

**EFFICIENT APPROACH TO VECTOR-VALUED PROBABILISTIC
SEISMIC HAZARD ANALYSIS OF MULTIPLE CORRELATED GROUND
MOTION PARAMETERS**

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EXECUTIVE SUMMARY

One of the key building blocks of performance-based earthquake engineering (PBEE), which has become a commonplace tool in the industry for assessing and designing the seismic performance of buildings and other structures, is the evaluation of the structural response to different level of earthquake ground shaking.

To increase the accuracy of the response prediction engineers have recently started to take full advantage of the computational capability of modern computers by building more realistic 2-Dimensional and 3-Dimensional numerical models. The response of these complicated models is better gauged by monitoring multiple response measures, such as maximum interstory drifts in both longitudinal and transverse directions and maximum story acceleration along the height of the structure. To predict the values of such response measures multiple intensity parameters of the ground motion in both horizontal directions of the ground motion (and sometimes vertical) are necessary. Hence, many researchers have recognized the need for estimating the *joint* site hazard of the predictor ground motion parameters of interest, which is a complicated task since such parameters are correlated.

The seminal methodology to evaluate joint seismic hazard, which was co-authored by one of the PIs of this proposal, was introduced in 1998 and was called Vector Probabilistic Seismic Hazard Analysis (VPSHA). Despite the long availability of the methodology, VPSHA has been rarely used in practice mainly because the two VPSHA computer programs in existence (Bazzurro, 1998; Thio, 2003) have scarce documentation, are limited to two parameters, and are unable to identify the scenarios that control the joint hazard via the so-called “disaggregation” procedure. These shortcomings have severely hindered their usage. To circumvent the problem, engineers have used scalar PSHA for single ground motion parameters that are combination of multiple ones (e.g., the geometric mean of the spectral accelerations at the first period of vibration in the two main horizontal directions of the building).

To help promoting the use of VPSHA, this study proposes an alternative, approximate method for computing the joint hazard that can be implemented with *any standard scalar* PSHA software. The scalar PSHA code of choice needs only be modified to provide the disaggregation of the *scalar* site hazard for all the parameters considered in the ground motion prediction equations used in the hazard evaluation (e.g., magnitude, distance, rupture mechanism, etc.). In addition, this method requires the covariance matrix of the ground motion parameters for which the joint hazard is sought, which for spectral quantities has become recently available in the literature. This indirect approach to VPSHA is computationally efficient, delivers the disaggregation of the joint hazard, and can accommodate up to four or five ground motion parameters with the current computer limitations without significant loss of accuracy. This study provides the methodology and an illustrative example for the evaluation of the joint hazard for three spectral acceleration quantities at a San Francisco site.

1 BACKGROUND AND MOTIVATION

After many years of incubation in the academia, performance-based earthquake engineering (PBEE) has become commonplace in the industry for assessing the response of buildings and other structures to seismic loading. Studies based on PBEE are used by a variety of stakeholders. Building owners, for example, use the results of these studies to decide whether to retrofit buildings and/or whether to and how much earthquake insurance to buy. Lending institutions utilize them to decide whether or not to make or deny a loan. Insurers use them to decide whether to underwrite earthquake insurance for a structure at a particular site and to determine an appropriate premium. Structural engineers use it to design structural components that withstand forces and control displacements induced by the target design ground motion with a margin of safety commensurate with that of good-performing code-compliant structures. A recent and important example where PBEE has been instrumental is the design of buildings taller than the maximum height of 240ft allowed by the building code in the main cities along the West Coast (e.g., Maffei and Yuen, 2007). Regardless of the application, it is critical that estimates of the likelihood of structure's response exceeding levels of severity, ranging from onset of damage to incipient collapse, be as accurate as reasonably possible.

The three essential aspects of PBEE are a) the evaluation of seismic hazard; b) the assessment of the response of the structure for any given level of ground shaking; and c) the computation of monetary losses to repair the structure in any state of damage predicted by the response analysis. The computation of collapse probability, in particular, is essential for quantifying the risk of fatalities. To increase the accuracy of the structure's response prediction engineers have recently started to take full advantage of the computational capability of modern computers by building more realistic 2-Dimensional (2D) and 3-Dimensional (3D) numerical models (e.g., see Figure 1). These computer models are subjected to many different ground motion accelerograms of different intensity to assess their performance. Statistical tools are usually utilized to provide functional relationships between the intensity parameters of the ground shaking and the measures of the response that are associated with undesirable levels of performance (e.g., life safety or collapse).

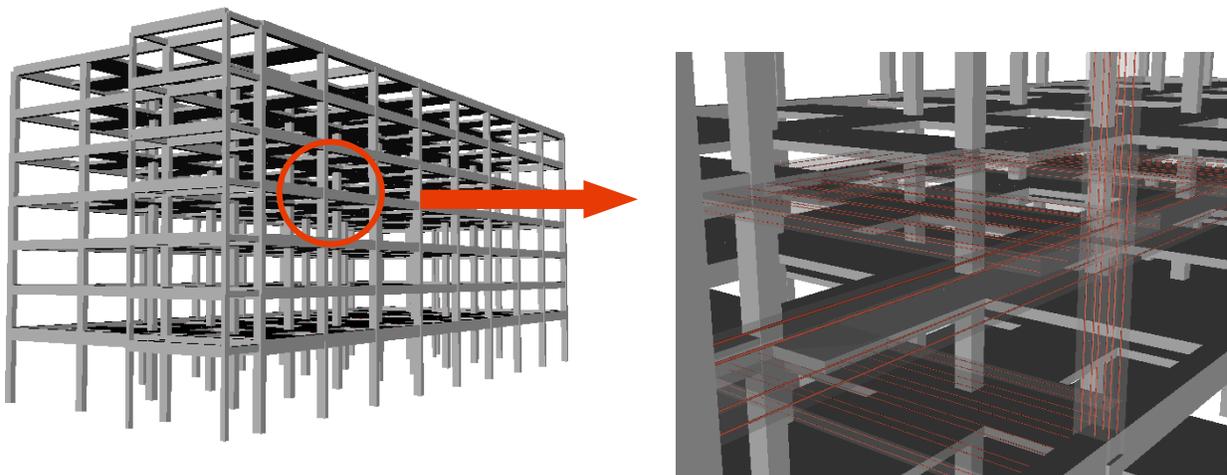


Figure 1. Three-Dimensional model of a 7-story reinforced concrete frame (left) with details of rebars in slabs and columns (right).

The complex response of these realistic models, however, is more accurately gauged by monitoring multiple response measures, such as maximum interstory drifts in both longitudinal direction, δ_l , and transverse direction, δ_r , and maximum absolute acceleration at each story. The maximum values of these measures are good indicators of the physical damage (e.g., collapse) expected in the structure and of the damage caused to contents. It is intuitive to understand that estimates of the maximum values of these

response measures are better predicted by a pool of intensity parameters of the ground motion in both horizontal directions (and sometimes vertical) rather than by a single parameter (e.g., horizontal Peak Ground Acceleration, PGA) as done in the past. For example a good predictor of δ_L may be the spectral acceleration at the first period of vibration in the structure's longitudinal direction $S_a(T_L)$ and, similarly, δ_T may be well predicted by $S_a(T_T)$, that is the spectral acceleration in the structure's transverse direction. The collapse of the building, however, is more likely to happen when *both* δ_L and δ_T and, therefore, $S_a(T_L)$ and $S_a(T_T)$ are large rather than when either one is large.

It is clear that a quantification of the *joint* hazard of both $S_a(T_L)$ and $S_a(T_T)$ is a must in such applications. Conventional scalar PSHA, however, computes the mean rate of occurrence (or exceedance) of $S_a(T_L)$ and $S_a(T_T)$ and other parameters separately, not jointly. How can an engineer combine this information to evaluate the likelihood that a building may collapse? The building response in the two directions and the parameters $S_a(T_L)$ and $S_a(T_T)$ —as well as others that may be utilized—are, in general, correlated and this aspect makes the computation of the joint site hazard not straightforward. The desired improvement in the accuracy of structure's response, therefore, comes at a price.

The seminal methodology to evaluate joint seismic hazard, which was co-authored by one of the PIs of this study, was introduced in 1998 (Bazzurro, 1998; Bazzurro and Cornell, 2001 and 2002) and was called Vector-valued Probabilistic Seismic Hazard Analysis (VPSHA). Despite its long availability, the VPSHA approach has been rarely used in practice mainly because the two VPSHA computer programs in existence (Bazzurro, 1998; Thio, 2003) have scarce documentation, are limited to two parameters, and are unable to identify the scenarios that control the joint hazard via the so-called “disaggregation” procedure. Disaggregation is used in practical applications, among other things, to identify the earthquake scenarios that drive the hazard and to select appropriate ground motion records for structural response estimation that are consistent with such scenarios. These shortcomings have severely hindered the usage of VPSHA. To partially circumvent the problem, engineers have used scalar PSHA for single ground motion parameters that are combination of multiple ones (e.g., the geometric mean of the spectral accelerations at the first period of vibration in the two main horizontal directions of the building) (e.g., Cordova *et al.*, 2001, Luco *et al.*, 2005a; Luco and Cornell, 2007). As is intuitively obvious, such a workaround is, however, less effective than considering single ground motion parameters separately in the response prediction and accounting for the *joint* hazard of such a pool of parameters when computing the risk.

To help promoting the use of VPSHA, this study develops and implements a methodology that allows the computation of the joint hazard using results from any standard, *scalar* PSHA software. This indirect approach to VPSHA is computationally efficient, delivers the disaggregation of the joint hazard, and accommodates up to four or five random variables (RVs) with the current computer limitations without significant loss of accuracy. A computer application that can be coupled with the results from any scalar PSHA code is provided to illustrate its use. The application of VPSHA will be illustrated in the subsequent section.

Note that the VPSHA methodology explained here could be easily coupled, for example, with the hazard and disaggregation results from USGS scalar hazard maps or from OpenSHA to produce the *joint* hazard estimates of different combinations of ground motion parameters at any site. It is possible to envision an interactive tool on the USGS website that would enable a user to input the coordinates (or ZIP code) of a building site, specify the periods of the first modes of vibration along the principal axes of the building (i.e., T_L and T_T), identify the direction of the building principal axes with respect to North, and obtain plots and tables with joint rates of exceedance (or “equaling”) of different pairs of values of $S_a(T_L)$ and $S_a(T_T)$. Some of the tables could, for example, list all the $S_a(T_L)$ and $S_a(T_T)$ pairs whose (joint) exceedance rate is equal to some common target value, such as 10% in 50 years. With the additional piece of information of the target exceedance rate (e.g., 10% in 50 years) and a pair of $S_a(T_L)$ and $S_a(T_T)$ values taken from one of the provided tables, the user could also ask for a plot of the disaggregated magnitude and distance pairs of the scenarios that most contribute to the exceedance of those values at the site. Such a tool based on

VPSHA could be very valuable for both design and performance evaluation of buildings and other structures.

2 SCOPE OF WORK

The relevance of this report to reducing losses from earthquakes in the U.S. is direct and relates to a more comprehensive evaluation of the seismic hazard at structure's site. Until now most seismic risk analyses were conducted by coupling hazard and response analyses via a single parameter as a pinch point between hazard and structure's response estimation. Such a link has been traditionally PGA or spectral acceleration at a given period in one of the two horizontal directions of the ground motion. Many researchers have shown (see, among others, Bazzurro, 1998; Luco and Cornell, 2000; Bazzurro and Cornell, 2002; Bazzurro and Luco, 2004 and 2005; Luco *et al.*, 2005a and 2005b) that using more than one ground motion parameter substantially improves the accuracy of the response prediction especially of mid-rise and high-rise buildings. Multiple ground motion parameters can theoretically be included in the hazard using the VPSHA approach but its practical implementation has had obstacles that this study intends to remove. The systematic use of VPSHA will undoubtedly increase the accuracy of seismic risk analyses of existing buildings and refine the design of new ones. An improved structural risk assessment of single structures is the first step in mitigating seismic risk for the inventory of buildings at large.

2.1 Objectives

The objective of this study is twofold:

1. Exploring the applications of a novel, very efficient, and versatile approach to VPSHA that is based on the appropriately utilizing standard results from scalar PSHA analyses and the variance-covariance matrix of the ground motion parameters for which the joint mean rate density (MRD) or equivalently, the joint mean annual rate (MAR) are sought. The implementation of this method removes the main obstacle that has prevented the widespread use of multiple response measures in response assessment of 2D and 3D representations of structures. This tool computes the joint hazard of multiple ground motion parameters to be utilized as predictors of structural response severity. Note that a USGS-sponsored collaborative study of the PIs of this study and of Dr. Nicolas Luco of USGS (Award No. 06HQPA001; Bazzurro *et al.*, 2007) focused on how PSHA results and, preferably, VPSHA results should be used in conjunction with structural response assessment techniques to correctly and accurately compute the risk of an undesirable structural performance (e.g., exceedance of life-safety conditions).
2. Developing a computer code to for the public domain to illustrate the procedure of converting a scalar PSHA code into a VPSHA one. This procedure is, in fact, completely general and can be repeated with other scalar PSHA codes.

3 VPSHA METHODOLOGY

The VPSHA approach implemented here is based on the tenable assumption of the joint lognormality of the correlated ground motion parameters (Baker and Jayaram, 2007) conditional on the characteristics of the causative event. . Recall that the same assumption has been exploited for essentially any ground motion prediction equation in existence. The joint MRD (for definition and details, see Bazzurro and Cornell, 2002) or, alternatively and equivalently, the Mean Annual Rate (MAR) of occurrence of a pool of ground motion parameters can be computed with the knowledge of the following input:

1. *Site-specific seismic hazard curves for all the ground motion parameters* (see Figure 2a).
The vector of ground motion parameter is denoted here as \mathbf{S} , where the bold character indicates that the quantity is a vector. This vector could include, for example, three parameters: the spectral

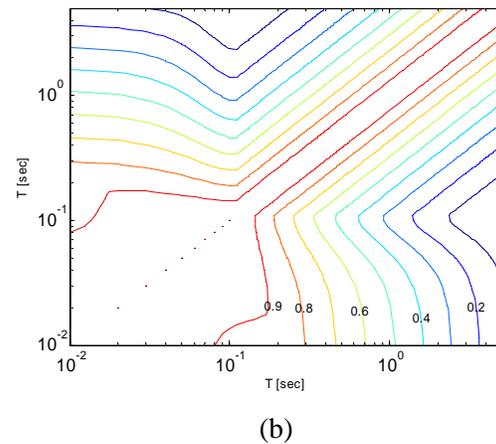
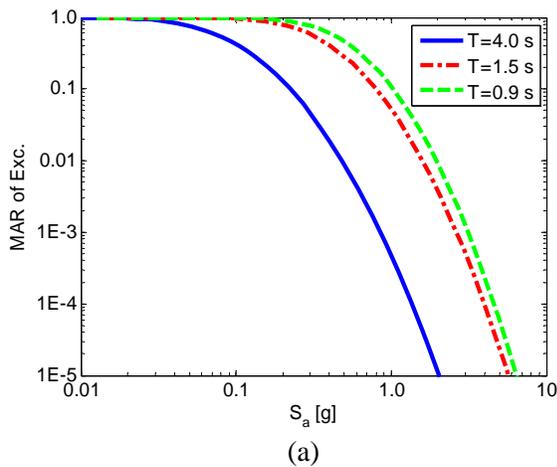
acceleration at two periods in one of the horizontal directions and at the first period in the orthogonal horizontal directions. These periods could correspond to the first- and second-mode of vibrations of a building in the longitudinal directions and to the first mode in the transverse direction. These three hazard curves can be obtained with any standard PSHA code. The simple example here is for a scenario event with moment magnitude (M) 7.1, source-to-site distance (R) of 11 km, and strike slip rupture mechanism. The mean rate of occurrence is assumed to be unity for illustration purpose. The structural periods of interest are 4.0, 1.5, and 0.9 sec.

2. *The pair-wise correlation matrix of all the ground motion parameters* (see Figure 2b). Inoue (1990) and Baker and Cornell (2006) have empirically derived the correlation structure for spectral accelerations with different periods and accelerogram orientations. The development of a correlation matrix for spectral accelerations for fault normal and fault parallel conditions is virtually identical (Baker, 2007). For this example, the pair-wise correlation matrix for the three spectral accelerations is shown in Table 1 below:

Table 1. Pair-wise correlation matrix of S_a (in logarithmic space) for T_s of 0.9s, 1.5s, and 4.0s.

	$T = 4s$	$T = 1.5s$	$T = 0.9s$
$T = 4s$	1.000	0.649	0.481
$T = 1.5s$	*	1.000	0.814
$T = 0.9s$	<u>sym</u>	*	1.000

3. *The disaggregation results from scalar PSHA* (see Figure 2c). The joint distributions of all the basic variables, \mathbf{X} , used in the ground motion prediction equation of choice (i.e., M , R , and all the other variables – such as the style of faulting, the directivity parameters, the distance to the top of the co-seismic rupture, and dip angle, that are needed to compute the level of ground motion for every earthquake rupture) conditional on the value of one or more ground motion parameters is a straightforward extension of the disaggregation results (e.g., based on M and R only) routinely available from standard scalar PSHA codes. As shown below, the necessary modifications are conceptually simple and involve only disaggregation of the site hazard in terms of additional RVs beyond the magnitude, M , the source-to-site-distance, R , etc. as done in the past.



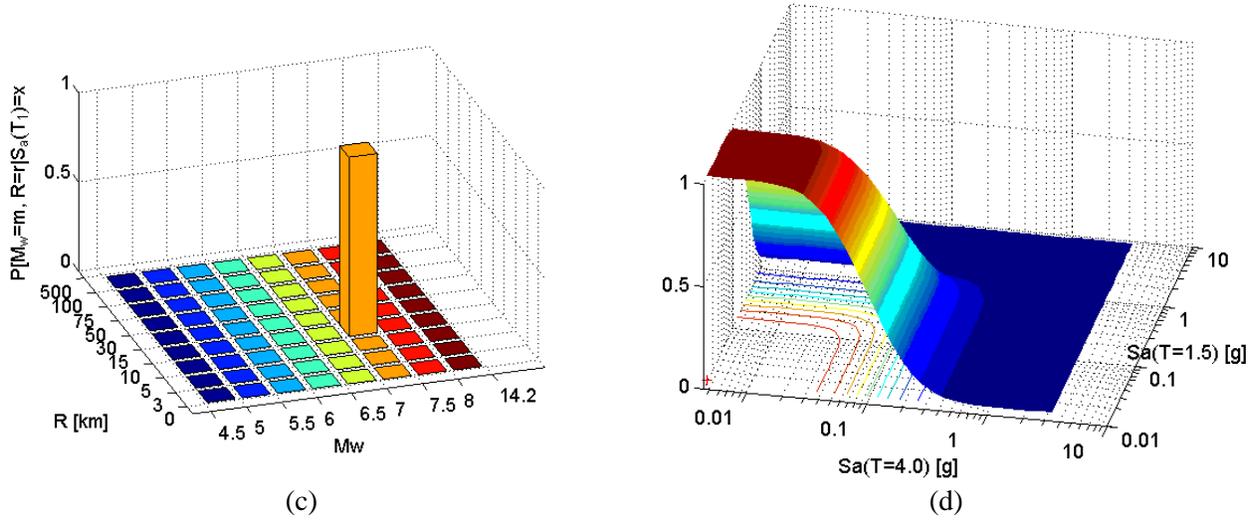


Figure 2. (a) Hazard curves for S_{a1} , S_{a2} and S_{a3} , here for the periods of 0.9s, 1.5s, and 4s; (b) correlation structure of spectral accelerations (in logarithmic space) at different periods in a random horizontal component of a ground motion, (c) disaggregation of $X|S_{a1}$ where $X=[M,R]$ here; and (d) a graphical representation of the output representing the MAR of exceeding a joint of $[S_{a1}; S_{a2}]$ simultaneously for the characteristic event scenario.

For illustration purposes Figure 2a through 2c show a graphical representation of the input and Figure 2d shows the joint hazard output that can be obtained in a 2D case. Note that the two other 2D representations of the joint MAR of $[S_{a1}; S_{a3}]$ and $[S_{a2}; S_{a3}]$ similar to that displayed in Figure 2d have been omitted for conciseness. Again, one of the distinct appealing qualities of this methodology is that it can be written as a standalone post-processor routine of a standard PSHA code. The accuracy of the results, however, could potentially be jeopardized by the selection of too wide bins during the discretization of the domain of each ground motion parameter (e.g., M and R). The accuracy issue, which is part of the parallel project supported by USGS 2008¹, will only be briefly addressed here.

To be concise, we present below some details of the methodology for the case of three spectral accelerations. This approach, which requires some basic matrix algebra, is scalable to a larger number of RVs and can include any other ground motion parameters (e.g., duration, near-source forward-directivity pulse period) for which correlation structure and prediction equations are available. For simplicity, here we will also treat the RVs as discrete rather than continuous quantities.

Let us denote with $\mathcal{S}=[S_{a1}; S_{a2}; S_{a3}]$ the vector of RVs for which we seek to obtain the joint hazard and with $MAR[S_{a1}; S_{a2}; S_{a3}] = MAR_{S_{a1}; S_{a2}; S_{a3}}(a_1; a_2; a_3)$ the mean annual rate of three spectral acceleration quantities S_{a1} , S_{a2} , and S_{a3} in the neighborhood of any combination of three values a_1 , a_2 , and a_3 , respectively. To be precise, note that $S_{a1}; S_{a2}; S_{a3}$ represent the natural logarithm of the spectral accelerations but the logarithm has been dropped here to avoid heavy notations. $MAR[S_{a1}; S_{a2}; S_{a3}]$ could, for example, denote the MAR that the spectral acceleration at the first and second modes of vibration in the longitudinal direction of a building and the spectral acceleration of the first mode in the transverse direction of a building assume values in the neighborhood of, say, 1.0g, 1.5g, and 0.8g, respectively. In an application, these spectral acceleration values could be related to the onset of an important structural limit state (e.g., collapse) found using a statistical analysis of the response of the structure to many ground motion records. Then, using simple probability theory and the theorem of total probability, one can write the following:

$$MAR[S_{a1}; S_{a2}; S_{a3}] = P[S_{a1} | S_{a2}; S_{a3}] \cdot P[S_{a2} | S_{a3}] \cdot MAR[S_{a3}] \quad (\text{Eq. 1})$$

where

¹ A communication from USGS has just informed us that funding for this already approved project did not materialize. Therefore, further investigation on VPSHA will be postponed.

- $P[S_{a1} | S_{a2}; S_{a3}] = \sum_{\mathbf{X}} P[S_{a1} | S_{a2}; S_{a3}; \mathbf{X}] \cdot P[\mathbf{X} | S_{a2}; S_{a3}]$ (Eq. 2)

is the conditional distribution of S_{a1} given S_{a2} and S_{a3} . This term can be numerically computed by conditioning it on the pool of variables, \mathbf{X} , in PSHA that appear in the selected ground motion prediction equation and integrating over all possible values of \mathbf{X} . Under the tenable assumption of joint lognormality of \mathbf{S} mentioned before, for every possible value of \mathbf{X} the quantity $P[S_{a1} | S_{a2}; S_{a3}; \mathbf{X}]$ can be computed simply with the knowledge of the variance-covariance matrix of S_{a1} , S_{a2} , and S_{a3} (e.g., Baker and Cornell, 2006) and the ground motion prediction equation of choice (e.g., Abrahamson and Silva, 1997). More mathematical details are provided below. $P[\mathbf{X} | S_{a2}; S_{a3}]$, which is the probability of \mathbf{X} conditional on the values of S_{a2} and S_{a3} , can instead be obtained via disaggregation and Bayes theorem as follows:

$$P[\mathbf{X} | S_{a2}; S_{a3}] = \frac{P[\mathbf{X}, S_{a2} | S_{a3}]}{\sum_{\mathbf{X}} P[S_{a2} | S_{a3}; \mathbf{X}] \cdot P[\mathbf{X} | S_{a3}]} = \frac{P[S_{a2} | S_{a3}; \mathbf{X}] \cdot P[\mathbf{X} | S_{a3}]}{P[S_{a2} | S_{a3}]} \quad (\text{Eq. 3})$$

where

- $P[\mathbf{X} | S_{a3}]$ can be derived using conventional scalar PSHA disaggregation.
- $P[S_{a2} | S_{a3}; \mathbf{X}]$, as for a similar term above, can be computed with only the knowledge of the variance-covariance matrix of S_{a1} , S_{a2} , and S_{a3} , and the attenuation relationship of choice.
- $P[S_{a2} | S_{a3}] = \sum_{\mathbf{X}} P[S_{a2} | S_{a3}; \mathbf{X}] \cdot P[\mathbf{X} | S_{a3}]$ can be evaluated as explained above.
- $MAR[S_{a3}]$ is the absolute value of the differential of the conventional seismic hazard curve for the scalar quantity S_{a3} at the site.

In more detail, the above conditional terms (i.e., $P[S_{a1} | S_{a2}; S_{a3}; \mathbf{X}]$ and $P[S_{a2} | S_{a3}; \mathbf{X}]$) can be obtained using the multivariate normal distribution theorem. More in general, if we call $\mathbf{S} = [S_{a1}, S_{a2}, \dots, S_{an}]^T$ the vector of the natural logarithm of the random variables for which the joint hazard is sought, then \mathbf{S} is jointly normally distributed with a mean vector ($\boldsymbol{\mu}$) and variance-covariance matrix ($\boldsymbol{\Sigma}$), i.e., in mathematical terms $\mathbf{S} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. By partitioning \mathbf{S} into 2 vectors $\mathbf{S}_1 = [S_{a1}, S_{a2}, \dots, S_{ak}]^T$ and $\mathbf{S}_2 = [S_{ak+1}, S_{ak+2}, \dots, S_{an}]^T$ where \mathbf{S}_2 comprises the conditioning variables (in the example above $\mathbf{S}_1 = [S_{a1}]$ and $\mathbf{S}_2 = [S_{a2}, S_{a3}]$), one can write the following:

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \end{bmatrix} \sim N \left(\begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix} \right) \quad (\text{Eq. 4})$$

For jointly normal distribution, the conditional mean and conditional variance can be determined as $\mathbf{S}_2 | \{\mathbf{S}_1 = \mathbf{s}_1\} \sim N(\boldsymbol{\mu}_{2|1}, \boldsymbol{\Sigma}_{2|1})$ (Eq. 5)

$$\boldsymbol{\mu}_{2|1} = \boldsymbol{\mu}_2 + \boldsymbol{\Sigma}_{21} \boldsymbol{\Sigma}_{11}^{-1} (\mathbf{s}_1 - \boldsymbol{\mu}_1); \quad \boldsymbol{\Sigma}_{2|1} = \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21} \boldsymbol{\Sigma}_{11}^{-1} \boldsymbol{\Sigma}_{12} \quad (\text{Eq. 6})$$

These two parameters can be used with the conditional distribution mentioned above (i.e., $P[S_{a1} | S_{a2}; S_{a3}; \mathbf{X}]$ and $P[S_{a2} | S_{a3}; \mathbf{X}]$). This VPSHA methodology can be used to evaluate the joint hazard at a designated site and for the vectors containing up to 4 or 5 ground motion parameters. The joint

hazard can also be disaggregated on the vector parameters \mathbf{X} (for example, M and R). The same concept can also be applied to other ground motion prediction equations (e.g., Chiou and Youngs, 2008). Note that the conditional probability distribution $P[S_{a2} | S_{a1}; \mathbf{X}]$ in the VPSHA calculation (see Eqns. 2 and 3) should be also carried out with the same ground motion prediction equation used in scalar PSHA calculation.

3.1 Validation of the Proposed VPSHA framework

To confidently apply the proposed vector-valued probabilistic seismic hazard analysis, the methodology will be validated in this section for the characteristic event case. The accurate joint hazard information for a given M and R can simply be determined using the ground motion prediction equation and the pair-wise correlation of the selected pair spectral ordinates. The selected event for comparison is for M 7.1, R of 11 km, and strike-slip rupture mechanism. The site is characterized as a soil type. The mean annual rate of occurrence is assumed to be unity. The structural periods of interest are 4.0 and 1.5 sec. The correlation value between the two spectral ordinates is 0.649. Using this information, the theoretical joint probability density function can be generated using the jointly normal distribution formula (Benjamin and Cornell, 1970). The theoretical joint hazard density is shown in Figure 3a. The dashed-dotted lines in Figure 3a were derived using the theoretical formula while the solid ones using the proposed VPSHA. In Figure 3a, we can observe that the shape as well as the centroids of the two approaches provide consistent joint hazard results. Note that the contour levels of the theoretical and the proposed method do not coincide only because they are at different levels. If the levels had been chosen to be the same, the results from the two approaches would have been, for all practical purposes, identical.

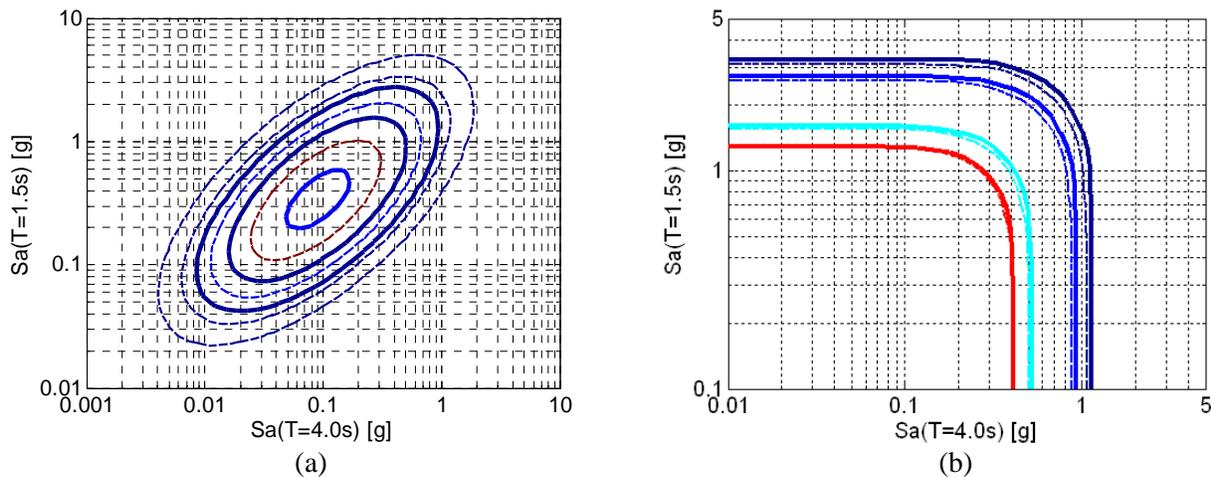


Figure 3. Comparisons of the joint hazards using the theoretical joint normal distribution (solid lines) and the proposed VPSHA framework in terms of (a) the mean rate of equaling and (b) the mean rate of exceeding.

The contour of the joint hazard in terms of MAR of exceeding pairs of $S_a(1.5s)$ and $S_a(4.0s)$ is illustrated in Figure 3b for the values of 0.02 (i.e., a mean return period, MRP, of 50 years) in red, 0.01 (100 years) in cyan, 0.001 (1,000 years) in blue, and 0.0004 (2,500 years) in black. As can be seen in Figure 3b, the proposed framework (solid lines) provides joint hazard results that are practically identical to those derived using the theoretical formula (thin dashed-dotted lines). Although the comparison performed here is for the characteristic event case, the conclusion drawn here is valid for a site with multiple faults generating events of different magnitude and rupture locations.

4 CASE STUDY

To illustrate the methodology explained in the previous section, a site located in downtown San Francisco is selected. The site and the location of the nearby faults are displayed in Figure 4a. The 5%-damped

elastic spectral accelerations considered are for the periods of 4.0s, 1.5s, and 0.9s. The scalar (or, equivalently in probabilistic jargon, marginal) hazard curves for these three S_a 's are shown in Figure 4b. For short we denote below $S_a(T_1=4.0s)$, $S_a(T_2=1.5s)$, $S_a(T_3=0.9s)$ as S_1 , S_2 , S_3 respectively. The pair-wise correlation matrix of these spectral accelerations is the same as the one shown in Table 1.

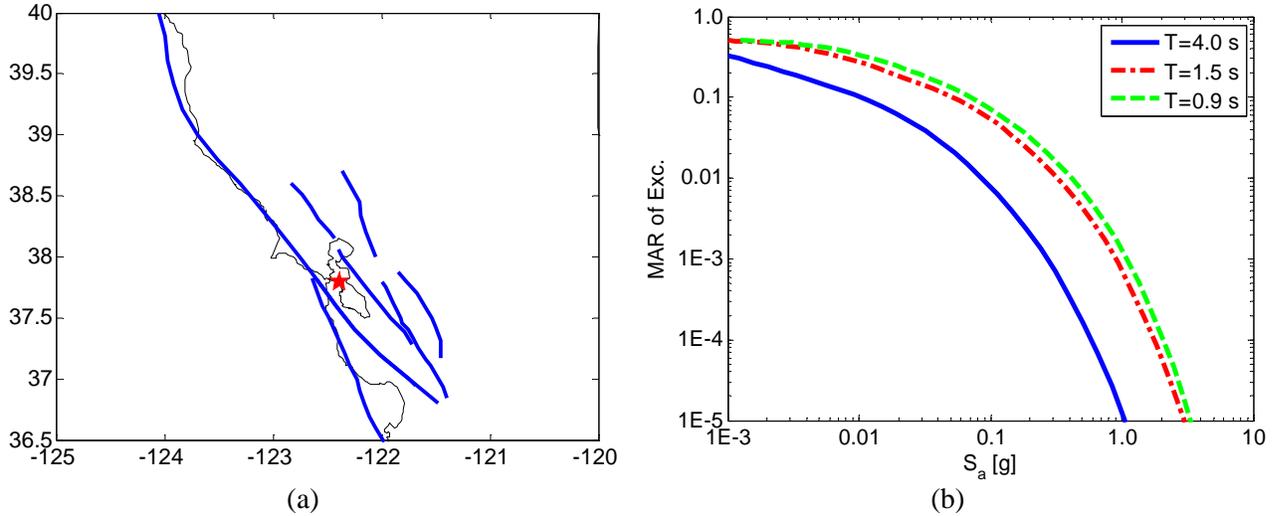


Figure 4. (a) Designated site in downtown San Francisco (red star) and the geologic faults considered in PSHA using a public-domain PSHA software (Abrahamson 2007), (b) the corresponding hazard curves for the oscillator periods of 4.0s (S_1), 1.5s (S_2), and 0.9s (S_3).

Following the methodology described in Section 3 along with the correlation matrix as well as the marginal hazard curves and its scalar disaggregation, the joint hazard curve for S_1 and S_2 can then be constructed. Figure 5a shows the joint hazard of S_1 and S_2 (denoted as $MAR_{S_1;S_2}$), here illustrated in terms of the MAR of “*equaling*” a pair value of S_1 and S_2 *simultaneously*. It should be noted that the spike shown at small S_1 and S_2 pairs in the MAR of equaling (Figure 5a) represents the rate of small and far away earthquakes that cause essentially no ground shaking at the site (strictly speaking, the rate of events that produce S_1 and S_2 values smaller than 0.0001 and 0.001, respectively). Similarly, the $MAR_{S_1;S_2}$ of *exceeding* a specified value of S_1 and S_2 *simultaneously* can be seen in Figure 5b. The $MAR_{S_1;S_2}$ of exceeding the zero values of S_1 and S_2 is simply equal to the sum of the mean recurrence rate of all the events (in this case, above the threshold M of 5.0 used in the PSHA calculations) generated by all the faults considered (Figure 4a).

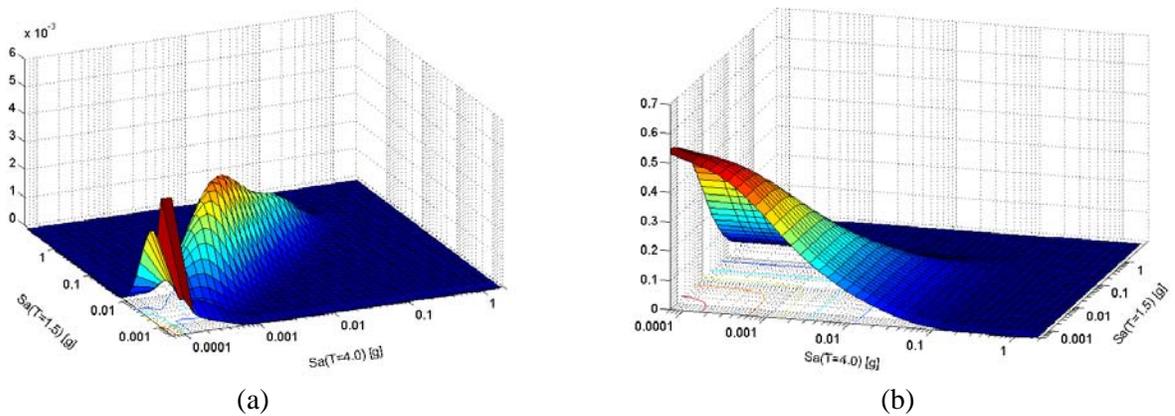
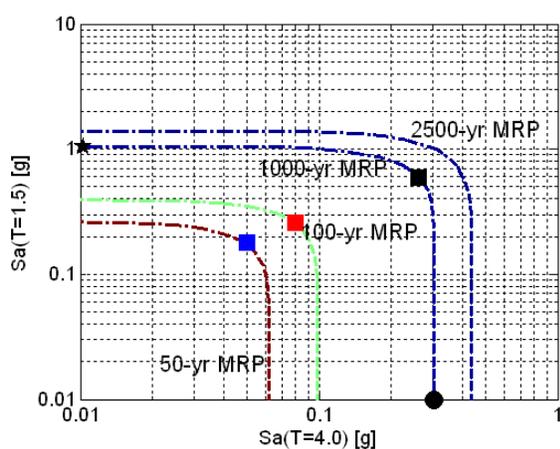


Figure 5. Joint hazard curve for S_1 and S_2 ($MAR_{S_1;S_2}$): (a) the MAR of *equaling* and (b) the MAR of *exceeding* a pair value of S_1 and S_2 simultaneously.

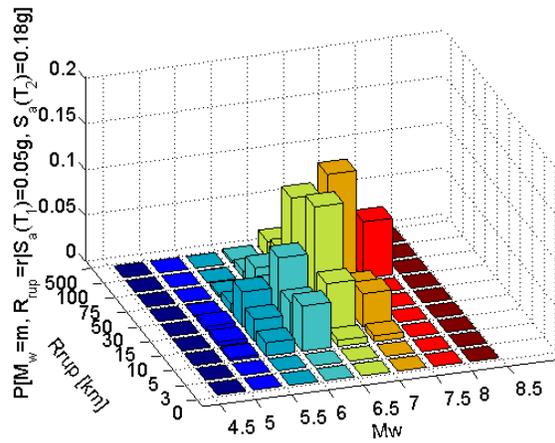
Furthermore, with the disaggregation of the joint hazard engineers can determine, for example, what the likelihood is that a specified pair of values of S_1 and S_2 *simultaneously* observed at a site may be caused by earthquake scenarios with different, M and R values. The illustration below demonstrates the M and R disaggregation results of the joint hazard associated with the $MAR_{S_1:S_2}$ of exceeding 0.02, 0.01, 10^{-3} , and 4×10^{-4} (i.e., the M and R parameters that cause the 50-, 100-, 1,000-, and 2,500-year mean return period (MRP) ground motion for this site, respectively). The contours of $MAR_{S_1:S_2}$ of exceeding the ground motion levels corresponding to 50-, 100-, 1,000-, and 2,500-year MRPs are shown in Figure 6a. All the S_1 and S_2 pairs on each one of these contours have the same likelihood of being exceeded at this site every year. This plot indicates also the five pairs of S_1 and S_2 values selected for the M - R disaggregation shown in Figure 6b to Figure 6f. Figure 6b presents the disaggregation of the pair ($S_1 = 0.05g$; $S_2 = 0.18g$) (blue square in Panel a) that is exceeded at this site once every 50 years, on average. Although many M - R scenarios can cause the occurrence of this S_1 and S_2 pair at the site, the largest contributions come from $M6.5$ - $M7.0$ events at a distance of 30 to 75km and by $M7.0$ - $M7.5$ events at a distance of 75-100km.

Figure 6c shows a similar M - R plot for the exceedance of the 100-year MRP pair of $S_1 = 0.08g$ and $S_2 = 0.26g$ (red square in Panel a). For comparison purposes, Figure 6d, Figure 6e, and Figure 6f illustrate the M - R disaggregation information for three pairs of S_1 and S_2 values that are exceeded, on average, once every 1,000 years at this site. Figure 6d considers a S_1 - S_2 pair that includes approximately² the scalar 1000-yr value of $S_2 = 1.00g$ (black star in Panel a) and very small value of S_1 , while Figure 6e refers, approximately, to the scalar value of $S_1 = 0.30g$ (black circle in Panel a) and a very small value of S_2 . Figure 6f is associated with an intermediate case where both values of S_1 and S_2 are of engineering significance, namely $S_1 = 0.26g$ and $S_2 = 0.59g$ (black square in Panel a). As can be seen from Panels d-f, the largest contributions to the occurrence of the “scalar” S_2 value (Panel d) come from events with smaller M and, therefore, shorter R than those that contribute the most to the scalar S_1 value (Panel e). This is to be expected given that, on average, only large magnitude events have enough frequency content at long periods such as 4.0s associated with S_1 . The disaggregation of the intermediate case displayed in Panel f falls in between the two extreme cases, as expected. This disaggregation information is useful to engineers to determine the representative earthquake ground motions at a site. This data has been made routinely computed in the past decade for scalar PSHA studies, and it is extended here for the vector-valued PSHA case.

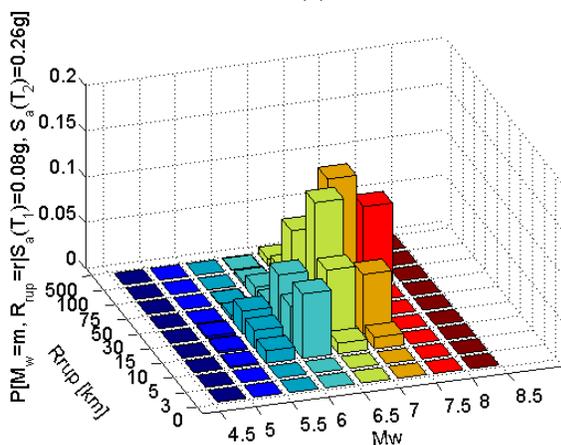
² Strictly speaking, the exact 1000-yr MRP scalar value of S_2 should be paired with a zero value of S_1 . However, the value of S_1 in this pair is so small (0.01) that difference is negligible. This statement can be checked by comparing the value of $S_2 = 1.00g$ from Figure 6a with the exact 1,000-yr value of S_2 in Figure 4b.



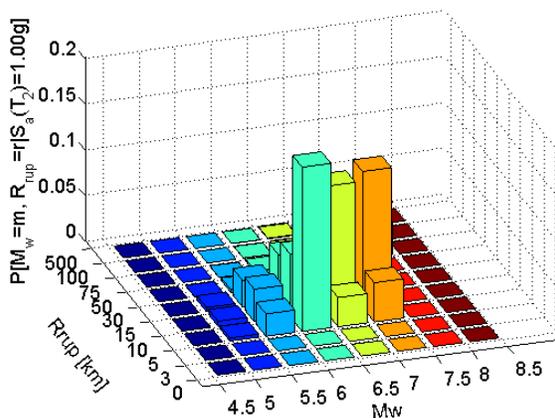
(a)



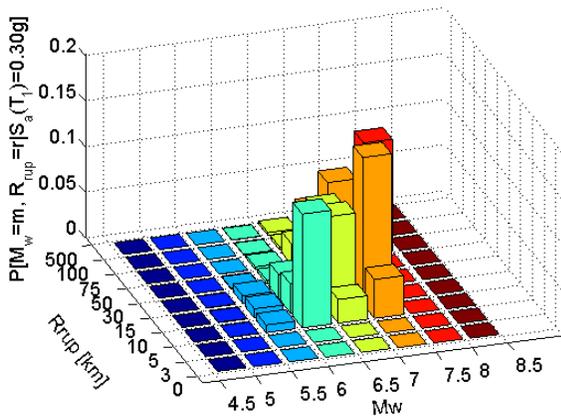
(b)



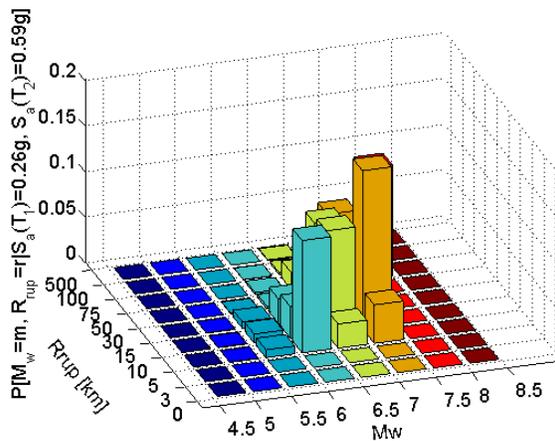
(c)



(d)



(e)



(f)

Figure 6. (a) Contours of the $MAR_{S_1:S_2}$ of exceeding S_1 - S_2 pairs with 50-, 100-, 1,000-, and 2,500-year MRPs. The five selected pairs of S_1 and S_2 for which the M - R disaggregation is performed are also shown. M - R disaggregation of the joint hazard of $S_1 = 0.05g$ and $S_2 = 0.18g$ with 50-yr MRP (b); the joint hazard of $S_1 = 0.08g$ and $S_2 = 0.26g$ with 100-year MRP (c); the scalar hazard of $S_2 = 1.00g$ (strictly speaking, the joint hazard of $S_2 = 1.00g$ and $S_1 = 0.01g$) with 1,000-year MRP (d); the scalar hazard of $S_1 = 0.30g$, (strictly speaking, the joint hazard of $S_1 = 0.30g$ and of $S_2 = 0.01g$) with 1,000-yr MRP (e); and the joint hazard of $S_1 = 0.26g$ and $S_2 = 0.59g$ with 1,000-yr MRP (f). The site is located in downtown San Francisco (Figure 4a).

The M - R disaggregation matrices of the joint MAR_{S_1, S_2} , of S_1 and S_2 , the MAR_{S_1, S_2} , itself, and the hazard curve of S_3 can be processed (see Eqs. 1-6) to obtain the joint hazard of S_1 , S_2 , and S_3 (denoted as MAR_{S_1, S_2, S_3}). The results shown in Figure 7a and Figure 7b are the joint MAR_{S_1, S_2, S_3} of *equaling* and *exceeding*, respectively. The surface of each plot represents the locus of all the possible combinations of $\{S_1, S_2, S_3\}$ triplets with the same joint MAR of equaling (Figure 7a) or exceeding (Figure 7b). The MAR of equaling or exceeding S_1 - S_2 pairs or S_1 - S_2 - S_3 triplets are, conceptually, very similar and the latter can be easily visually recognized as an extension of the former. For example, the iso-surfaces representing the MAR of *exceeding* the values of 0.10, 0.01, and 10^{-3} (i.e., 10-, 100-, and 1,000-year MRPs) shown in Figure 7b in red, blue, and green, respectively, is clearly a 3D extension of the 2D curves shown in Figure 6a which are also visible in Figure 5b.

It is emphasized here again that the procedure in Eqs. 1-6 can be applied to evaluate the joint hazard of four or more ground motion parameters. It is simply a recursive procedure that can be implemented to expand the dimensions of the joint hazard calculation. The only limitations in the applicability of this methodology are only practical, merely in the computer memory and disk storage capacities, not theoretical. For a large number of variables and smaller size bins the storage requirements can become extremely large.

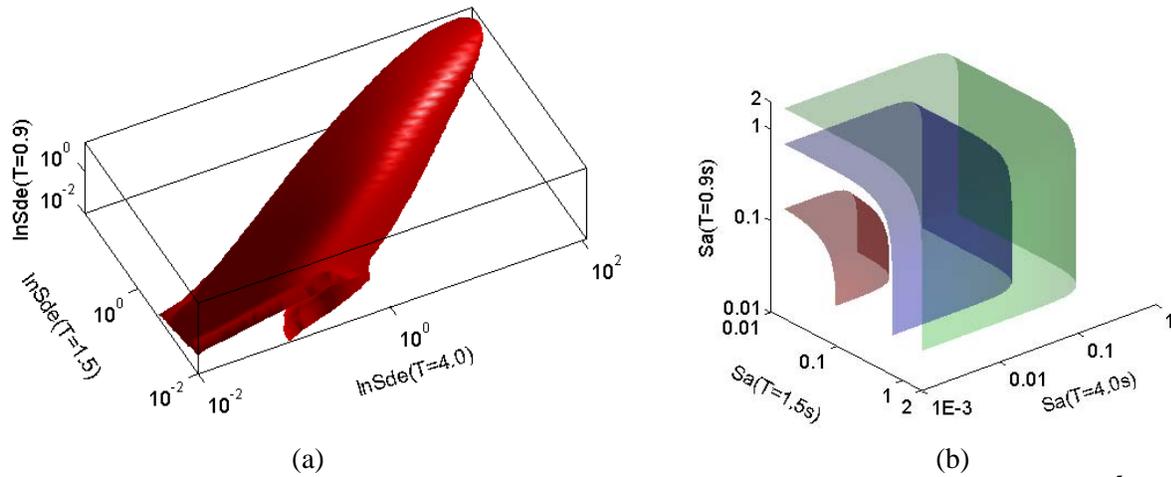


Figure 7. Joint hazard of MAR_{S_1, S_2, S_3} . (a) Iso-surface representing the MAR of *equaling* the value of 10^{-5} , and (b) iso-surface representing the MAR of *exceeding* the values of 0.10, 0.01, and 10^{-3} (i.e., 10-, 100-, and 1000-year mean return periods) shown as red, blue, and green respectively.

4.1 Simplified Approach

This section illustrates the effect of the conditional disaggregation (i.e., $P[\mathbf{X} | S_a]$) on a joint VPSHA framework discussed in Section 3. The study here is to explore the differences in the VPSHA results when the $P[\mathbf{X} | S_a]$ is simplified to $P[\mathbf{X}]$, which is shown in Figure 8. This simplification, although rigorously speaking incorrect, would simplify the computation of the joint hazard tremendously. In this section we aim at understanding how pervasive the impact of this simplification is on the accuracy of the joint hazard estimates.

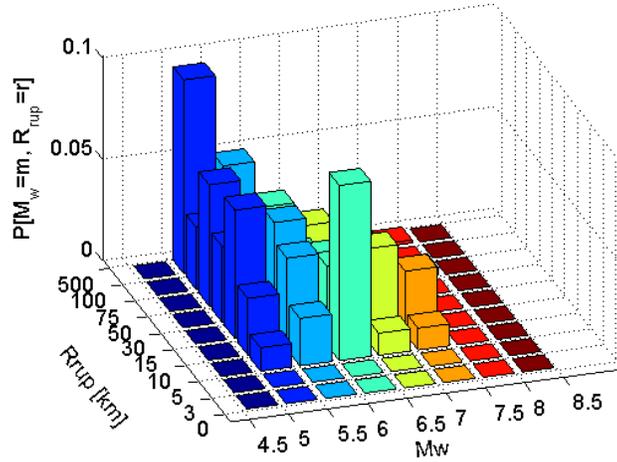


Figure 8. Seismic-source contribution for the San Francisco site ($P[X]$).

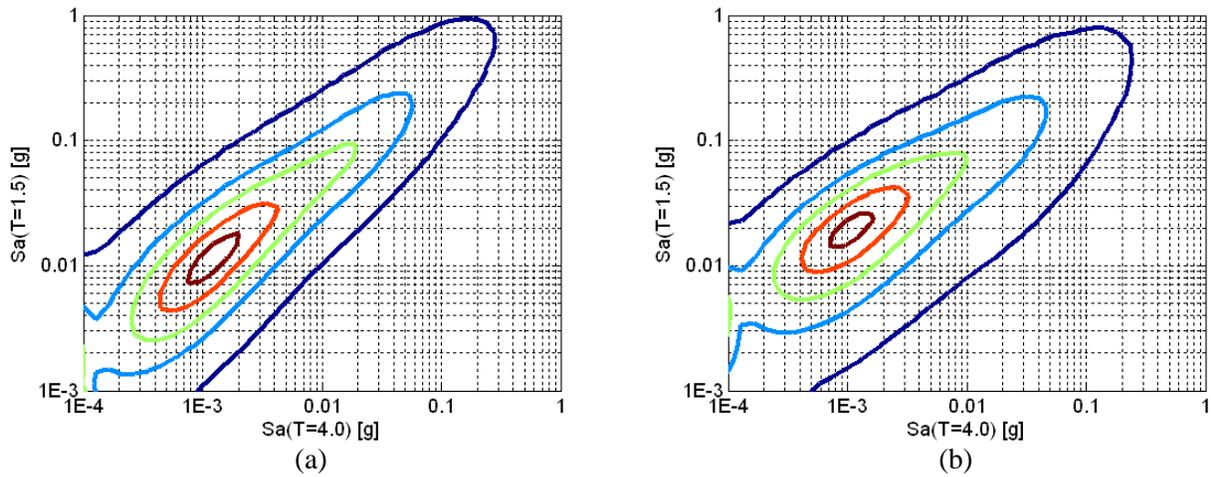


Figure 9. Joint $MAR_{S_1:S_2}$ of equaling for a site in downtown San Francisco. The $MAR_{S_1:S_2}$ on the left (a) is computed correctly using the disaggregation conditional on spectral ordinates, while the sub-figure on the right (b) is obtained by omitting the conditioning in $P[X|S_1]$.

For the San Francisco site considered before, Figure 9 shows contours of the MAR of equaling different S_1 - S_2 pairs computed using the approach in Eqs. 1-6 (Panel a) and the simplified approach above (Panel b). As can be seen from Figure 9, omitting the conditioning part of the disaggregation results in a different shape of the $MAR_{S_1:S_2}$ as well as different centroids. The contours of the simplified version (Figure 9b) are wider, as expected given that a source of correlation has been disregarded. Figure 10 shows more clearly the impact of assuming $P[X|S_a] \cong P[X]$ on conditional distributions of S_1 on S_2 for different levels of S_1 . The conditional distributions based on the simplified approach are wider and have smaller median values than the “correct” ones. The differences in the median values are larger at a higher S_1 ground motion level.

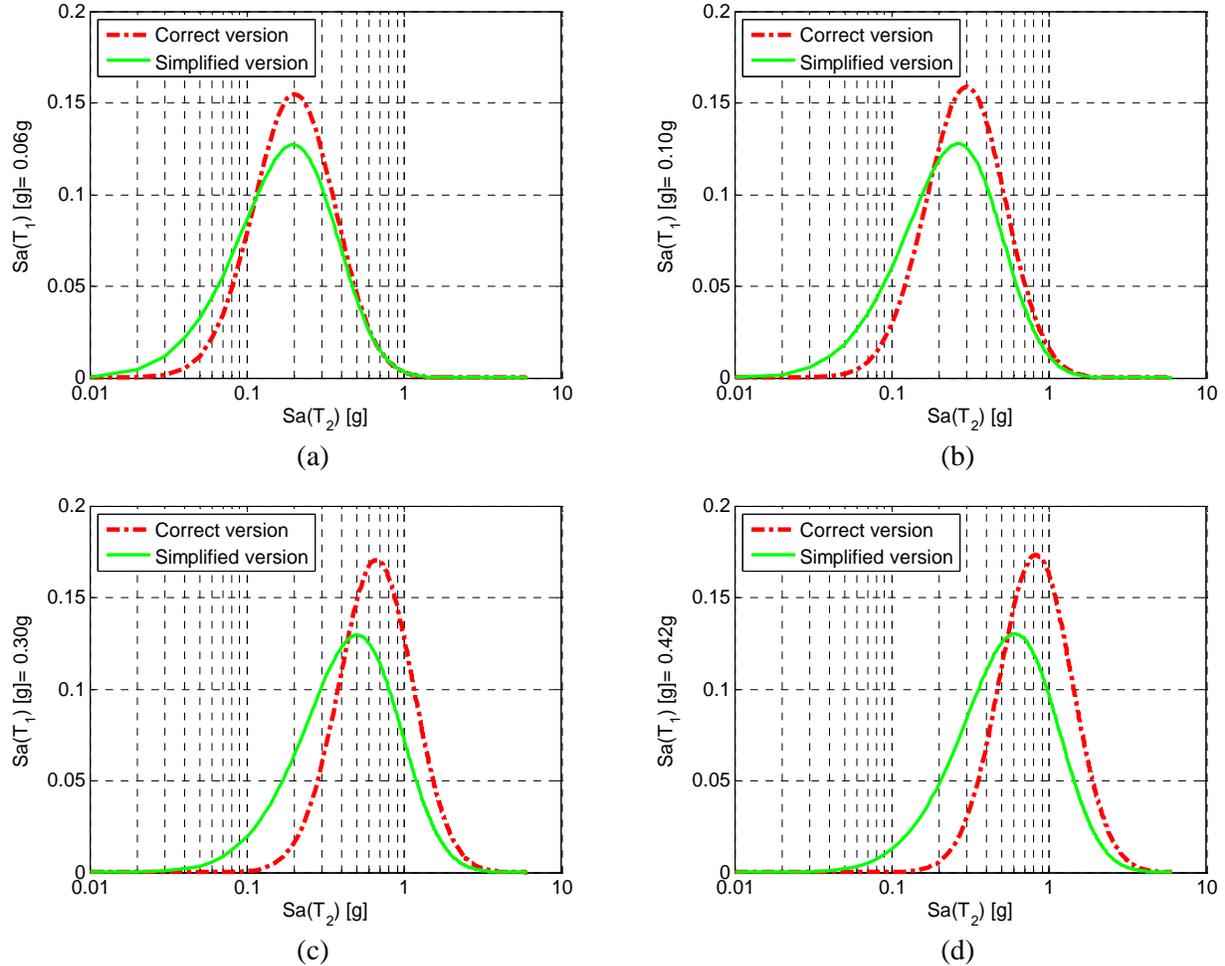


Figure 10. Conditional probability distribution of S_2 on S_1 equal to (a) 0.06g, (b) 0.1g, (c) 0.3g, and (d) 0.42g (i.e., 50-, 100-, 1000-, 2500-mean return periods) using the framework discussed in Section 3 (dashed-dotted lines) and the simplified approach (solid lines).

Recall that conceptually this difference depends primarily on the surrounding fault condition. An easy way to prove it mathematically is to reverse the conditioning, namely $P[\mathbf{X}|S_a] = (P[S_a|\mathbf{X}] P[\mathbf{X}]) / P[S_a]$. For the limit case of a single M and R scenario (e.g., the characteristic event case discussed earlier), the conditioning can simply be ignored without losing computational accuracy. This is because $P[S_a|\mathbf{X}] = P[S_a]$ in the equation above and, therefore, $P[\mathbf{X}|S_a] = P[\mathbf{X}]$. In general, the more significant the contribution to the site hazard from multiple scenarios (i.e., M , R , etc.), the more inaccurate the joint hazard from the simplified approach. Hence, we can conclude that the simplified approach, although computationally appealing, should perhaps only be used in those cases where the site hazard is clearly dominated by one event.

4.2 Effect of the disaggregation bin size on the joint hazard estimates

Although the mathematical framework shown in Section 3 is rigorous, the VPSHA results based on Eqs. 1-6 are still numerically approximate because during the disaggregation multiple events of slightly different M and R values are included in the same bin. A direct integration of the joint distribution of all the ground motion parameters generated by any event of given M and R as done, for example, by Bazzurro and Cornell (2002), would avoid this numerical approximation. Note that the direct integration, however, is also affected by a numerical approximation given by the size of the M and R bins used during the integration (here, 0.1 units in M and 2km in R). This latter source of approximation is not matter of investigation here.

The approximation introduced by the size of the M and R bins during disaggregation is above and beyond the one introduced by bin size during integration.

It is intuitive to expect that the bin size of M , R , and of any other variables accounted for in the disaggregation may have an influence in the accuracy of the joint hazard. The joint hazard results shown so far are based on magnitude bins that are 0.25-unit wide. For example, all the events with magnitude between, say, M 6.0 to 6.25 will be included in the same bin which is represented by its centroid, namely, 6.125. If the bin is very wide, say, $1.0M$ interval, then the results may tend to depart from those based on direct integration. Small bin sizes, although theoretically preferable, are not practical because of the increase in the computational time and in the memory and/or data storage requirements. Guidance on bin size selection can be obtained from inspecting the ground motion prediction equation to determine the relative changes in ground motion caused by changes in the input parameters such as M and R .

Figure 11, which displays contours of the $MAR_{S_1:S_2}$ of equaling S_1 and S_2 pairs for the same site considered used above, intends to explore the issues of different M and R bin sizes. The different panels show the results obtained using M and R bins of different sizes in the disaggregation calculation. For the four cases, the details of the M and R bins are:

- i. Eight M bins from 4.5 to 8.5 with 0.5 M intervals, and nine R bins equal to 0, 3, 5, 10, 15, 30, 50, 75, 100, 500 km;
- ii. Five M bins from 4.5 to 8.5 with 1.0 M intervals, and four R bins equal to 0, 10, 30, 100, 500 km;
- iii. Two M bins from 4.5 to 8.5 with 2.0 M intervals, and two R bins equal to 0, 30, 500 km; and
- iv. One M bin ranging from 5.0 to 8.5 and two R bins equal to 0, 34.5, 500 km

The M and R bin sizes for the last case were determined to produce the M and R centroid values of 6.75 and 17.25km, respectively. These two values seem to be the representative values based on the M - R disaggregation shown in Figure 6. Interestingly, Panels a through c of Figure 11 display relatively similar contours. In the last case where only one bin has been selected (Figure 11d), the contours are tighter than the others. Although not shown here, the $MAR_{S_1:S_2}$ of equaling S_1 and S_2 pairs obtained by considering 0.25-unit wide M bins are almost identical to those shown in Figure 9a for Case i.

As can be seen from Figure 11, Case i results are very similar to those of Case ii with slight differences in the region near the Mode (i.e., the value at which its probability density function attains its maximum value). In general, Case i seems to produce $MAR_{S_1:S_2}$ of equaling S_1 and S_2 pairs that are more “pointy” around the modal value than those of Case ii and even more so than those in Case iii. Only at rarer (i.e., extreme) seismic intensity levels, the results of the three cases are more similar. The extremely coarse disaggregation bin case (Case iv) produces the contours that are noticeably different from the others illustrating that the a very coarse bin disaggregation does not produce accurate estimate of the joint hazard. The differences in the joint hazard estimates discussed above can be more easily appreciated by inspecting Figure 12, which displays the conditional probability density function of S_2 on S_1 at different levels of S_1 . Again the effect of the disaggregation bin size is briefly shown here. A more detailed analysis will be conducted in a parallel project in a near future.

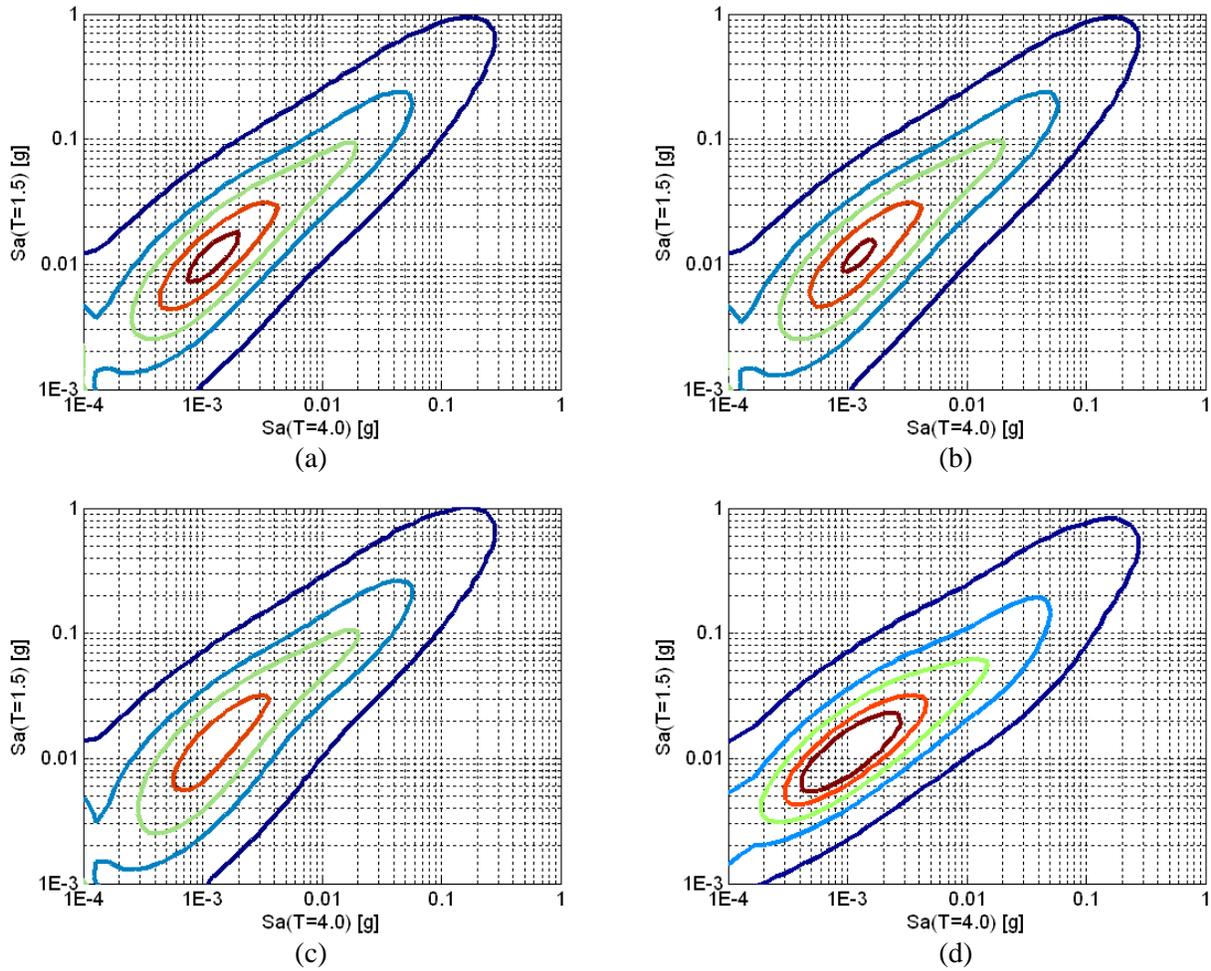


Figure 11. Joint MAR_{S_1,S_2} of equaling S_1 and S_2 pairs for a site in downtown San Francisco. The MAR_{S_1,S_2} was computed using (a) M bins 0.5-unit wide between 4.5 and 8.5 and R bins equal to 0,3,5,10,15,30,50,75,100,500 km, (b) M bins 1.0-unit wide between 4.5 and 8.5 and R bins equal to 0,5,15,50,100,500 km, (c) M bins equal to 4.5, 6.0, 8.5 and R bins equal to 0,20,100,500 km, and (d) one M bin from 4.5 to 8.5 and one R bin from 0 to 100 km.

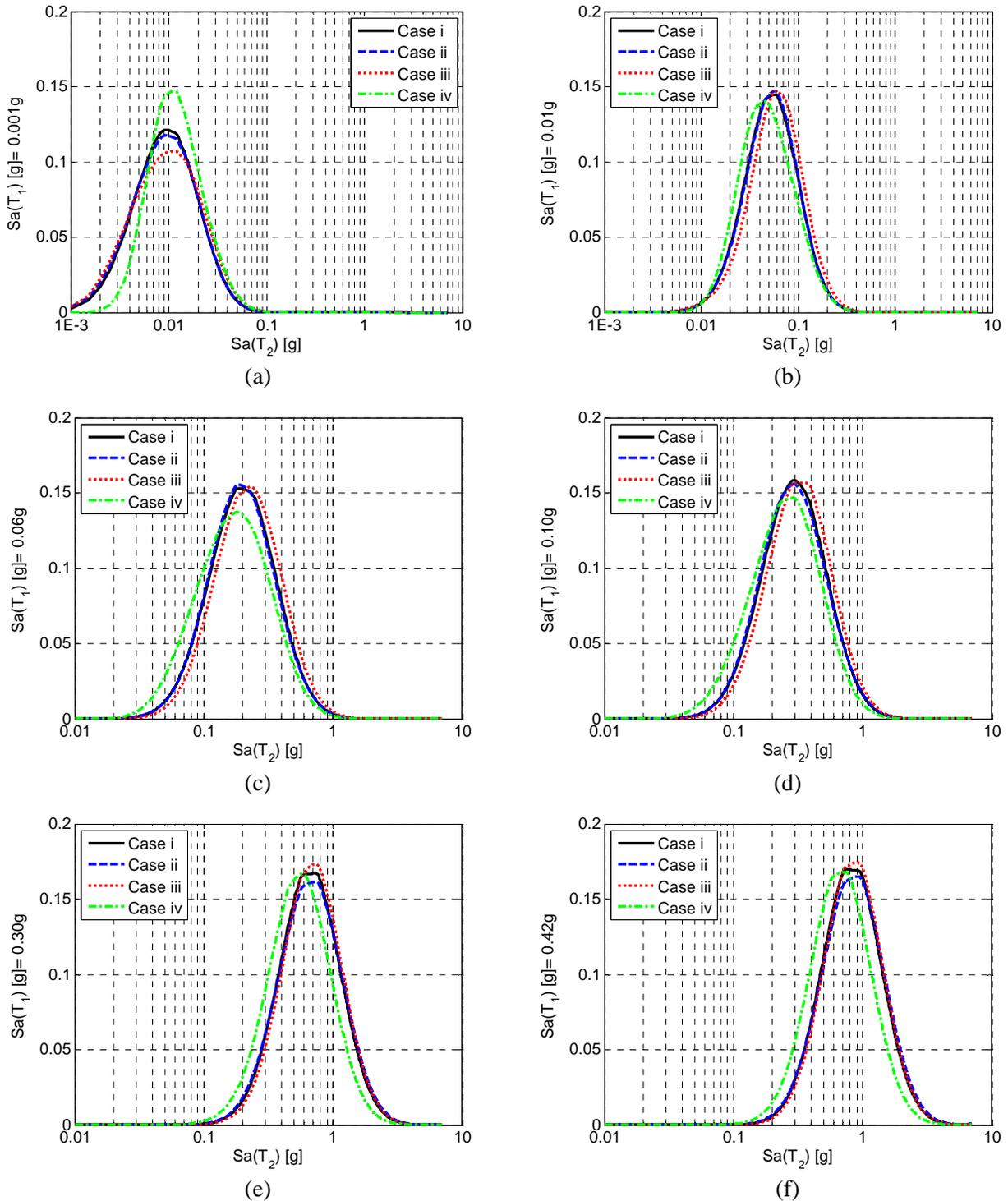


Figure 12. Conditional probability distribution of S_2 on S_1 equal to (a) 0.001g, (b) 0.01g, (c) 0.06g, (d) 0.1g, (e) 0.3g, and (f) 0.42g (i.e., the values corresponding to MRPs of 3-, 10-, 50-, 100-, 1,000-, 2,500 years at this site) using the four different disaggregation bins. (Note that the conditional probability distribution shown here is obtained from the joint $MAR_{S_1;S_2}$ of equaling shown in Figure 11 and been normalized to have a unit area.)

5 DISCUSSION AND CONCLUSIONS

In recent years, the site-specific seismic hazard analysis has become a common tool in engineering society to determine the ground motion amplitude at a designated site (e.g., the ground motion level associated with 2500-year mean return period). Conventionally this computation is only done for a scalar ground motion intensity measure—such as peak ground acceleration or spectral acceleration—although the methodology for computing the joint (i.e., vector) hazard analysis has been proposed almost 10 years ago (Bazzurro, 1998; Bazzurro and Cornell, 2001 and 2002). The joint hazard calculation has not been adopted in the engineering seismology community mainly due to the lack of a computational tool to perform the vector-valued seismic hazard analysis (VPSHA). This report presents a methodology that can be used to construct the VPSHA by combining the results obtained from the scalar PSHA as opposed to performing a direct integration of the jointly normal distribution, as originally proposed.

This novel approach to VPSHA is appealing for a number of reasons.

- It does not require writing a complex VPSHA code. It can be implemented by making limited modifications to any standard PSHA code of choice. The results from the PSHA analyses will then need to be processed according to the methodology shown here to produce the joint hazard sought.
- It does have the potential to incorporate many more variables than the standard direct integration approach. At the time of this writing, given the current limitations in memory and storage, we believe that the procedure presented here could handle up to five or six ground motion parameters as opposed to two or at most, three as the computer programs available based on the standard approach. More numbers of variables can be incorporated as the memory and storage space in affordable computers keep growing at a significant rate.
- It does provide the disaggregation of the joint hazard as a standard output.

Joint hazard estimates can guide, for example, structural engineer to understand the potential seismic events at a site for multi-mode dominated structures when two or more intensity measures are often required to ensure the accuracy in seismic risk calculation. We hope that this tool will facilitate the adoption of VPSHA in applications outside the academia.

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