Including nonlinear site effects
in broadband deterministic wavefield models

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Proposal Category: Integration and Theory
Interdisciplinary Focus Groups: Ground Motion
Seismic Hazard Analysis
Disciplinary Group: Seismology
SCEC CEO Focus Area: Implementation Interface
1. Introduction
The selection of the appropriate methodology for the prediction of soil nonlinearity in strong ground motion is based on the anticipated strain amplitude, and while nonlinear methods are necessary to capture large-strain phenomena such as permanent deformations or liquefaction, there currently exist uncertainties regarding the conditions under which nonlinear analyses are necessary, the complexity of the nonlinear model to be employed, and the development methodology of the input model parameters for implementation. These difficulties, further aggravated by the lack of well-documented validation studies, have thus far prohibited the wide adoption of the nonlinear analysis procedures by the engineering design and seismological communities alike. Resolving the physics of sediment nonlinear behavior in strong seismic shaking requires a good estimate of the input motion, and perhaps the most reliable source of information is provided by downhole arrays in seismically active areas [1-28]. Nonetheless, comprehensive validation and uncertainty analysis of approximate and elaborate nonlinear site response methodologies on the basis of downhole geotechnical array data involves a two-fold impediment: (i) the scarcity of near-surface geotechnical information usually results in predictions of surface ground motion that poorly compare with weak motion observations, a discrepancy further aggravated for strong ground motion that is usually associated with nonlinear, and potentially irreversible material deformations, and (ii) the paucity of design-level records in Southern California does not allow a statistically significant number of strong motion downhole recordings to be compiled to allow a statistically sound modeling and parametric uncertainty analysis of nonlinear site response. In this work, the former is addressed by implementing a weak motion seismogram inversion algorithm to improve the resolution of the continuum in our simulations, and the latter by generating a suite of synthetic ground motions and enable the investigation of a wide spectrum of site conditions and ground motion characteristics.

The long term goal of this work addresses the pressing need for establishment of guidelines that will allow credible predictions of nonlinear site response based on an efficient and cost-effective framework. Applied on a regional basis for the Los Angeles (LA) sedimentary site conditions, we here present results of a comprehensive uncertainty assessment study that was conducted under the auspices of SCEC2 (FY2006), opting to determine: (i) the site conditions and ground motion characteristics under which nonlinear analyses should be conducted; (ii) the adequate complexity of the nonlinear model to ensure computationally-efficient simulations; and (iii) the geotechnical site investigation and input parameter development methodology, to ensure cost-effective parameter acquisition. For further information, the reader is referred to [29, 30].

2. Site conditions in the LA Basin
Accounting for the lack of geotechnical and strong motion data recordings alike, a statistically significant number of 3-component broadband ground motion time-histories were simulated for the purpose of this study for strike-slip fault rupture scenaria at 33 discrete points on a 100x100km² square grid (Figure 1a).

Figure 1. (a) Plan view of station locations where 3-component broadband seismogram synthetics were computed for the purpose of this study; (b) Downhole array stations in the Los Angeles basin, where seismogram inversion has been employed; and (c) 1D crustal velocity and density models, extracted from the SCEC community velocity model.

The alternative 1D-crustal velocity and density models were compiled using available near-surface geotechnical data, downhole array seismogram inversion profiles [28], and data extracted from the Southern California Earthquake Centre (SCEC) community velocity model (CVM) at the locations of 7 downhole geotechnical arrays in the LA basin. The location of the arrays and the crustal models used in the ensuing for the synthetic ground motion simulations are shown in Figures 1b and 1c. In particular for the near-surface site conditions, high-resolution velocity, attenuation and density profiles were estimated by means of waveform inversion of downhole array recordings, using the optimization technique developed by Assimaki et al [28, 31].
Referred to as hybrid global-local optimization, the numerical scheme comprises a genetic algorithm in the wavelet domain coupled in series to a nonlinear least-square fit in the frequency domain, which improves the computational efficiency of the former while avoiding the pitfalls of using local linearization techniques for the optimization of multi-modal, discontinuous and non-differentiable functions. The best-fit variation of shear wave velocity, attenuation and density with depth estimated for the La Cienega SMGA in the LA basin is depicted in Figure 2.

Figure 2. Velocity ($V_s$), attenuation ($Q$), density ($\rho$) and nonlinear parameter profiles, evaluated by means of downhole array seismogram inversion at the La Cienega SMGA in the LA basin (depicted in Figure 1).

3. Broadband ground motion synthetics

Successively, broadband ground motion time-histories have been computed for the alternative 1D crustal velocity models and strike-slip fault rupture scenario corresponding to events of magnitude $M_w = 3.5-7.0$ and depth-to-fault geometry $d=2.5-6$ km; for each scenario, 3-component time-histories were evaluated by means of the hybrid low/high frequency method with correlated random source parameters [32]. In the complete formulation of the hybrid low/high frequency method, the evaluation of low-frequency synthetics ($< \sim 1$Hz) is based on 3D-velocity structures, and the broadband synthetics are evaluated for 1D models, while the two frequency ranges are combined using matched filtering at a cross-over frequency of 1Hz. The source description, common to both the 1D and 3D synthetics, is based on correlated random distributions for the slip amplitude, rupture velocity, and rise time on the fault. A typical realization of the normalized slip distribution, rupture and rise time for a strike-slip event of magnitude $M_w=6.5$ are shown in Figure 3.

For the purpose of this study, broadband ground motion synthetics were initially computed on the surface of the linear elastic 1D crustal velocity models, and successively, the ground surface motion was deconvolved to the level of the lowest downhole instrument (typically $\approx 100$ m) at each array and the estimated incident waveform was propagated to the surface by means of the alternative approximate and elaborate nonlinear constitutive models investigated. Typical crustal ground surface linear elastic synthetics are depicted on Figure 3b. Note that this approach allows the computationally-efficient integration of nonlinear site response methodologies in three-dimensional deterministic rupture simulations, conditioned -however- on the existence of well-defined criteria that determine the nonlinearity susceptibility of near-surface site conditions; acknowledging that currently established site classification schemes often fail to describe the anticipated soil behaviour as a function of the incident ground motion characteristics, the long term objectives of our work involve the refinement of currently established site classification schemes to quantify the level of anticipated strain and the required nonlinear model as a function of the soil profile variability and ground motion amplitude and frequency content.

4. Modelling and parametric uncertainty in nonlinear site response

In the ensuing, we used the ground motion synthetic accelerograms to assess the relative modelling and parametric uncertainty in nonlinear site response predictions and identify the optimally computationally-efficient and cost-effective model to be implemented as a function of the site conditions and ground motion characteristics. For this purpose, the alternative models were validated for low strain conditions ($\gamma<10^{-3}$) by comparison with available in-situ downhole array data (e.g. Figure 4), the relative modelling uncertainty was assessed for strong synthetic ground motions by comparison with the most credible model formulation identified based on published data, and the parametric uncertainty of each model was finally assessed by systematically randomizing the model parameters. The two families of site response analysis approaches that were investigated in this work are the equivalent-linear and nonlinear methods.

**Equivalent-linear models:** In the equivalent-linear (EQL) iterative approach [33], viscoelastic simulations are performed in each iteration, a characteristic level of strain is defined as a fraction of the peak strain exerted by
each layer of the profile, the material properties are selected in the beginning of each iteration to be consistent with the levels of strain computed in the previous iteration, and the algorithm progresses until convergence. This stepwise analysis procedure, formalized into the computer program SHAKE [34], is currently the most widely used analysis package for 1D strong motion site response calculations. In this study, we investigated range of applicability of the original EQL formulation [34], and the EQL frequency-dependent model proposed by Assimaki & Kausel [35].

Nonlinear models: We investigated the prediction accuracy associated with the multi-linear, the hyperbolic, the Ramberg-Osgood (R-O) [36-37], and the modified hyperbolic [38-39] idealized monotonic stress-strain models. To describe the unloading and reloading soil response to transient loading, we implemented the original [40], extended and generalized Masing rules for cyclic loading, as well as the multi-yield plasticity model developed by Prevost [41]. The Masing rules were implemented by means of a mathematical model comprising \( N \) ideal elastoplastic springs in parallel (Figure 4), whose number and corresponding stiffness and Coulomb resistance values are in each case selected to fit the idealized model behavior (\( \tau = f(\gamma) \)). Based on this mathematical representation originally proposed by Iwan [42] and Mroz [43], the multi-linear shear stress-strain behavior for \( N \) elastoplastic springs at a strain amplitude \( \gamma \) is:

\[
\tau = \sum_{i=1}^{N} k_i \gamma + \sum_{i=1}^{n} \tau_{ci}
\]

where \( k_i \) is the shear stiffness of the \( i^{th} \) element, \( \tau_{ci} \) is the critical slipping (Coulomb) stress of the \( i^{th} \) element, \( n \) is the number of elastoplastic elements that remain elastic upon the application of a strain increment, and \( \tau \) is the estimated level of shear stress at a given level of strain amplitude \( \gamma \).

In order to fit the nonlinear model parameters to the experimental data of soil modulus degradation and damping vs. shear strain amplitude, we implement a global stochastic search (genetic algorithm) whose objective function simultaneously minimizes the square error between the measured and theoretically predicted data as follows:

\[
\sum_{i=1}^{N} \left[ w_i \left[ O_i(\gamma_i) - G(P, \gamma_i) \right] \right]^2 + \sum_{j=1}^{N} \left[ w_j \left[ O_j(\gamma_j) - \xi(P, \gamma_j) \right] \right]^2
\]

where \( w_i \) and \( w_j \) are the weight coefficients of the global search, \( O_i(\gamma_i) \) and \( O_j(\gamma_j) \) are the \( i^{th} \) and \( j^{th} \) experimental points for the modulus degradation and damping curves at \( \gamma_i \) and \( \gamma_j \) strain amplitudes correspondingly, and \( G(P, \gamma_i) \) and \( \xi(P, \gamma_j) \) are the corresponding predicted values as a function of the model parameters \( P \).

Successively, the measured low-strain, non-hysteretic, frequency independent damping was implemented in the time domain simulations based on the rheology formulation of a generalized Maxwell body [44], modified here as follows:

\[
\tau = \sum_{i=1}^{N} k_i \gamma + \sum_{i=1}^{n} \tau_{ci}
\]
$$\tau(t) = G \left[ \gamma(t) - \sum_{k=1}^{N} \zeta_k \right]$$

where $G$ is the shear modulus given by the nonlinear model, and $\zeta_k$ are memory variables that correspond to the solution of the following first order set of differential equations, with $\tau_k$ being the relaxation times and $w_k$ being the weight coefficients:

$$\tau_k \frac{d\zeta_k(t)}{dt} + \zeta_k(t) = w_k \gamma(t)$$

The accuracy of low strain damping modeling depends on the accuracy of estimation of $\tau_k$ and $w_k$, and for the purpose of this project, we implemented the empirical interpolating algorithm proposed by [45]. Comparison of the measured and idealized modulus degradation and damping vs. shear strain amplitude by means of the different nonlinear models, as well as examples of strong motion site response predictions at the La Cienega SMGA in Southern California are shown in Figure 5 for a $M_w5.1$ event recorded on ground surface and at 18m depth, and in Figure 6 for two synthetic time histories corresponding to an $M_w5.0$ and an $M_w6.5$ events at distance 5km from the fault. Note that for the near-fault site shown in Figure 6, the deviation in the predicted surface ground motion increases with increasing ground motion intensity, and when subjected to incident motion generated by a $M_w6.5$ event, linear elastic (or EQL) formulations overestimate the predicted PGA by 150% relatively to the nonlinear formulations. Comparing the alternative models investigated here:

a. In the **EQL model formulation**, the mathematical simplicity of linear analysis is preserved and the evaluation of the nonlinear constitutive parameters is avoided; nonetheless, their use is prohibited for calculations that involve permanent deformations, deep/soft sedimentary sites, or liquefiable profiles;

b. **Nonlinear constitutive models** can simulate soil behavioral features unavailable in the EQL formulation such as inelastic stress-strain relationships and pore-pressure generation, which are critical for the prediction of large strain and ground failure problems. The advantages and disadvantages of each model are briefly described in the ensuing:

(i) the **multi-linear model** can be calibrated to match exactly the experimental modulus degradation at all levels of strain, yet upon application of the extended Masing criteria, the predicted damping is found to overestimate the laboratory measured data at high levels of strain (>0.1%);

(ii) the **hyperbolic model** has only one independent parameter (reference strain $\gamma_r$), which, once obtained by fitting the modulus reduction curve, results in a strain-dependent damping ratio that underestimates experimental data at low strain (<0.05%) and overestimates them at high strain (>0.1%);

(iii) the **R-O model** has three independent parameters ($\gamma_r$, $\alpha$ and $r$) and is therefore much more flexible to be calibrated in fitting the experimental data both for the modulus degradation and damping ratio curves; nonetheless, in the R-O formulation, the shear strain increases in proportion to the shear stress, a drawback that prohibits the use of the model for large levels of strain at which the soil is anticipated to reach the level of shear strength of the material; and

(iv) while the **modified four-parameter hyperbolic model** has even more versatility to fit the experimentally-obtained shear stress vs. shear strain relationship over a wide range of strain amplitudes, the resulting damping ratio cannot be expressed in closed form due to the complexity of the model, and the resulting fitted parameters need to be properly constrained to allow their physically sound interpretation.

Figure 5. Comparison of nonlinear parameters and ground motion predictions of linear visco-elastic and nonlinear model predictions at depth $z=0\text{m}$ and $z=-18\text{m}$ for an $M_w5.1$ event recorded at the La Cienega SMGA array on 04/26/2001.
Figure 6. Comparison of ground motion predictions of linear visco-elastic and nonlinear models, for a synthetic strike-slip $M_w=5$ event (left) and an $M_w=6.5$ event (middle) at distance 5km from the fault; the predicted stress-strain hysteresis loops by the alternative nonlinear models investigated is also shown on the right.

Typical comprehensive results of our investigation are finally shown in Figure 7, where the ensemble of nonlinear site response simulations are categorized based on the event magnitude, and the predicted level of PGA that deviates from the linear elastic site response estimated ground motion is plotted as a function of the station distance from the surface projection of the fault. As can be readily seen, the linear elastic models may be used to adequately simulate the ground surface response for low magnitude events and far-field sites, while the uncertainty in the predicted amplitude among the alternative nonlinear formulations increases proportional to the magnitude and inversely proportional to the epicentral distance.

Figure 7. Deviation of predicted peak ground acceleration (PGA) by means of the alternative nonlinear models investigated in this work from the linear elastic solution for the La Cienega site conditions, for a series of synthetic ground motions corresponding to events $M_w$3.5-7.5, as a function of epicentral distance.

5. Future Work

Selection of the appropriate methodology for prediction of soil nonlinearity in strong ground motion is based on the anticipated strain amplitude, and while EQL formulations have been shown to produce reasonable results for low strain amplitudes, nonlinear methods are necessary to capture phenomena such as irreversible deformations and pore pressure coupling. The accuracy of nonlinear site-response analyses, however, depends on the constitutive model used, and elaborate constitutive models require numerous parameters which must be determined through lab tests and/or field tests; in turn, this additional effort involved to develop the required parameters, often limits their frequency of use.

Future research tasks associated with this work will focus on the development of quantitative guidelines for the optimal prediction of nonlinear site response addressing the following issues: (i) under what conditions (site conditions and ground motion characteristics) are nonlinear analyses necessary? (ii) which =model is adequately complex to simulate nonlinear site effects for a given site subjected to strong ground motion?; and (iii) how should the nonlinear input parameters be selected?


