima line is associated to the occurrence of a singularity in the signal. In the limit of small scales a maxima line converges to the time of occurrence of the singularity and the wavelet modulus along the maxima line behaves as

$$|W(s, \tau_o)| \sim A\ s^h \tag{4}$$

This provides a means to detect singularities and to extract their local Hölder exponent h. A power law fit to the wavelet modulus along maxima lines based on Equation (4) produces stable exponents for the synthetic data sets and strong motion recordings we have analyzed.

When the temporal density of singularities is high, the maxima lines interfere and it is impractical to analyze each singularity individually. Nevertheless, statistical properties of the singularities can be extracted

[Arneodo, 1996], in particular the *singularity spectrum*, $D(h)$, defined as the fractal dimension of the set of occurrence times of all singularities of exponent h. This global measure is based on an approach that borrows concepts from thermodynamics and statistical physics, and involves the computation of a partition function over maxima lines and the characterization of its power law behavior in the limit of small scales. This method was described in more detail by Ampuero [2008]. The singularity spectrum characterizes multi-fractal signals that contain a range of singularity exponents h. In contrast, a mono-fractal signal has a single exponent, so its singularity spectrum is very narrow.

The presence of noise in the signal generates spurious WTMM of small amplitudes. To retain the subset MB_s of strongest WTMM at scale s we set a threshold relative to the largest WTMM at the given scale:

$$\mathrm{MB}_s = \{\tau_o \in \mathrm{M}_s \text{ such that } |W(s,\tau_o)| \geq \max_{\tau}(|W(s,\tau)|)/B(s)\} \tag{5}$$

where $B(s)$ is a scale dependent relative threshold. If the signal-to-noise ratio is high, as in the case of the strong motion data, a constant B-value is sufficient. We typically worked with $1 < B < 5$. Our results on the exponent do not depend on the choice of B.

The wavelet scale s is related to the Fourier period τ_s, a concept more usual to seismologists, by $\tau_s = s\,\tau_m/(1\,\text{sec})$, where τ_m is the central period of the Fourier spectrum of the mother wavelet of scale $s = 1$ sec. Here we adopted the complex second derivative of a Gaussian as the mother wavelet, for which $\tau_m = 2.5$ sec. For convenience we present W as a function of central period τ_s instead of scale s.

3 Results

We present results of the WTMM analysis of synthetic waveforms from dynamic rupture simulations and of strong motion recordings of the 2008 Chino Hills, 2004 Parkfield and 1999 Chi-Chi (Taiwan) earthquakes.

3.1 Analysis of Dynamic Rupture Simulations

We performed 2-D in-plane dynamic rupture simulations using a boundary integral equation method in spectral form with a slip weakening friction law [Cochard and Rice, 2000; Rubin and Ampuero, 2007]. The initial stresses contained a singularity of the form $\sigma \sim |x|^{\nu}$, where x is the along-strike position and $\nu < 0$ is the stress singularity exponent (Figure 1-b). Ruptures were initiated near one end of the fault and propagated mostly unilaterally (Figure 1-a). The interaction between the rupture front and the stress singularity radiates a singular wave phase that can be visually identified in the seismic potency acceleration (Figure 1-c at $t = 0.3$ sec), a quantity proportional to far-field velocity. From a set of simulations with stress exponents ν ranging from -0.6 to 0 the singular exponent μ of the radiated strong phase, measured by the WTMM technique on the seismic potency acceleration (Figure 1-d), is virtually equal to the singularity exponent of the initial stress [Ruiz-Paredes et al., 2009] (Figure 1-e). This suggested a potential for inferring the character and distribution of stress singularities from strong motion recordings.

3.2 Analysis of Strong Motion Recordings

Strong motion data for the 2008 Chino Hills and the 2004 Parkfield earthquakes were downloaded from the Center for Engineering Strong Motion Data (CESMD) and the California Integrated Seismic Network (CISN), and data for the 1999 Chi-Chi (Taiwan) earthquake from the Consortium of Organizations for Strong Motion Observation Systems (COSMOS). An example of the WTMM applied to a recording of the 2004 Parkfield is shown in Figure (2). The strongest energy appears between 2 and 8 Hz. Localized energy released around 6 seconds suggests the presence of a secondary source consistent with Allmann and Shearer [2007]. At a finer scale, the main event itself is composed of at least two sub-events (Figure 2-b).

Figure (2-d) shows the WTMM along the strongest maxima lines for one strong motion record of the Parkfield 2004 earthquake. The behavior at frequencies above 4 Hz is well described by a single exponent $h \approx 2.1$. This is confirmed by the thermodynamic approach [Arneodo, 1996; Ampuero, 2008]: the singularity spectrum is very narrow (Figure 2-right). The h exponents show variability across stations but no systematic dependence on epicentral distance and azimuth (Figure 3). We observed an averaged $\bar{h} \sim 2.2, 1.9$ and 1.8 for Parkfield, Chino Hills [Ruiz-Paredes et al., 2009] and Chi-Chi earthquakes respectively, but the inter-event variability is smaller than the intra-event variability. These observations suggest that a mono-fractal process with $h \approx 2$ is controlling the high frequency wavefield.

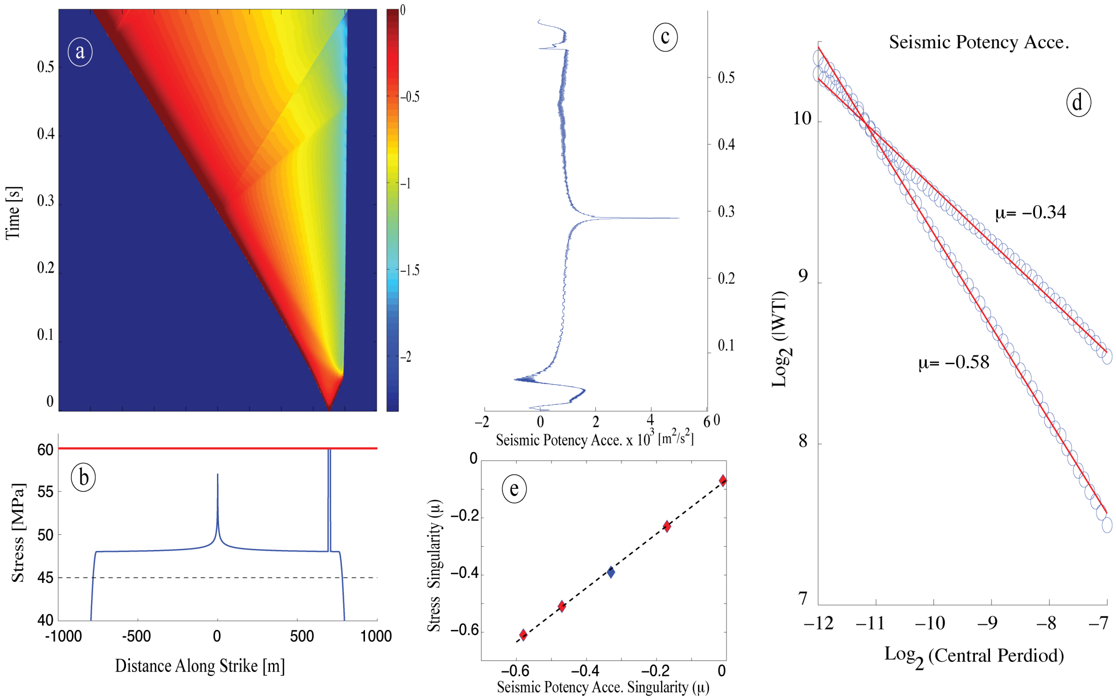

Figure 1: a) Space-time evolution of a dynamic rupture simulation (logarithm of the slip rate). b) Heterogeneous initial stress distribution, containing a singularity $\sigma \sim |x|^{\nu}$ with $\nu = -0.34$. c) Seismic potency acceleration. d) WTMM of the seismic potency acceleration along maxima lines and singularity exponent μ obtained by power law regression based on Equation 4 (red solid line) for two cases: $\mu = -0.34$ and -0.58. e) Singularity exponent of seismic potency acceleration (μ) as a function of exponent ν of the initial stress singularity.

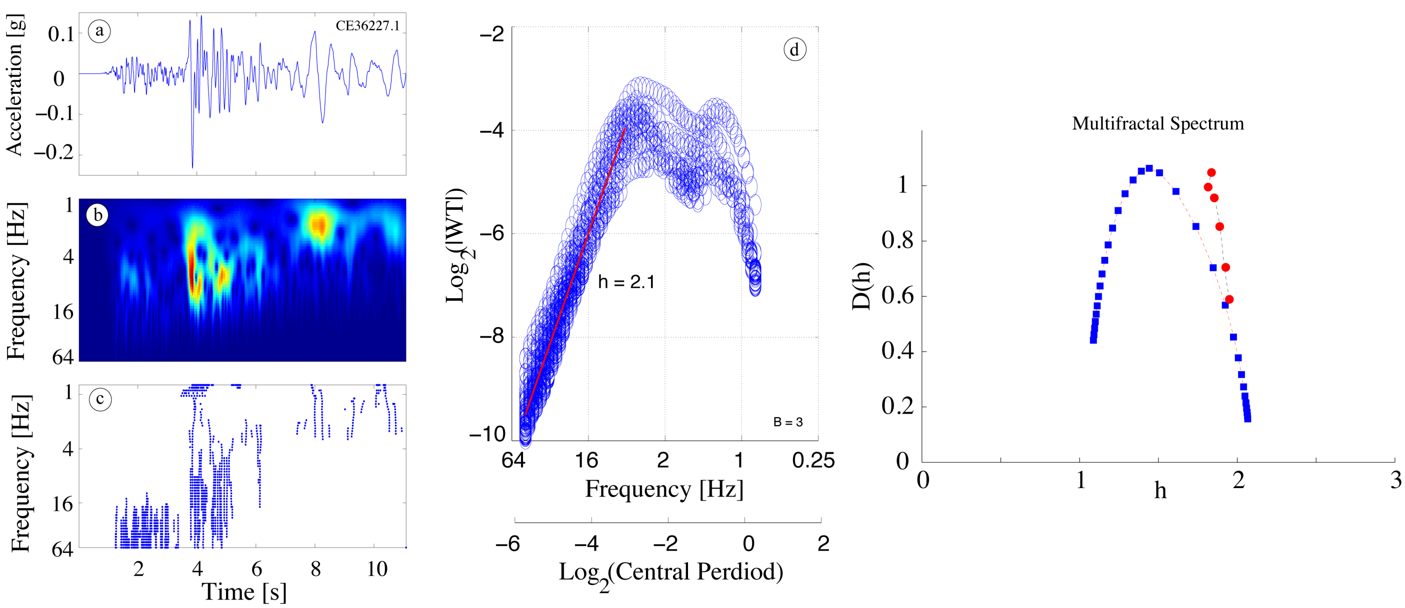

Figure 2: a) A strong motion record from the 2004 Parkfield earthquake. b) Its Wavelet Transform. c) Its skeleton (WTMM lines) thresholded with $B = 3$. d) WTMM along the strongest maxima lines. The singularity exponent $h = 2.1$ is obtained by using Equation 4 (red solid line). Right: The singularity spectrum of a strong motion recording (red circles) is very narrow compared to the singularity spectrum of a multi-fractal signal (blue squares).

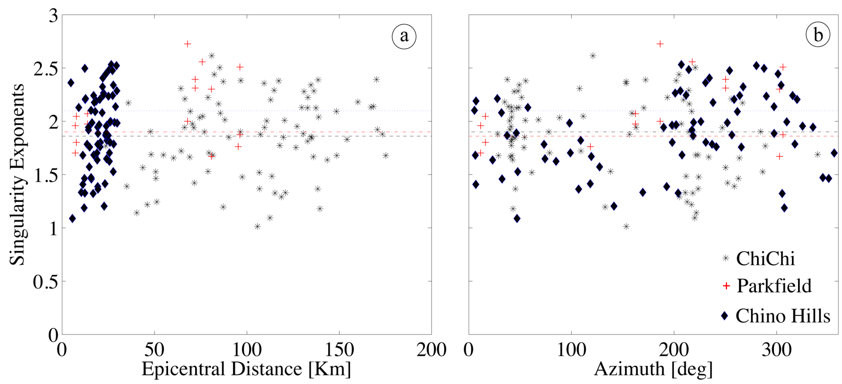

Figure 3: Singularity exponent for strong motion recordings at stations in the Parkfield motivated by previous work on the analysis of the Parkfield strong motion data [Allmann and Shearer, 2007]. We limited strong motion recordings to stations up to 40Km epicentral distance in the case of the Chino Hills and up to 200Km for the 1999 ChiChi earthquake. a) h as a function of the epicentral distance and b) azimuth.

If this behavior is attributed to a source effect, in the context of our dynamic rupture model an acceleration waveform with $h = 2$ corresponds to initial fault stresses with positive odd exponent, $h_\sigma \sim 3$, inconsistent with a stress singularity. The high frequency range we considered, beyond the so-called f_{max}, is more likely dominated by attenuation. Synthetic tests including attenuation with constant Q show a significant curvature of the WTMM along maxima lines as a function of frequency that is inconsistent with the power-law decay highlighted by the WTMM technique. The exponent $h = 2$ is equivalent to f^{-5} spectral decay in displacement seismograms. A source effect related to smoothing at the scale of the process zone would lead to f^{-3} in slip-weakening models and to f^{-4} in rate-and-state friction models.

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