2004 Annual Report

Hypothesis

Slip variability due to dynamic processes of earthquake ruptures produces heterogeneous strain drops and heterogeneous stress; therefore, stress in the crust is heterogeneous and can be described in a statistical manner. Short wavelength stress amplitudes can be large in comparison to long wavelength spatial averages of stress. This stress heterogeneity can bias which points fail as earthquakes; the bias will be towards stress orientations aligned with the tectonic stress rate tensor. If stress heterogeneity is large, which we believe it is, then background seismicity will primarily reflect the orientation of the stress rate tensor not the spatially averaged deviatoric stress tensor.

Results

We produced synthetic earthquake catalogs from numerical models of the crust (3D grids) that included three types of stress at each point: 1) A spatially and temporally uniform background tectonic stress, $\boldsymbol{\sigma}^{Background}$, 2) a spatially heterogeneous but temporally uniform, heterogeneous stress due to the summed effect of past earthquakes, $\boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x})$, 3) and a spatially uniform but linearly increasing with time secular tectonic stress that brings points to failure, $\dot{\boldsymbol{\sigma}}^{Secular}\left(t - t_{LastMajorEarthquake}\right)$. $\dot{\boldsymbol{\sigma}}^{Secular}$ is assumed to be spatially uniform and temporally constant for the space and time window of interest and can be estimated by converting strain rates from GPS measurements into stress rates. Hence, the total deviatoric stress tensor is:

$$\boldsymbol{\sigma}(\mathbf{x},t) = \boldsymbol{\sigma}^{Background} + \dot{\boldsymbol{\sigma}}^{Secular}\left(t - t_{LastMajorEarthquake}\right) + \boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x}) \tag{1}$$

See Figures 1 and 2 for a visual idea of our model.

$\boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x})$ was modeled statistically in the following manner. We used a random number generator $R(x,y,z)$ with a Gaussian distribution (white noise) in the three spatial dimensions to generate the 5 independent components of the deviatoric stress tensor. We then used low-pass filters that have no characteristic length scale (fractals) by fractionally integrating with respect to space. The formalism for this filtering can be expressed as follows:

$$\begin{aligned}\boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x}) &= \mathbf{FT}^{-1}_{\text{3-D}}\left[\hat{\boldsymbol{\sigma}}^{Heterogeneous}(\mathbf{k})\right] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{\boldsymbol{\sigma}}^{Heterogeneous}\left(k_x,k_y,k_z\right)\exp i2\pi\left(xk_x + yk_y + zk_z\right)dk_x dk_y dk_z\end{aligned} \tag{2}$$

where

$$\begin{aligned}&\hat{\boldsymbol{\sigma}}^{Heterogeneous}\left(k_x,k_y,k_z\right) \\ &= \boldsymbol{\sigma}_0^{Heterogeneous}\left(k_x^2 + k_y^2 + k_z^2\right)^{-\alpha/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(x,y,z)\exp - i2\pi\left(xk_x + yk_y + zk_z\right)dxdydz\end{aligned} \tag{3}$$

$\left(k_x^2 + k_y^2 + k_z^2\right)^{-\alpha/2}$ is a power law filter in vector wavenumber space, and $\boldsymbol{\sigma}_0^{Heterogeneous}$ is a constant tensor that scales the absolute size of the stochastic internal stress. We examined results for various levels of fractional filtering, including $\alpha = 0.0$ (no filtering), $\alpha = 0.35$, $\alpha = 0.5$, and $\alpha = 1.0$ (one full integration). Recent work by Liu and Heaton [2003 in preparation] on the variability of slip in space on faults suggests that $\alpha = 0.35$, which is equivalent to a 0.35 integration of the white noise with respect to space, may be compatible with observations of rupture vs. length scaling.

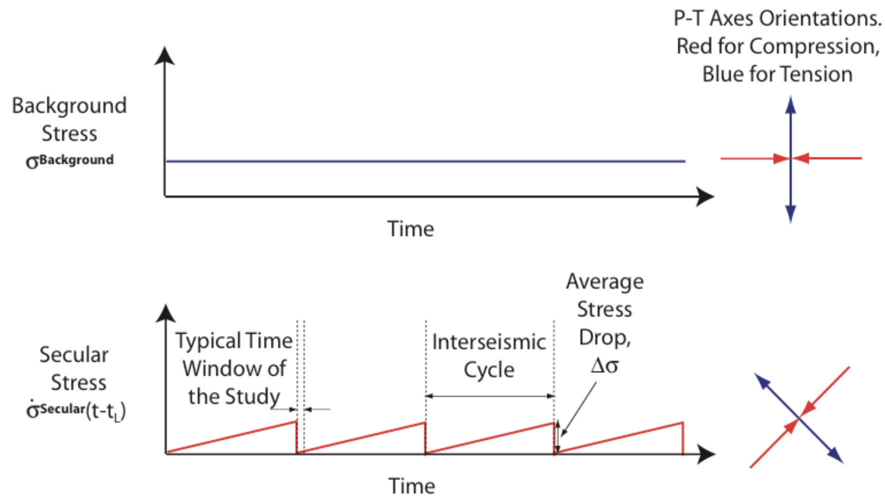

Figure 1. A simplistic view of the earthquake cycle. Spatially uniform stress is composed of a temporally constant, background stress, and a time varying, secular stress. The secular part grows with time then is reset to a lower level during a major earthquake. The three main points we would like to bring out of this figure are: 1) The typical time window of our simulations will be small in comparison to the interseismic cyle, 2) therefore, we will make the approximation that the secular stress rate is constant in time, 3) and last that the background stress may have a different orientation from the secular stress rate.

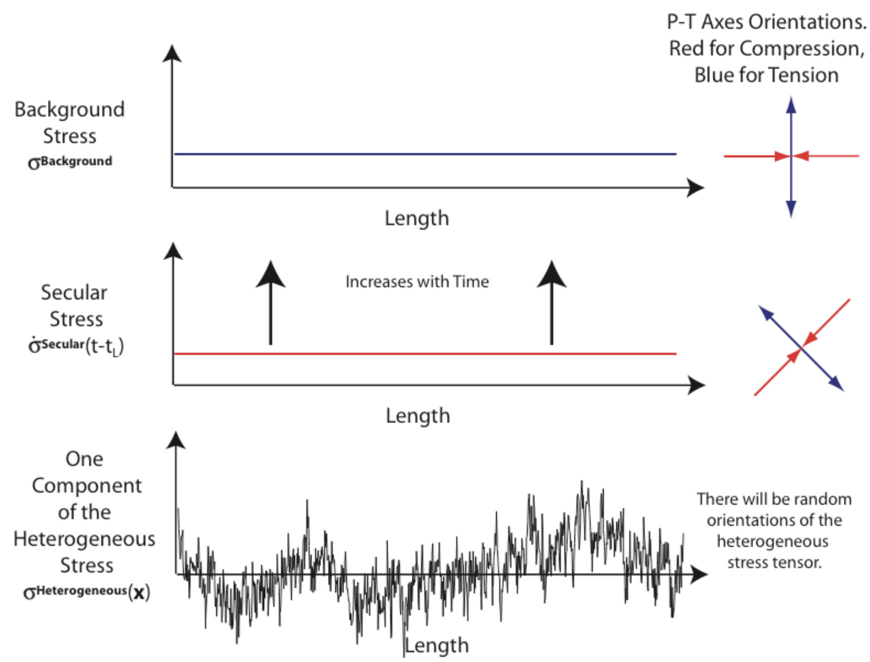

Figure 2. This shows the three quantities of our stress tensor as a function of space. Both the background and secular stress are uniform in space. The heterogeneous quantity is modeled with random statistical fluctuations as a function of space. The heterogeneous stress tensor will have random orientations as a function of space.

To create failures in our grid, i.e., synthetic earthquakes, we used a plastic yield criteria, the Hencky-Mises yield condition [*Housner and Vreeland*, 1965]. This criteria is a measure of the maximum shear stress regardless of orientation, and once a threshold value is exceeded, failure occurs. The measure used is $I_2^{'}$, the second invariant of the deviatoric stress matrix, $\boldsymbol{\sigma}(\mathbf{x},t)$. When

$$I_2^{'} = \frac{2}{3}\sigma_0^2 \tag{4}$$

where σ_0 is the yield stress in uniaxial tension, we have failure and a focal mechanism is calculated. If the summation of $\boldsymbol{\sigma}^{Background}$ and $\boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x})$ at a particular point in the grid is,

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} \tag{5}$$

then we know what $I_2^{'}$ at time = 0 will be

$$I_2'(t=0) = \sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + 2\left[\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2\right] \tag{6}$$

Now let's add some tectonic stress. If the tectonic stress rate as calculated from GPS is,

$$\dot{\boldsymbol{\sigma}}^{Secular} = \begin{pmatrix} 0 & \dot{\sigma}_{xy} & 0 \\ \dot{\sigma}_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{7}$$

then at some time later, Δt, the total deviatoric stress at that point in the grid will be,

$$\boldsymbol{\sigma} + \dot{\boldsymbol{\sigma}}^{Secular}\Delta t = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} + \dot{\sigma}_{xy}\Delta t & \sigma_{xz} \\ \sigma_{xy} + \dot{\sigma}_{xy}\Delta t & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} \tag{8}$$

and $I_2^{'}$ at Δt will be,

$$I_2'(t=\Delta t) = \sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + 2\left[\left(\sigma_{xy} + \dot{\sigma}_{xy}\Delta t\right)^2 + \sigma_{yz}^2 + \sigma_{xz}^2\right] \tag{9}$$

Hence, the change in $I_2^{'}$ from time = 0 to time = Δt is

$$\Delta I_2' = I_2'(t=\Delta t) - I_2'(t=0) = 4\sigma_{xy}\dot{\sigma}_{xy}\Delta t + 2\dot{\sigma}_{xy}^2\Delta t^2 \tag{10}$$

We then ask, which points will be most likely to fail between time = 0 and Δt? In other words, which points will have the largest $\Delta I_2'$? Even in the case, where $\dot{\tau}_{xy}\Delta t$ is small, we find that the cross term, $4\sigma_{xy}\dot{\sigma}_{xy}\Delta t$, biases which points will fail. When the stress rate pulls out a cross term that explicitly depends on stress in the same direction as the stress rate, it increases the likelihood that points with orientations aligned with the stress rate will fail. This bias grows as the $Ratio = \dfrac{Standard\ Deviation\left(\boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x})\right)}{Norm\left(\boldsymbol{\sigma}^{Background}\right)}$ increases because as the relative magnitude of the random heterogeneity increases, the orientation of the failure stress tensor will be more influenced by any $\boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x})$ aligned with $\dot{\boldsymbol{\sigma}}^{Secular}$ rather than $\boldsymbol{\sigma}^{Background}$. Interestingly, this is an effect we see numerically, in our results.

After producing synthetic earthquake catalogs, we examined the results a few different ways. Some of them include: 1) Plotting the focal mechanisms to compare their orientations to real data. See Figure 3. 2) Plotting the P-T stress axes on an equal area plot to compare with real data. See

Figure 4. 3) Then inverting the focal mechanisms with a standard stress study tool, Andy Michael's program *slick*.

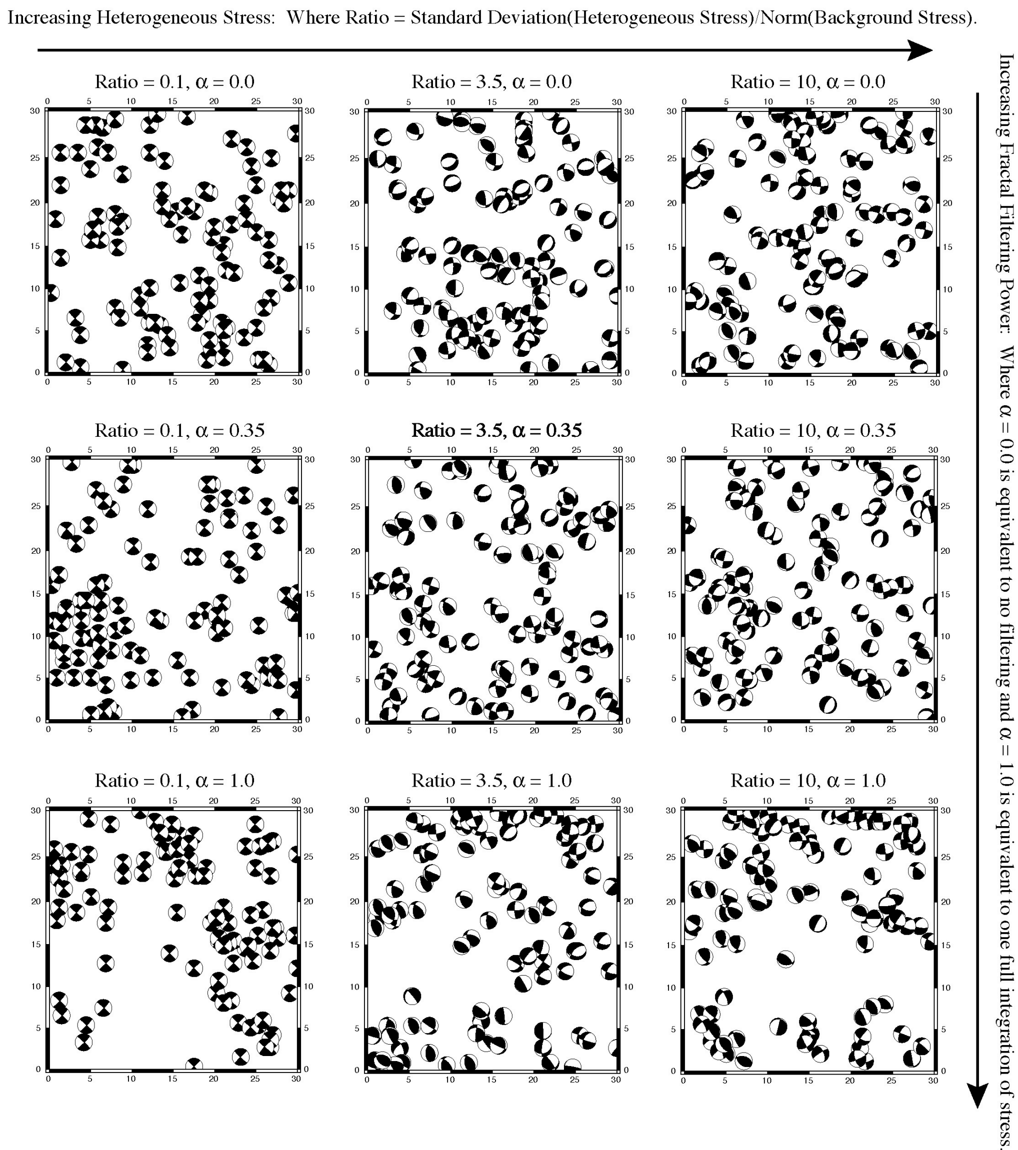

Figure 3. In each box, the first 100 focal mechanisms are plotted for a particular set of simulation parameters. Stress heterogeneity, or the *Ratio* increases from left to right, and the fractal wavelength filtering of stress heterogeneity increases from top to bottom. First, as stress heterogeneity increases, the heterogeneity of focal mechanism orientations increase. The more heterogeneous cases of $Ratio = 3.5$ or $Ratio = 10.0$ seem to better correlate with real data. Second, as fractal wavelength filtering of the heterogeneous stress increases, the earthquakes increasingly clump together spatially. From Jiang and Heaton [2003 in preparation] we know that $\alpha = 0.35$, is most compatible for rupture vs. length scaling. Given this evidence and our next figure, Figure 4, we suggest that the center panel, $Ratio = 3.5$ and $\alpha = 0.35$ (not taking into account pre-existing faults) is most representative of the real Earth.

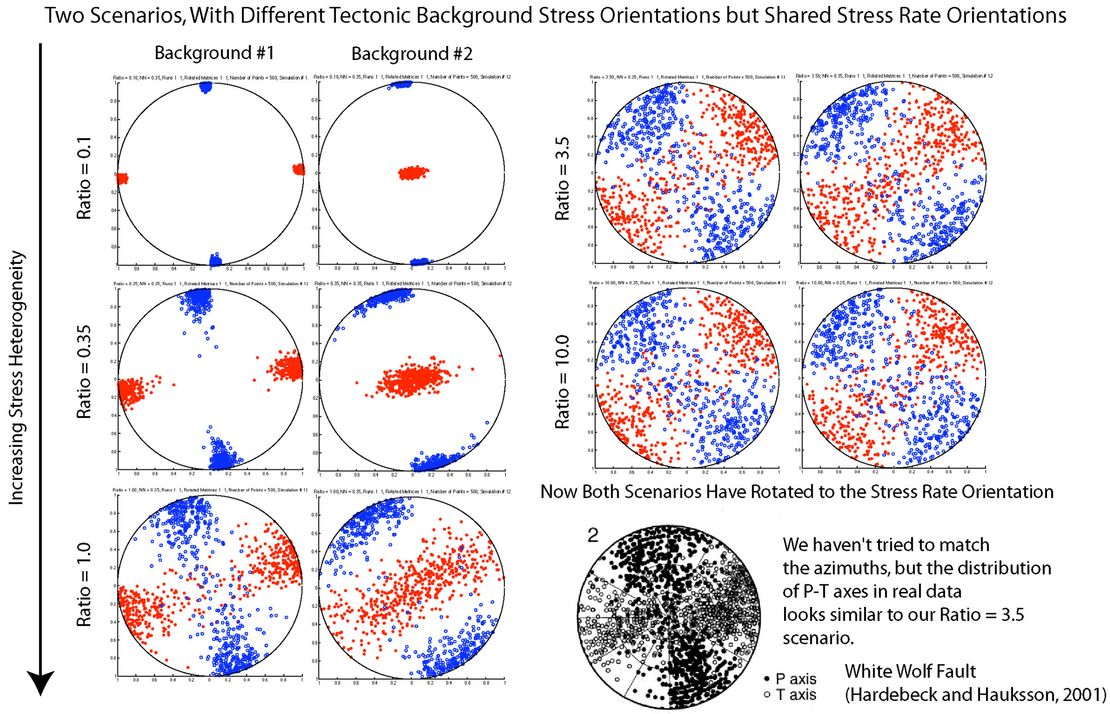

Figure 4. Demonstration of what happens to P-T plots as stress heterogeneity increases, and comparison to real data. We start with two scenarios that have different $\boldsymbol{\sigma}^{Background}$ orientations. For small $\boldsymbol{\sigma}^{Heterogeneous}(\mathbf{x})$, i.e., $Ratio \ll 1$, the points that fail have orientations very similar to $\boldsymbol{\sigma}^{Background}$. However, as $Ratio$ increases the two scenarios P-T axes begin to rotate toward the stress rate tensor orientation, $\dot{\boldsymbol{\sigma}}^{Secular}$. Since the two scenarios have the same $\dot{\boldsymbol{\sigma}}^{Secular}$ applied, the end result for $Ratio \gg 1$, looks very similar. Interestingly, as $Ratio$ increases, the heterogeneity in P-T axes also increases. When one compares this to the White Wolf Fault [Hardebeck and Hauksson, 2001], one finds that a $Ratio \approx 3.5$ is very suggestive of real data.

Conclusion

We find the following: 1) Heterogeneous stress follows logically from dynamic slip variations in earthquake ruptures. 2) Heterogeneous stress in our models can reproduce the focal mechanism and P-T axes heterogeneity seen in real data, yielding additional evidence for stress heterogeneity in the real Earth. 3) Standard inversions of our synthetic focal mechanism catalogs demonstrate that heterogeneous stress affects the final inverted stress tensor. In the presence of significant stress heterogeneity, the stress tensor solution orientation tends to align more with the tectonic stress rate than with the tectonic background stress. This indicates that studies which use focal mechanism inversion tools may need to be reinterpreted.