Non-parametric Hawkes models with strike angle covariates

James Molyneux, Frederic Paik Schoenberg

Abstract: Earthquake focal mechanisms have been posited to have predictive value for forecasting future seismicity. For strike-slip earthquakes, aftershocks should occur roughly along the estimated mainshock strike. However, errors in such strike angles is considerable. We compare the degree to which estimated strike angles forecast the direction of future seismicity to that of uniformly distributed angles and to strike angles estimated based on previous seismicity. The fit of non-parametrically estimated Hawkes models using the estimated strike angle that best fits the post-mainshock set of events for each mainshock is compared to that of corresponding models that exclude these estimates. Strike angle estimates are shown to have marginal predictive value for forecasting the direction of future seismicity, but no more than the better-fitting of a uniformly distributed angle and its complement.

Methods

Strike angle estimates

Each earthquake with a focal mechanism, referred to as mainshocks hereafter, has two nodal plane strike angles, denoted \( \phi \), for the ith mainshock. The strike angles for each mainshock are compared to uniformly random angles, denoted \( \phi_i \) and two strike angles estimated using a set of earthquakes, \( \hat{\phi}_i \), which occur before each mainshock and within a distance of \( a(M) = 10 \times 10^{a(M - 3.2)} \) from each mainshock. The two estimation methods used are:

- Deming regression, denoted \( \hat{\phi}_i \), estimates the slope of the line minimizing the orthogonal distances from the line to each earthquake in \( S_i \).
- Minimum absolute angle, denoted \( \hat{\phi}_i \), estimates the slope of the line minimizing the absolute values of the angles to each earthquake in \( S_i \).

Since each mainshock has two strike angles, we also evaluate angles which are orthogonal to \( \hat{\phi}_i \), \( \hat{\phi}_j \), and \( \hat{\phi}_k \).

Evaluating strike angles

Two fitting criteria are used to evaluate how well each strike angle estimate and its orthogonal component fit seismicity occurring after each mainshock.

- Root mean square error (RMSE), where errors are defined as the orthogonal distance between the line whose slope is derived from the strike angle estimates and the events occurring after mainshock within a distance of \( a(M) \).
- Mean absolute value of the angles formed by the line whose slope is derived from the corresponding strike angle estimate and the same set of events occurring after each mainshock.

When considering estimated strike angles and their orthogonal components, we indicate the best fitting strike angle for each mainshock and for each estimation method with a tilde (~).

Non-parametric Hawkes models

Non-parametric Hawkes models are used to compare the predictive performance of point process models with strike angle estimates to those without. The conditional intensity function for these models is \( \lambda(t, \hat{\phi}_i, \phi) \), \( \lambda(t, \hat{\phi}_j, \phi) \), \( \lambda(t, \hat{\phi}_k, \phi) \), and \( \lambda(t, \phi) \), where \( \phi \) is the strike rate, \( \hat{\phi}_i, \hat{\phi}_j, \hat{\phi}_k \) are densities governing the temporal and spatial triggering, and \( \phi \) dictates how the productivity of aftershocks depends on the mainshock magnitude. Here, \( \hat{\phi}_i \) is the time of the ith event, \( \hat{\phi}_j \) is the epicentral location, \( \phi_i \) is the magnitude, and \( \phi \) is the best fitting strike angle from the ith-double-couple.

Deviance residuals and Voronoi deviance residuals

Deviance residuals and Voronoi deviance residuals are used to compare the fit of models which include or exclude the best nodal plane strike angles. Let \( \hat{\phi}_i \) indicate a bin in a rectangular grid or cell in the Voronoi tessellation of the mainshock locations. The deviance for two different intensity estimates, \( \hat{\phi}_i \) and \( \hat{\phi}_j \), over cell \( \hat{\phi}_i \) is

\[
R_i(K) = \sum_{(i,v,c) \in K} \log \left( \lambda(t, \hat{\phi}_i, \phi_i) \right) - \int_{\hat{\phi}_i} \lambda(t, \hat{\phi}_i, \phi_i) \, d\phi \cdot \sum_{(i,v,c) \in K} \log \left( \lambda(t, \hat{\phi}_i, \phi_i) \right) - \int_{\hat{\phi}_j} \lambda(t, \hat{\phi}_i, \phi_i) \, d\phi
\]

Positive deviance residuals imply model \( \hat{\phi}_i \) provides a better fit to the data in the given region and negative values imply the opposite. The total deviance for the competing models is \( \sum_i R_i(K) \) with values close to zero indicating minimal difference in fit of the two competing models.

Results

Figure 1: Estimated strike angle performance

Figure 2: Estimated intensities

Figure 3: Deviance residuals

Figure 4: Voronoi deviance residuals

Discussion

Figure 1 shows the mean ± 1.96 standard errors of the RMSE or mean absolute angle for the 330 mainshocks. Each pane also includes the angle \( \hat{\phi}_i \) which represents the best angle for each mainshock based on the post-mainshock seismicity.

- Figure 1 (a) & (c) indicate that nodal plane strike angles fit post-mainshock events slightly better in terms of RMSE and substantially better in terms of mean absolute angle compared to strike angles estimated based on previous seismicity.
- Including a second, orthogonal estimate for \( \hat{\phi}_j \) and \( \hat{\phi}_k \), as shown in Figure 1 (b) & (d), and using the angle that fits the data best for each mainshock, as was done for \( \hat{\phi}_i \), led to a substantial improvement in both \( \hat{\phi}_j \) and \( \hat{\phi}_k \) with each fitting better than \( \hat{\phi}_i \).
- In fact, \( \hat{\phi}_i \) aftershock seismicity no better than \( \hat{\phi}_i \), the latter of which was simply the better-fitting of a random, uniformly distributed angle and its complement.
- The forecasting ability of \( \hat{\phi}_i \) is too be entirely explained by the fact that the better-fitting of the nodal plane and its complement is chosen as the estimated nodal plane.
- Comparing the results of each evaluation method to \( \hat{\phi}_i \) provides a sense of the amount of error inherent in the strike angle estimates.

For instance, Figure 1 (d) shows the mean absolute angle to aftershocks for \( \hat{\phi}_i \) was 29.49° and that of \( \hat{\phi}_i \) was 35.55°, thus the nodal plane estimate includes about 6.6° more uncertainty, on average, beyond what can be explained by the variability in the aftershock events alone.

Figure 2 displays the intensities of a saturated model, which includes \( \hat{\phi}_i \) in the triggering function, and a null model, which does not.
- Both intensity functions appear to be similar with the exception that the saturated model appears slightly more diffuse in regions of relatively low intensity.

Figures 3 & 4 show the deviance residuals and the Voronoi deviance residuals when comparing the null and saturated models.
- Both figures indicate that including \( \hat{\phi}_i \) led to an improvement in model fit which indicates some potential advantages of using strike angle estimates based on estimated nodal planes in earthquake forecasting.
- Whether the improvement shown comes from the accuracy of the estimated nodal planes or from choosing the better-fitting strike angle of the two nodal planes is unclear.
- The total deviance of Figure 3 is 6% in favor of the saturated model.

Contact information

James Molyneux
jmo@ucla.edu

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Data

Earthquake data for the Southern California region is contained in two earthquake catalogs, one of which features focal mechanism information. Both catalogs are constrained to contain earthquakes that are shallow (depth ≤ 75 km) with magnitudes ≥ 3.25.

Focal mechanism catalog:
- Contains 330 earthquakes from 1999 to 2016 with strike-slip nodal plane orientations.
- Features events with variance reduction, a measurement of focal mechanism quality, in excess of 40.

General earthquake catalog:
- Location qualities of C or better determined by SCEC.

References

[References list]

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