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COUPLING OF THE RANDOM PROPERTIES OF THE SOURCE AND THE GROUND MOTION FOR THE 2004 PARKFIELD EARTHQUAKE

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Based on the superposition of seismic waves and the Central Limit Theorem, we have laid the basis for an unified picture of earthquake variability from its recording in the ground motions to its inference in source models. This theory stipulates that the random properties of the ground motions and the source for a single earthquake should be both distributed according to the Lévy law. Our investigation of the random properties of the source model and peak ground acceleration (PGA) of the 1999 Chi Chi earthquake confirms this theory (Lavallée and Archuleta, 2005; for a summary and figures see also: http://www.scec.org/core/public/showNugget.php?entry=2118). As predicted by the theory, we found that the tails of the probability density functions (PDF) characterizing the slip and the PGA are governed by a parameter, the Lévy index, with almost the same values close to 1. The PDF tail controls the frequency at which extreme large events can occur. These events are the large stress drops—or asperities—distributed over the fault surface and the large PGA observed in the ground motion. Our results suggest that the frequency of these events is coupled: the PDF of the PGA is a direct consequence of the PDF of the asperities. As far as we know, this is the only model available that unifies the random properties of the source to an important parameter (PGA) of the ground motions.

The 2004 Parkfield earthquake is the best-recorded earthquake in history for the density of near-source data. It provides an ideal candidate for evaluating and validating the theory discussed above. For this purpose, we used several source models computed for the Parkfield earthquake by Custodio *et al*. 2005. All the source models used in this study are based on a method to invert the kinematic source parameters developed by Liu and Archuleta (2004). The compiled source models differ by the number and the location of the stations used in the inversion. For each source, we compile the parameters of the stochastic model and compare them to the random properties of the PGA. We found that that the tails of the probability density functions (PDF) characterizing the PGA are governed by a parameter, the Lévy index with a value close to 1. For several source models, the computed Lévy index is in good agreement with this value. Our results suggest that all source models are not equivalent in term of their random properties. The values of the stochastic parameters depend on the location of a number of stations used in the inversion. Thus, this study provides the basis to compare, validate and optimize computed source models by comparing the random properties of the source to the random properties of the ground motions.

The results of this project were presented at the SCEC 2005 Annual Meetings, and the AGU 2005 fall meeting. The results related to this research project are reported in the following papers:

Lavallée, D., and R. J. Archuleta, Coupling of the random properties of the source and the ground motion for the 1999 Chi Chi earthquake, *Geophys. Res. Lett*., **32**, L08311, doi:10.1029/2004GL022202, 2005.

Lavallée, D., S. Custodio, P. Liu, and R. J. Archuleta. On the random nature of earthquake processes: A case study the 2004 Parkfield earthquake. Proceedings and abstracts of the 2005 SCEC Meeting, Palm Springs, Ca, 142-143, 2005.

Lavallée, D., S. Custodio, P. Liu, and R. J. Archuleta. On the Random Nature of Earthquake Source and Ground Motion: the 2004 Parkfield Earthquake. *EOS Trans. AGU*, **86** (52), Fall Meet. Suppl., Abstract S13B-0200, 2005.

Lavallée, D., P. Liu, R. J. Archuleta. Stochastic model of heterogeneity in earthquake slip spatial distributions. *Geophys. J. Int*., in revision, 2005.

During the period going from 12/01/04 to 01/21/05, in addition to the 2005 annual meeting, I participated to two workshops sponsored by SCEC —the Broadband Ground Motion Simulations (Jan. 28, 2005) and 3D Rupture Dynamics Code Validation Workshop (Sept. 11, 2005). Following a request to the SCEC community by Mark Benthien to provide materials for the SCEC booth at the Fall

AGU meeting, I provided a movie about rupture propagation under heterogeneous conditions. During the meeting, the movie was on display with other SCEC projects.

1   Computation of the parameters of the stochastic models for the 2004 Parkfield earthquake:

For the dip and strike slip spatial distribution of the 2004 Parkfield earthquake (Custodio *et al.*, 2005), the following procedure has been performed (fore details see Lavallée and Archuleta, 2003, 2005; and Lavallée *et al.*, 2005).

1- The power spectrum is computed for each of the horizontal layers of the dip and strike slip. For both, the dip and the strike slip, the mean power spectrum of the horizontal layers has been computed —see Figure 1. For each distribution, the spectrum shows that there are no dominating wave numbers, which suggests that the data cannot be reduced to—or understood as— a combination of several periodical functions. The curves illustrated in Figure 1 show that all the wave numbers contribute to the slip variability but also that the weight of the wave numbers follows approximately a trend given by a decaying power law. The values of the scaling exponents v are reported in Table 1.
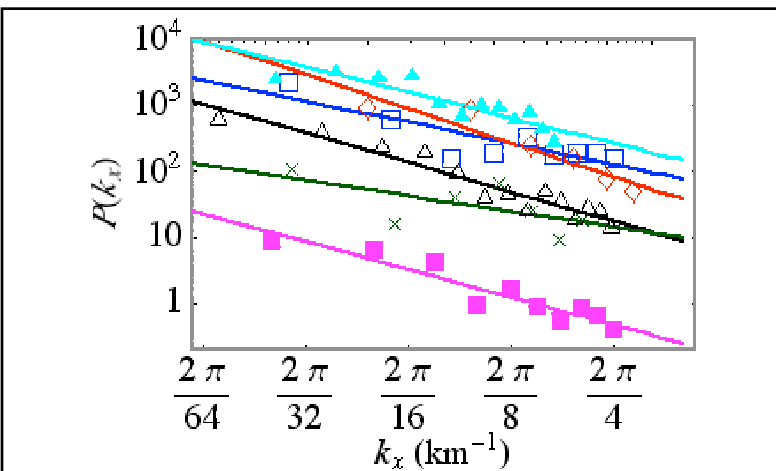

Figure 1: The mean power spectrum $P(k_x)$ as a function of the wave number k_x and the best straight line that fits the log-log curve are reported for the strike slip of the 2004 Parkfield earhquake (■). For comparison, the corresponding curves for the 1995 Hyogo-ken Nanbu (△), the 1979 Imperial Valley (×), 1989 Loma Prieta (□), 1994 Northridge (◊), and the 1999 Chi Chi (▲) earthquakes are also illustrated. These results suggest that the scaling behavior is observed for scale length that ranges from 2 to 72 km.

2- Each layer of the slip spatial distribution is filtered in the Fourier space in such a way that the resulting field has a mean power spectrum behavior that follows a flat curve (white noise). We assume that the resulting field corresponds to a field of random variables of magnitude X and compute the probability density function (PDF) of X. The (discrete) PDF is illustrated in Figure 2 for the strike slip.
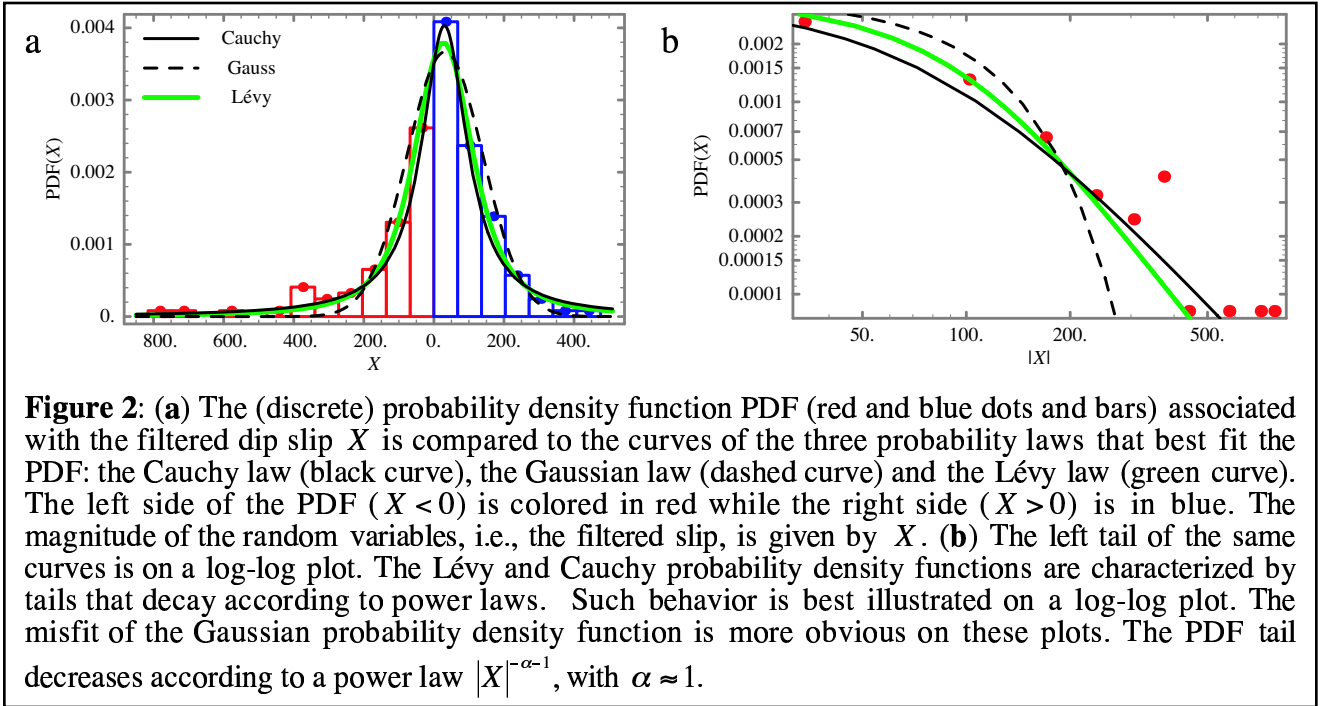
3- We then proceed to determine what theoretical probability law will provide the best fit to the probability density function PDF of X. Three candidates are considered: the Gauss law, the Cauchy law and the more general Lévy law (Uchaikin and Zolotarev, 1999). (The Gauss law and Cauchy law are characterized by two parameters whereas the more general Lévy law require four parameters. Note also that the Gauss law and Cauchy law are special cases of the Lévy law.) The curves of the Gaussian, Cauchy and Lévy law that best fit the PDF are illustrated in Figures 3.   The parameters of the Gaussian, Cauchy and Lévy law are reported in Table 1. The results presented in Table 1 are in good agreement with the stochastic parameters computed for several other earthquakes (see Lavallée and Archuleta, 2003, 2005; and Lavallée *et al.*, 2004). In particular, the parameter α takes values very close to the values reported for the Imperial Valley, Loma Prieta, Northridge and Chi Chi earthquakes. (The parameter α, with $0 < \alpha \leq 2$, controls the rate of falloff of the PDF tails.)

2   The Central Limit Theorem, the principle of superposition of linear waves and the consequences for the statistical properties of the ground motion:

The formulation of the slip and pre-stress variability in term of the Lévy random variables has fundamental consequences on the radiation field generated by the rupture motions. Note that the pre-stress is also related to the same random variables that characterize the slip distribution through a filtering but with a different scaling exponent (see Andrew 1980; Mai and Beroza, 2002). According to the superposition of (linear) seismic waves and the Central Limit Theorem, statistical properties of the

ground motion for a single earthquake should be also distributed according to a Lévy law (see Figure 3). Our investigation of the statistical properties of the source model and PGA of the 1999 Chi Chi earthquake confirms this hypothesis (Lavallée and Archuleta, 2005; see also my 2003 SCEC annual report).

Figure 2: (**a**) The (discrete) probability density function PDF (red and blue dots and bars) associated with the filtered dip slip X is compared to the curves of the three probability laws that best fit the PDF: the Cauchy law (black curve), the Gaussian law (dashed curve) and the Lévy law (green curve). The left side of the PDF ($X < 0$) is colored in red while the right side ($X > 0$) is in blue. The magnitude of the random variables, i.e., the filtered slip, is given by X. (**b**) The left tail of the same curves is on a log-log plot. The Lévy and Cauchy probability density functions are characterized by tails that decay according to power laws. Such behavior is best illustrated on a log-log plot. The misfit of the Gaussian probability density function is more obvious on these plots. The PDF tail decreases according to a power law $|X|^{-\alpha-1}$, with $\alpha \approx 1$.

The PGA amplitude estimated at different stations as a function of the closest distance between the station and the fault rupture is usually a function of the distance (for instance see Figure 3 in Lavallée and Archuleta, 2005; see also my 2003 SCEC annual report). However, for stations located between a distance of 0 and 11 km, the distributions of |PGA| values is not significantly affected by the distance. Thus, we can assume that variations in the statistical properties (or parameters of the probability law) are rather uniform (or independent of the distance) for stations located between 0 and 11 km. Nevertheless, to test the effect of the location of the stations, we computed the PDF of the PGA for stations located between a distance of 0 and 5 km, for stations located between a distance of 0 and 7.5 km, and for stations located between a distance of 0 and 11 km. Assuming that the PDF of the (absolute value of the) PGA can be approximated by a Lévy law, we compute the parameters of the Lévy law that fit the PDF curves. The results are reported in Table 2 and illustrated in Figures 4 and 5.

The values computed for α are close to 1 for almost all the PDF reported in Table 2. The values are in good agreement with the values of α computed for both the dip and strike slips (see table 1).

These results show that the Lévy law provides an accurate description of the PDF associated to the PGA. Furthermore, these results suggest that the rate of decrease of the PDF tails of PGA —controlled by α— is (almost) invariant for stations located between 0 to 11 km. The Central Limit Theorem and the principle of superposition of seismic waves provide the physical rationales for the observation of rate of decrease of the PDF tails in the source and the PGA. The PDF tail controls the frequency at which extreme large events can occur. These events are the large stress drops—or asperities—distributed over the fault surface and the large PGA observed in the ground motion. Our results suggest that the frequency of these events is coupled: the PDF of the PGA is a direct consequence of the PDF of the asperities. As far as we know, this is the only model available that unifies the statistical properties from the source to an important parameter (PGA) of the ground motion.
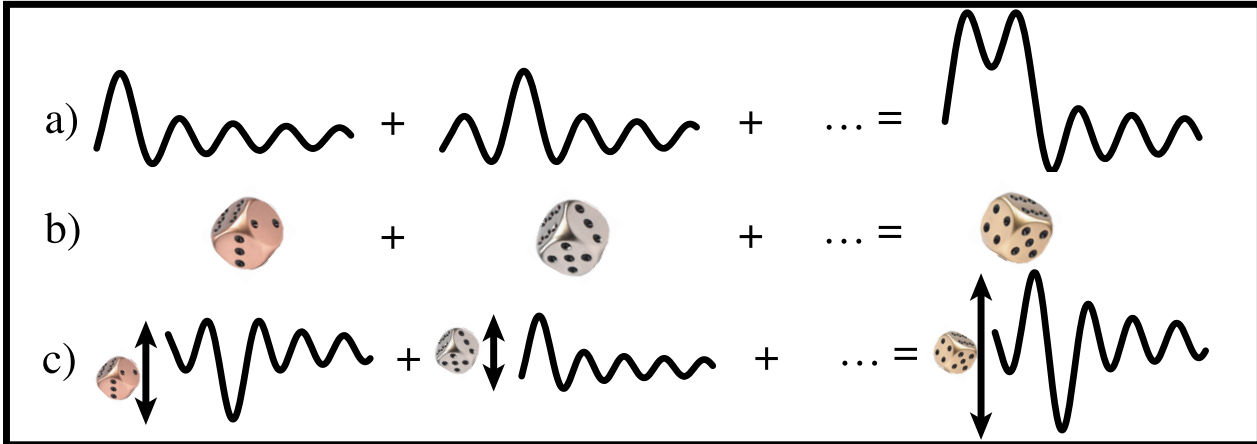
Figure 3: (**a**) During an earthquake, the stress is locally released, causing the fault to slip and generating an elastic wave in the surrounding medium. Assuming that the medium is linear, the signal observed at a given station can be understood as the sum of these waves. This is the principle of superposition for the linear wave equation. (**b**) Similarly, a sum of random variables (or dice) distributed according to a Lévy law is also a Lévy random variable. This is essentially the Central Limit Theorem. One important consequence of the Central Limit Theorem is that one of the four parameters of the Lévy law, the Lévy index α, remains invariant under the operation illustrated in (b). That is, a sum of Lévy random variables characterized by a parameter α will give a random variable with the same α value. (**c**) We assume that the wave generated by the slip has an amplitude proportional to the slip. If the slip values are distributed according to a Lévy law, so will be the wave amplitudes. Thus the principle of superposition of linear waves and the Central Limit Theorem imply that the signal observed at a given station will have also an amplitude distributed according to a Lévy law. As a consequence of this formulation, the PDF tail of the slip and of ground motion metrics (such as PGA) will decrease with a power law characterized by the same parameter α.
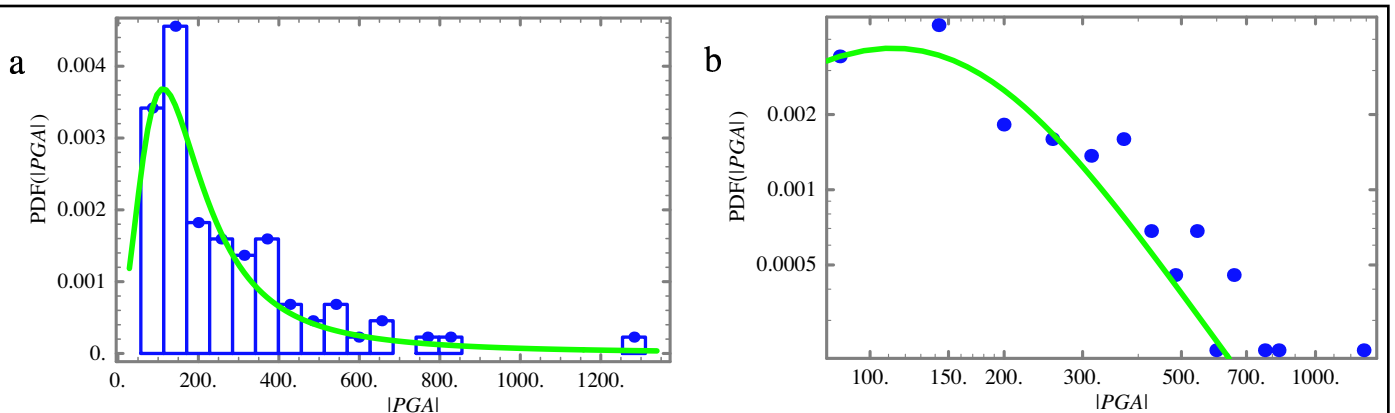

Figure 4: (**a**) The PDF of the |PGA| for the stations located between a distance of 0 to 7.5 km is compared to the curves of the Lévy law that best fit the PDF (green curve) — see Table 2. The variable X corresponds to the absolute value of the PGA. (**b**) The positive tails of the curves are plotted on a log-log scale. The PDF tails decrease according to a power law $\left|\text{PGA}\right|^{-\alpha-1}$ with α close to 1.
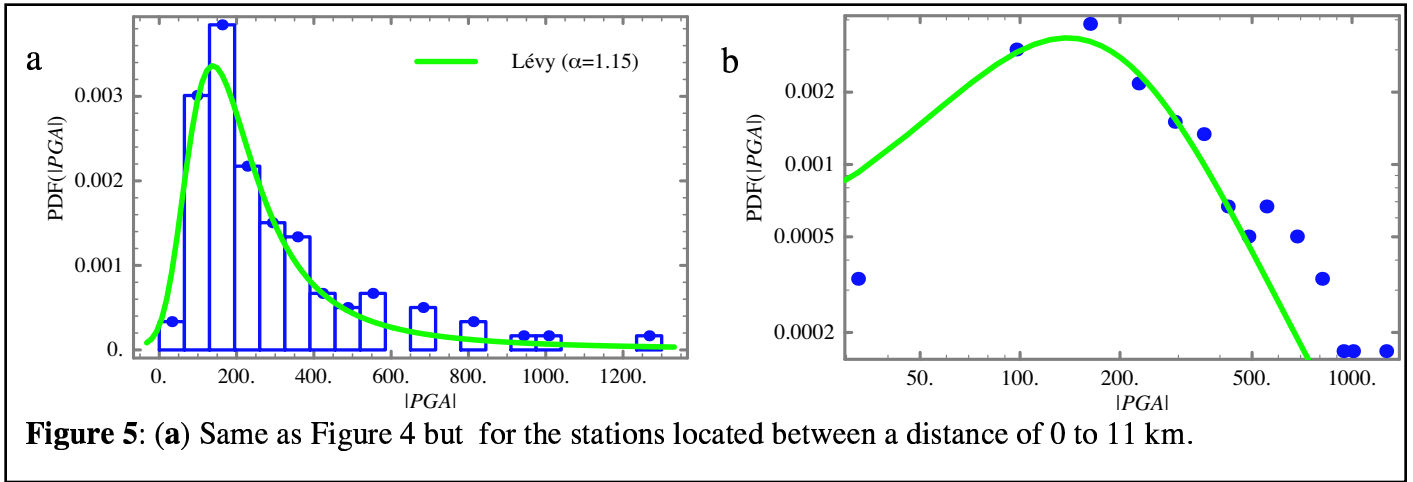
Figure 5: (**a**) Same as Figure 4 but for the stations located between a distance of 0 to 11 km.

Table 1: Parameters of the stochastic model for the dip and strike slip of the Parkfield earthquake. The parameter ν is the scaling exponent of the power spectrum (Figure 1). The parameters of the Lévy law that best fit the PDF(X) in Figure 2 are given.

	Scaling Exponent	Gauss law		Cauchy law		Lévy law			
	ν	μ	σ	γ	μ	α	β	γ	μ
Dip slip	0.92	10	171	101.	4.	1.26	0.0	387	14
Strike slip	1.40	25.	86.	64.	23.	1.11	0.0	93.	20

Table 2: Parameters of the Lévy law that best fit the PDF of the |PGA|.

Location of the stations (km)	Levy law			
	α	β	γ	μ
0-5	1.00	1.0	79.3	-13453.0
0-7.5	1.04	1.0	89.7	1470.5
0-11	1.15	0.93	140.1	593.0

References

Andrews, D. J. A stochastic fault model 1. Static case. *J. Geophys. Res.*, **78**, p. 3867-3877, 1980.

Custodio, S., P. Liu, P., and R. J. Archuleta, Prediction of near-source ground motion for the 2004 Mw 6.0 Parkfield earthquake: effects of using different data sets in the inversion, *Seism. Res. Lett.*, **76**, 211, 2005.

Lavallée, D., and R. J. Archuleta, Coupling of the random properties of the source and the ground motion for the 1999 Chi Chi earthquake, *Geophys. Res. Lett.*, **3 2**, L08311, doi:10.1029/2004GL022202, 2005.

Lavallée, D. and R. J. Archuleta. Stochastic modeling of slip spatial complexities for the 1979 Imperial Valley, California, earthquake. *Geophys. Res. Lett.*, **30** (5), 1245, doi:10.1029/2002GL015839, 2003.

Lavallée, D., P. Liu, R. J. Archuleta. Stochastic model of heterogeneity in earthquake slip spatial distributions. *Geophys. J. Int.*, in revision, 2005.

Mai, P. M., and G. C. Beroza. A spatial random-field model to characterize complexity in earthquake slip. *J. Geophys. Res.*, *107* (B11), 2308, doi:10.1029/2001JB000588, 2002.

Uchaikin, V. V., and V. M. Zolotarev. Chance and Stability. VSP, Utrecht, The Netherlands. 570pp, 1999.