

# A Flexible and Memory-Efficient Displacement-Based Approach to Modeling Attenuation in Wave Propagation

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Attenuation in the form of energy losses due to internal friction is a major factor in wave propagation and ground motion simulation problems. These effects change the amplitude and dispersion characteristics of seismic waves, especially over large distances or in highly dissipative media. Attenuation is usually characterized in terms of the quality factor,  $Q$ , and represented by means of viscoelastic models made of springs and dashpots. A recently introduced model, called the BKT model (after authors Bielak, Karaoglu and Taborda), proposed the use of only three mechanical elements in parallel, two Maxwell elements (each made of a spring and a dashpot connected in series) and a Voigt element (consisting of a spring and a dashpot connected in parallel). The BKT model showed very good adherence to intended values of constant  $Q = Q_0$  in the frequency domain, and its implementation in the time domain required a reduced number of memory variables compared to other methods used in simulation. Though accurate and efficient, the implementation of the original BKT model required a fixed pre-computed set of auxiliary variables that had to be interpolated during running time for arbitrary  $Q_0$  target values, and was limited to frequency-independent  $Q$ . In this work, we expand the BKT model introducing a more flexible implementation in which the model parameters are set in run time using empirical expressions formulated based on a numerical optimization of the model's fit with the target  $Q$ . We also extend the BKT model by adding a third Maxwell element to improve accuracy permit the simulation of frequency dependent  $Q = Q(f)$ . The addition of a third Maxwell increases memory usage but the model still retains its memory-efficiency in comparative terms with respect to other alternative implementations. More important is the fact that this expansion allows us to model frequency dependent  $Q$ , a critical factor in deterministic physics-based earthquake simulations done at frequencies of engineering interest ( $f > 1$  Hz). In this work we present initial tests of the new BKT models with respect to semi-analytical solutions of idealized problems 1D and 3D, including comparisons with an equivalent frequency-wavenumber implementation. The results show that the model behaves well.