An Equivalent Linear Approach to Material Nonlinearity in Regional Simulations on a Reduced Domain

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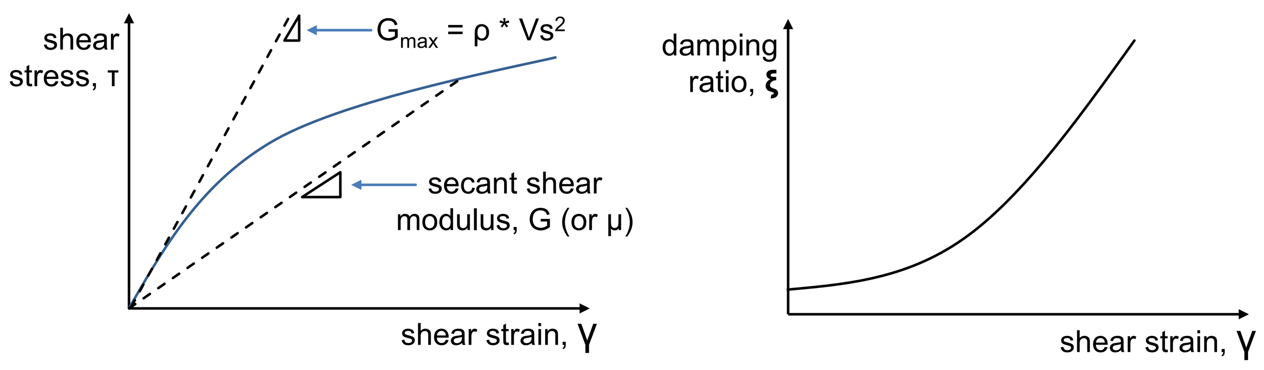
1A. Equivalent Linear Method: Motivation

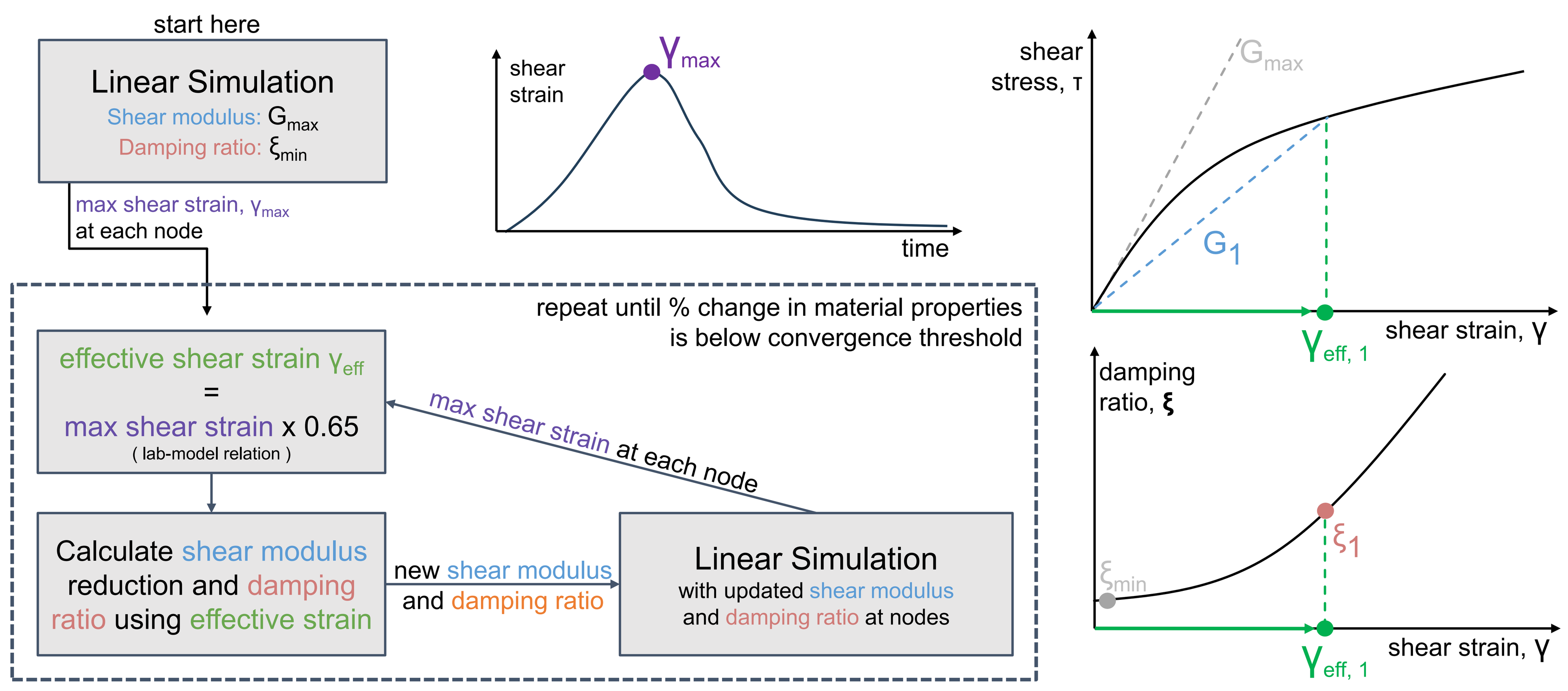
At frequencies of engineering interest (e.g. ~10 Hz), for the large earthquakes (e.g. M6+) that drive the bulk of the risk, soils in soft sedimentary basins are known to experience significant ground deformations, consequently exhibiting non-negligible nonlinear behavior.



SW4 (Seismic Waves, 4th Order), the finite difference code at the heart of EQSIM, simulates **linear** wave propagation to fourth order accuracy.

The equivalent linear method (EQL), which produces a linear approximation of nonlinear material behavior, allows SW4 to account for the softening of soils at large strains without losing the strengths of linear simulation.

1B. Equivalent Linear Method: Iterative Procedure



The equivalent linear method follows an iterative procedure, where material properties are updated between iterations of the same simulation following material curves generated by Darendeli (2001).

1C. Scalar Strain Measure

EQL uses a scalar measure of strain to compute material property changes. A common choice is the maximum shear strain on the coordinate frame rotated 45 degrees from the principal axes:

$$\gamma_{max}(\mathbf{x},t) = \epsilon_3(\mathbf{x},t) - \epsilon_1(\mathbf{x},t),$$
$$\text{given that } \epsilon_3 \geq \epsilon_2 \geq \epsilon_1$$

However, this is computationally slow, as it requires solving an eigenvalue problem at every time step, for every grid point.

We propose a 5x faster approximation for maximum shear strain, which uses the 2nd invariant of the strain deviator tensor (J_2):

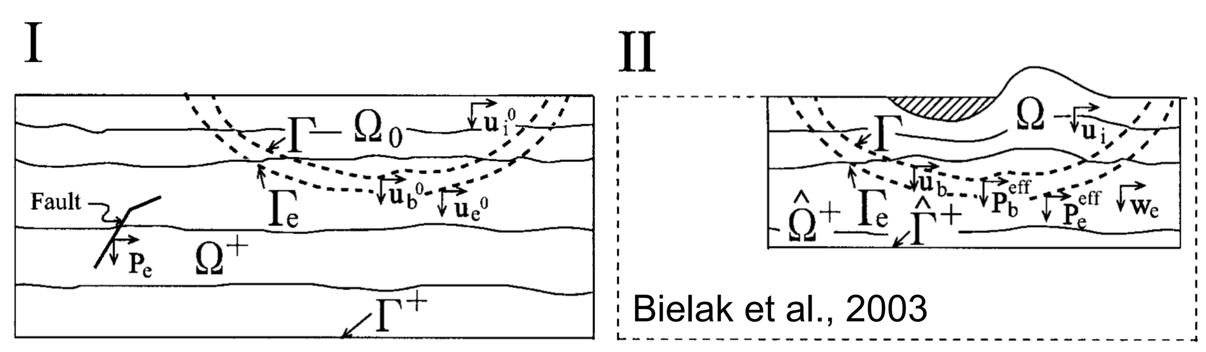
$$\gamma_{max}(\mathbf{x},t) \approx \sqrt{4J_2(\mathbf{x},t)}$$
$$J_2(\mathbf{x},t) = 1/3 \cdot \text{tr}(\epsilon(\mathbf{x},t))^2 - 1/2 \cdot [\text{tr}(\epsilon(\mathbf{x},t))^2 - \text{tr}(\epsilon(\mathbf{x},t)^2)]$$

2A. Domain Reduction Method: Motivation

Regional scale models in SW4 are composed of billions of grid points, so iterating simulations on such models is too computationally expensive.

Instead, we borrow the already existing concept of domain reduction (as originally proposed by Bielak et al., 2003) from the EQSIM framework.

In the domain reduction method (DRM), a distant earthquake source is fully represented by forces prescribed on the boundary of the reduced domain.



The goal is to implement the DRM in SW4, so it can be used in combination with the equivalent linear method in SW4 to iterate only on the near-surface, where we expect to see the most nonlinear soil behavior.

2C. Use-case Flowchart

Linear simulation over full domain

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External Python script to extract relevant displacement data in HDF5 format

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Import DRM data into SW4 to run linear and nonlinear simulations on reduced domain

2B. Computing DRM Forces in SW4: Time-Stepping Algorithm

Elastic wave equation (displacement formulation) ⟶ $\rho\vec{u}_{tt} = \nabla \cdot \tau(\vec{u}) + \vec{f}(\vec{x},t)$

For more information on the SW4 finite difference formulation, see Sjögreen and Petersson (2012).

Premise:
We have a displacement field (which we know the value of at every grid point for every time step) that we would like to apply to the boundaries of our reduced domain in such a way that outgoing waves can still exit the domain without reflecting. We accomplish this task by adding to the force term in the time stepping algorithm.

Within one time step:

1. SW4 predicts the displacement for the next time step,

$$\rho\frac{\vec{u}_p^{n+1} - 2\vec{u}^n + \vec{u}^{n-1}}{dt^2} = L_h(\vec{u}^n) - \sum_m L_h(\vec{\alpha}_m^n) + \vec{f}^n$$

$\vec{u}_p^{n+1}$ is the only unknown variable, which is the predicted displacement in the next time step that we are solving for. $L_h()$ is an operator for the discretized spatial derivatives in the wave equation, and $\vec{\alpha}_m^n$ represent the m number of memory mechanisms.

We can superimpose our desired displacement field on the linear simulation by adding to the vector field of forces, $\vec{f}^n$.

$$\vec{f}_{DRM}(\vec{x},t_n) = \rho\frac{\vec{u}_{DRM}^{n+1} - 2\vec{u}_{DRM}^n + \vec{u}_{DRM}^{n-1}}{dt^2} - L_h(\vec{u}_{DRM}^n) + \sum_m L_h(\vec{\alpha}_{m,DRM}^n)$$

For the above expression, everything on the right hand side is known. $\vec{u}_{DRM}$ represents the known displacement field we are trying to superimpose, which we know the value of at all relevant grid points for all relevant time steps.

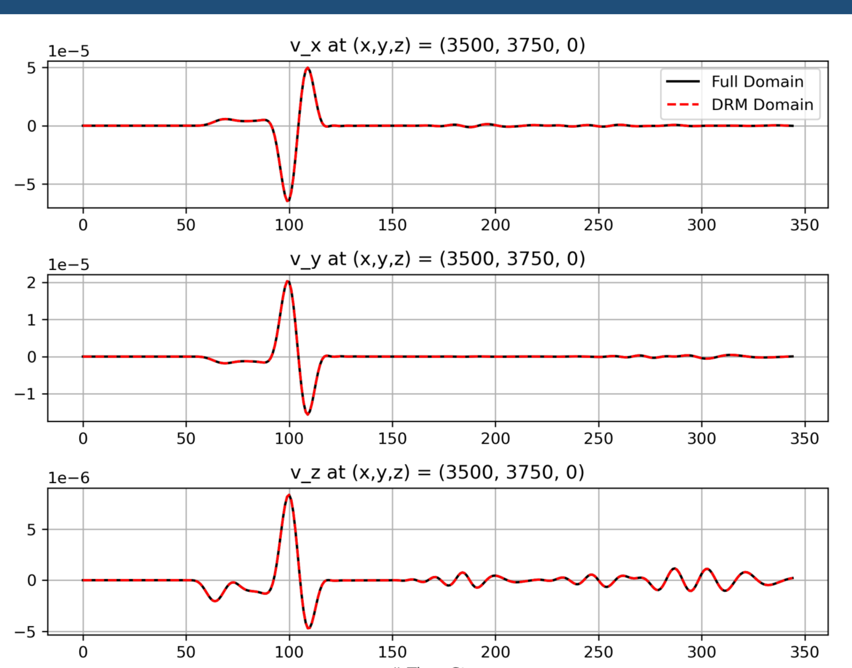
2. SW4 adds a corrector term to the predicted displacement,

$$\vec{u}^{n+1} = \vec{u}_p^{n+1} + \frac{dt^4}{12\rho}[L_h((\vec{u}^n)_{tt}) - \sum_m L_h((\vec{\alpha}_m^n)_{tt}) + (\vec{f}^n)_{tt}]$$

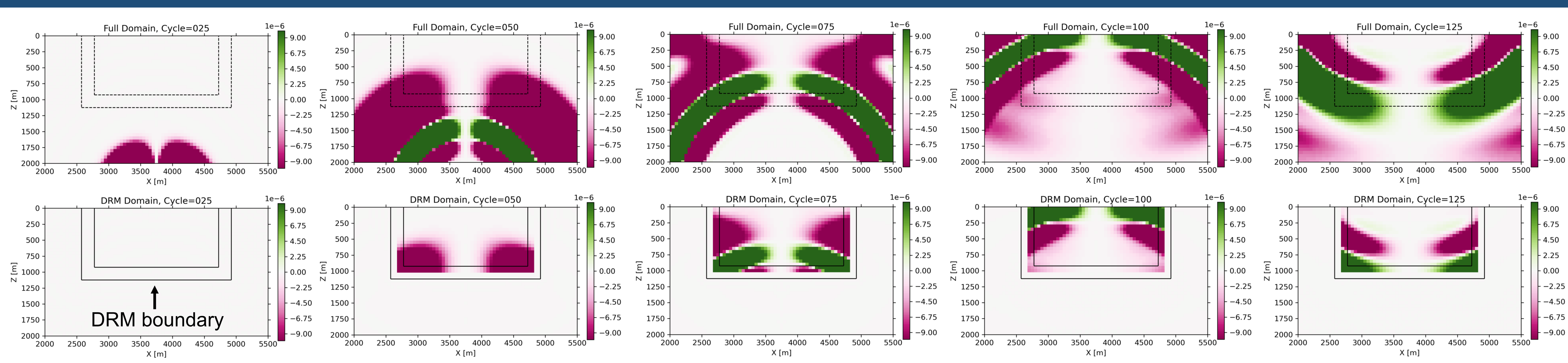
We once again add to the force term to account for the displacement field we want to superimpose. The right hand side of the following expression is composed purely of values that are known or computable from our known displacement field.

$$(\vec{f}^n)_{tt,DRM} = \sum_m L_h\left((\vec{\alpha}_{m,DRM}^n)_{tt}\right) - L_h((\vec{u}_{DRM}^n)_{tt})$$

3A. Verification of DRM Implementation in SW4



Comparison of velocity time series at surface.



Full domain vs. DRM domain velocity waveform comparison on the X-Z plane at Y=3500m in a simple homogeneous example.

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