1. Summary

- Goal: Quantifying the transition from distributed events to a system-size failure in Acoustic Emission (AE) data.
- Data: Several data sets (Panel 2) from different experiments (Refs. 1,2,3,4,5) provide information on the failure processes in different rock types (granite, sandstone), loading conditions (constant load rate vs. servo-controlled to maintain constant AE rate) and sample properties (pre-existing notch vs. intact sample).
- Method: Each data set is considered a realization of a coalescence process (Panels 3,4) where earlier failures tend to merge with later ones. The merging of two existing failures can only occur via a new failure that connects the two. Healing is ignored and a global coalescence failure structure progressively grows in size by merging with other failures. Mergers (failures that unify earlier failures) are defined as the nearest-neighbors of the coalescing events with respect to a given space-time-magnitude distance (Panel 3). A catalog of AE events is represented with this simplified framework by a time-oriented tree, whose vertices are the catalog events with the root in the last event.

In this work we show that:
- The coalescence index $C = 1 - b$, where $b$ is the branching number for vertex $i$. The index $C$ is always equal to 1 or less than 1. It quantifies the intensity of coalescence: $C = 1$ (or $b = 0$) corresponds to no coalescence, $C < 0$ ($b > 1$) indicates equality of coalescence and event occurrence rates, and $C < 0$ ($b > 1$) indicates that coalescence dominates generation of new events.

2. Acoustic Emission Data

Constant displacement rate samples (1,2,3). Triaxial compression experiments [6,7,8,9,10] were performed on granite samples of 4–5 cm in diameter and 10–11 cm in height, with a constant displacement rate of 20 μm/min. The experimental conditions favored the formation of localized fractures zones in two Westerly granite samples. No large-scale imperfections were identified in sample Wgp01. Large-scale imperfections in the Wgp07 sample were introduced by two 15 mm deep saw-cut notches inclined at 30° to the loading axis.

3. Coalescence process reconstruction

We consider an AE sequence as a realization of a coalescence process. Specifically, we assume that recorded cracks merge with other cracks. A merger of two existing cracks can only occur via a new crack that connects the two existing ones (there are no alien mergers). Healing is ignored. A crack can only grow in size by merging with other cracks.

We start with a catalog of AE events (cracks), each of which is characterized by its occurrence time $t_i$, location $(x_i, y, z)$ and magnitude $m_i$ (amplitude). A coalescence process is represented by a tree $T$ that is a collection of vertices $v$ connected by edges $e = (v_i, v_j)$.

The vertices (leaves and internal ones) correspond to individual AE events, the edges correspond to coalescences (mergers) of events, and the length of edge $e = (v_i, v_j)$ equals the proximity between events that correspond to the vertices $v_i$ and $v_j$ (Panel 4).

This reflects the assumption that cracks can only merge via new cracks. A coalescence tree $T$ for a given catalog of events is constructed by finding for each event a later merger event that merges with it at the instant of occurrence and connecting them in a tree. Since all connections are oriented from an earlier event to a later event and since all events become connected, the resulting graph is a time-oriented tree. Different rules of mergers lead to different coalescence trajectories (different trees $T$) in this study, we use the nearest-neighbor approach for identifying event's merger. Specifically, we define a proximity of event $i$ to a later event $j$ as

$$
\eta_{i,j} = \frac{t_j - t_i}{t_j - t_i} 10^{\text{loss}}, \quad t_j > 0, \quad \infty, \quad t_j < 0,
$$

where $t_i$ is the respective inter-event time, $z_i$ is the spatial distance between events, $d_i$ is the facsimile dimension of event locations, and $b$ is the parameter of exponential approximation to the size distribution of cracks (Gutenberg-Richter law's b-value). For every event $i$ in the catalog we find its nearest neighbor $j$ according to the proximity (1). This event $j$ is called the merger of $i$. A coalescence tree $T$ for an AE loading is constructed by stacking each event in a catalog to its nearest-neighbor merger in time.

4. Coalescence statistics

4.1 Coalescence index $C$: For vertex (event, crack) $i$ is defined as $C = 1 - b$, where $b$ is the branching number for vertex $i$. The index $C$ is always equal to 1 or less than 1. It quantifies the intensity of coalescence: $C = 1$ (or $b = 0$) corresponds to no coalescence, $C = 0$ ($b = 1$) indicates equality of coalescence and event occurrence rates, and $C < 0$ ($b > 1$) indicates that coalescence dominates generation of new events.

4.2 The number of unmerged events (cumulative coalescence index). A useful statistic, closely related to the coalescence index of Sect. 4.1, is the cumulative number $M(t)$ of unmerged events up to instant $t$. Notice that $M(t)$ is the number of unmerged events $C$ for events that occurred up to instant $t$. It also can be represented via the cumulative number $N(t)$ of events up to instant $t$ by the cumulative number $M(t)$ of (merged) events up to $t$. To get $M(t)$, the cumulative number $N(t)$ of events up to instant $t$ is divided by the number of events $N(t)$.

5. Results: Stages of failure process

5.1 Coalescence representation of an AE sequence – an illustration. The synthetic sequence consists of six events and coalescence failures structure progressively grows in size by merging with other failures. Mergers (failures that unify earlier failures) are defined as the nearest-neighbors of the coalescing events with respect to a given space-time-magnitude distance (Panel 3). A catalog of AE events is represented with this simplified framework by a time-oriented tree, whose vertices are the catalog events with the root in the last event.

5.2 Cumulative number $N(t)$ of events (a) and cumulative number $M(t)$ of mergers (green). Notice that $M(t)$ is approximately equal to $M(t)$ and both are increasing prior to the main failure (red vertical line) in all experiments.

Acknowledgement

We thank Thomas Goebel and David Lockner for sharing the AE data. The work was supported by SCEC (projects 16023 and 17065), and NSF (EAR 1722561 and EAR 1723033).