Towards High Order Octree-Based Finite Element Earthquake Simulations

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Abstract

Recent advances in the use of octrees highlight the advantages and drawbacks of this data structure when used in conjunction with the finite element method to solve large-scale wave propagation problems. Challenges in the incorporation of topographical features and rheologies that rely on strains are among the difficulties found in low-order and nonconforming formulations. Some researchers have proposed approximations to deal with topography and plasticity models, but results vary, and systematic analyses of the accuracy of these efforts are inconclusive. Guaranteed improvement in performance in any implementation involves incrementing the polynomial order of the field approximation, as in most standard finite element codes. In this work we explore and present preliminary results of simple approaches to use octree-based meshes for obtaining conforming meshes using cubes, tetrahedral, and pyramidal elements, which facilitate the incorporation of topographies and high-order elements without losing the semistructured character of octrees and the inherent advantages in memory usage.

Introduction

Advances in high-performance computing and numerical methods have made possible realistic and complex simulations of regional and global wave propagation. Simulations provide a deep understanding of the effect of heterogeneity, topography, and site effects in several regions, whereas at global scale researchers using the spectral element method have been able to compute synthetics with resolutions of 20 seconds or less. Finite elements with octree-based mesh (OBM) implementations are one of the many approaches that offer a compromise between meshing simplicity and performance.

Low-order finite elements, typically used with OBM, require more than 12 points per wavelength to model the wave propagation accurately. Nevertheless, when large Vp to Vs ratios are involved in the calculation, accuracy might drop due to spurious modes inherent in the displacement formulation.

Non-conforming octree-based meshes, e.g., the Quake CMU Hercules implementation, enforce the continuity of displacements via Lagrange multipliers. Even though the latter has proven to be effective in some instances, it does not allow increments in the order of approximation of the field under consideration automatically. Here we review a simple approach to transform the nonconforming mesh into a conforming one, by inserting additional degrees of freedom and pyramidal and tetrahedral elements.

Octrees and Meshing Approach

Octrees are an approach to subdivide a 3D domain into octants. Their counterpart, the quadtree, divides a bidimensional space into rectangles. One of the advantages of the octree is that it can be visualized as a tree structure, whose leaves are the octants that can represent a mesh (see figure 1) to be used in any domain numerical method that can take advantage of the structure.

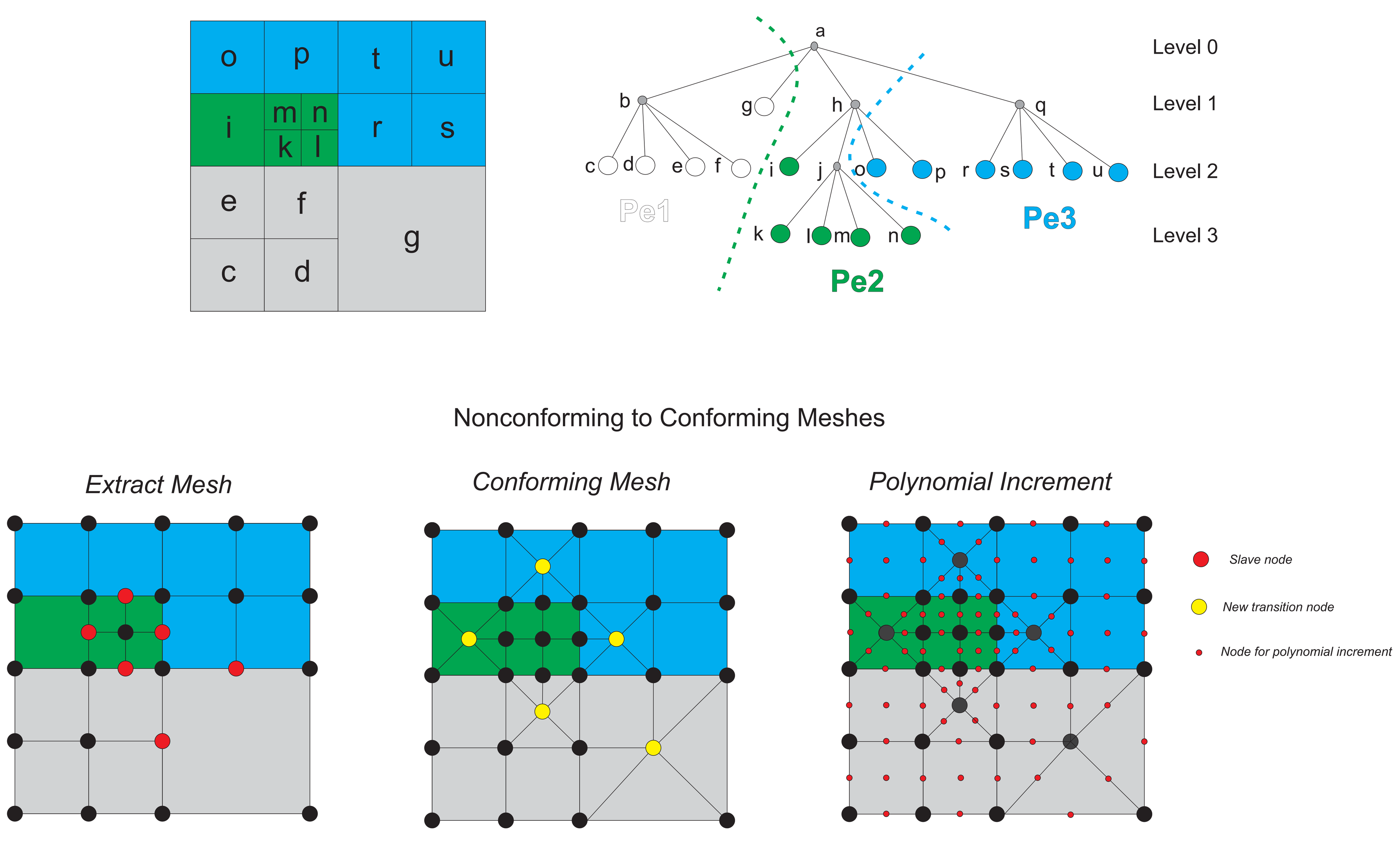

Figure 1. Octree structure and domain meshing approach. A) Extract mesh. B) Identify transition elements and add a new node to generate additional elements. C) Addition of degrees of freedom

Transition Elements

In order to enforce continuity in the OBM, we analyzed possible transitions, the creation of new elements, and the insertion of degrees of freedom. We use the master cube in figure 2 as a reference.

To avoid slave degrees of freedom, we need to guarantee that all nodes on the edges and faces are contained in contiguous elements. Some examples of the possible configurations are depicted in figure 3.

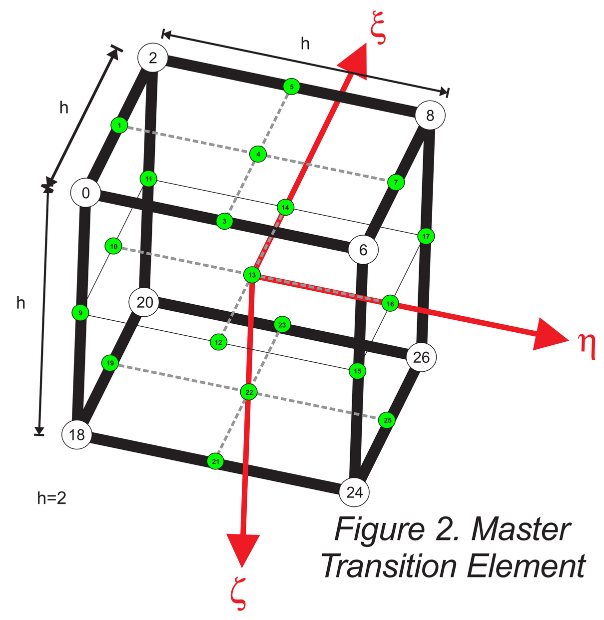

Figure 2. Master Transition Element

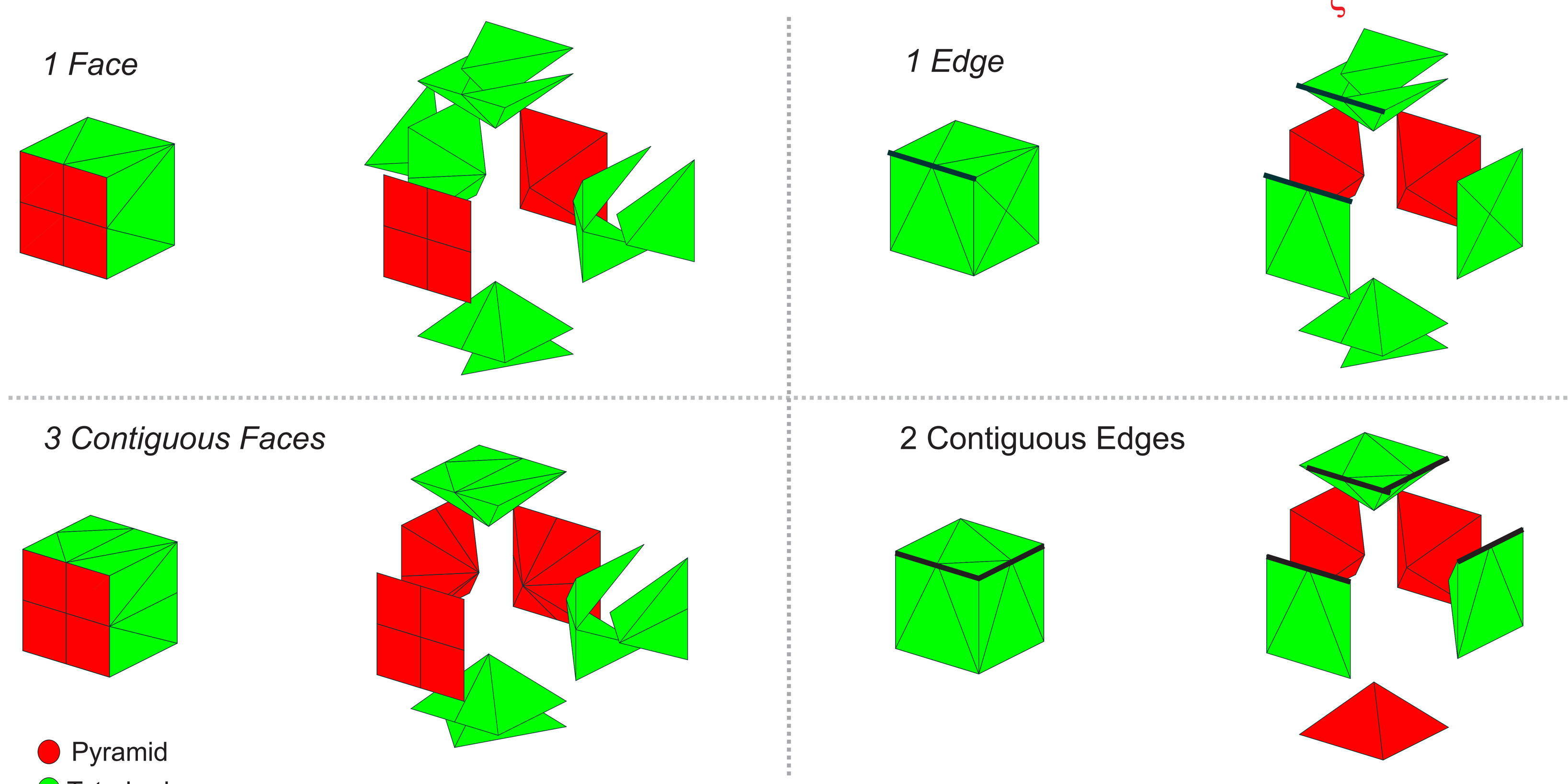

Figure 3. Some transition element decomposition.

The enumaration of all cases is error prone. In practice, any transition octant can be divided into six pyramids which are decomposed into tetrahedra depending on the nodes in the pyramid´s base.

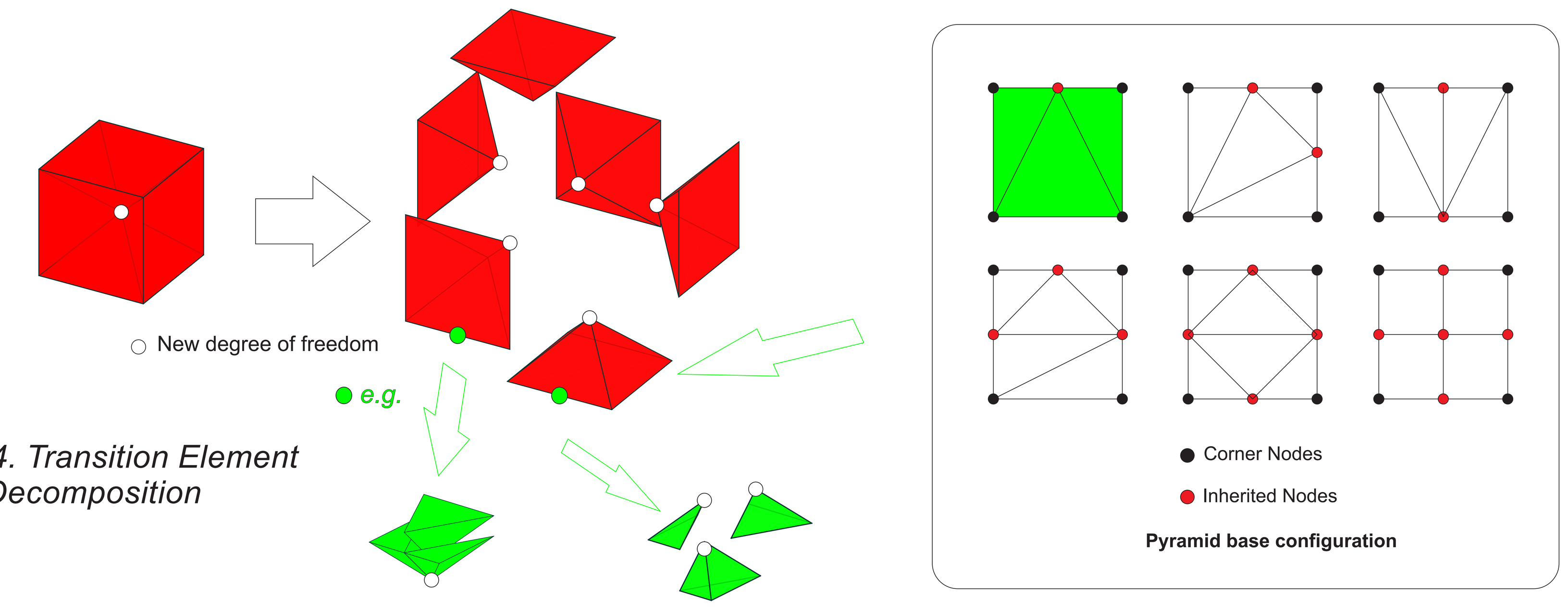

Figure 4. Transition Element Decomposition

Figure 5 illustrates a spherical low-frequency mesh obtained following the methodology outlined in figure 3. Note that the free surface is not represented correctly, but in some cases the tetrahedra helped smooth the staircase boundary.

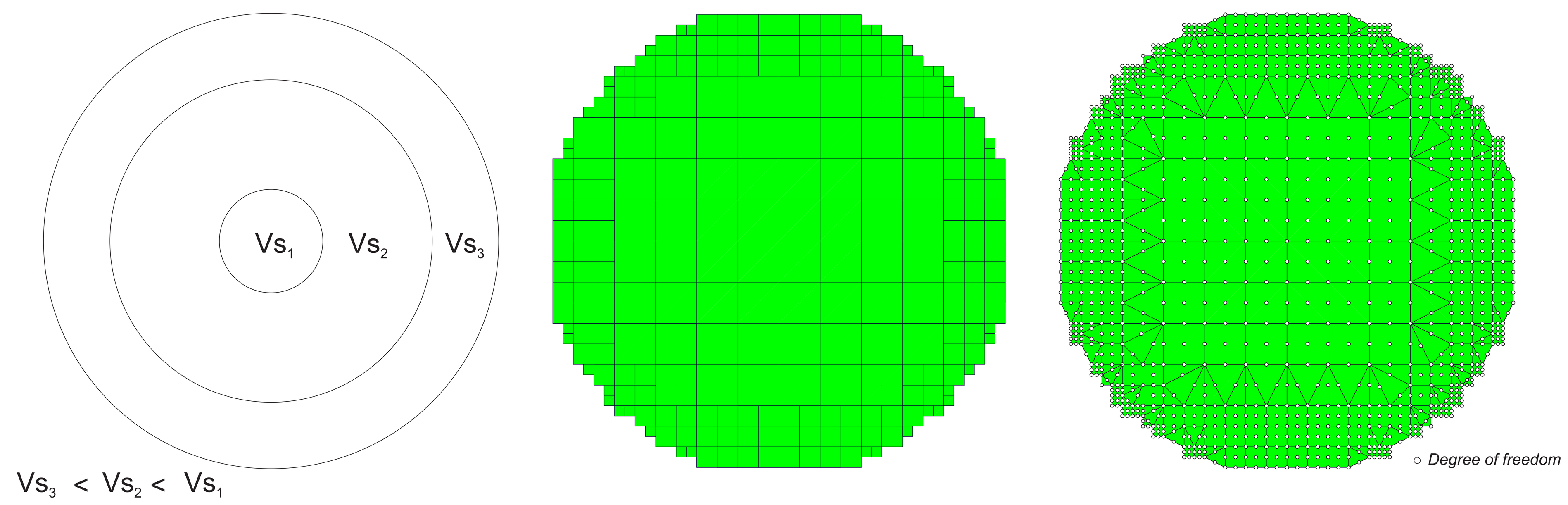

Figure 5. Improved meshing.

Comments and Ongoing Efforts

- The advantages of the elements resulting from an octree-based mesh remains. Only a new set of element stiffness stencils are necesary.
- Testing and verification of conforming meshing is in progress.
- A marching cubes algorithm is being evaluated as a means of improving the surface topography.
- Finite element solver with improved elements that account for large Vp/Vs ratios.
- Coupled fluid-solid interfaces.

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