Abstract
We develop and train a Generative Adversarial Neural Operator (GANO) to produce realistic three-component acceleration time histories on-demand, conditioned on moment magnitude, rupture distance, Vs30, and tectonic environment type of earthquakes. We test the architecture by training the GANO on 50K subduction and shallow crustal events of M 4.5-8.0 from the KIK-Net strong-motion dataset. Quantitative comparison of our model residuals to empirical data for Fourier power spectra, duration, and Arias intensity suggest so far that the synthetic time histories capture the data variability in the frequency range of engineering applications. Applications of the framework include: (i) generation of design ground motions for earthquake scenarios not adequately represented in empirical datasets, (ii) generation of risk-targeted ground motions, and (iii) spatial interpolation of recorded ground motions for the risk analysis of distributed infrastructure systems; and (iv) generation of high frequency components in broadband ground motion simulations.

1. Generative Adversarial Neural Operators (GANO)
Neural operators (NOs) are generalized NNs that learn mappings between infinite dimensional function spaces. Assume $\mathcal{A}$ and $\mathcal{U}$ are defined on bounded domains $\mathcal{D} \subset \mathbb{R}^n$ and $\mathcal{D} \subset \mathbb{R}^m$ respectively. Next, suppose there exists a nonlinear mapping $G^j: \mathcal{A} \rightarrow \mathcal{U}$ and observation pairs $\{s_1, u_i\}_{i=1}^N$, where $s_i$ are independent and identically distributed random variables.

Recognizing that $s_i = G^j(s_j)$ may be contaminated with noise, a neural operator approximates $G^j$ through a finite number of parameters so that $G^\theta \approx G^j$. Neural operators are comprised of three parts:

Lifting: Extracts features from input functions
Iterative Kernel Integration: Iterates a recurrent NN on feature space
Projection: Mapping from feature space to output function

Fourier neural operators (FNOs) parameterize the integral kernel directly in the Fourier space. As such, they are up to $10^3$ faster compared to traditional PDE solvers; and achieve superior accuracy compared to previous learning-based solvers under fixed resolution.

This work uses a recently developed FNO, the U-Net based FNO (U-NO), which allows for much deeper models than the original FNOs, thus providing highly parameterized neural operators for learning maps between function spaces. The U-NO is further integrated in a new architecture called generative adversarial neural operator (GANO), which generalizes the GAN framework and allows for the sampling of functions by learning push-forward operator maps in infinite-dimensional spaces. GANO consists of two main components, a generator neural operator and a discriminator neural functional. The inputs to the generator are samples of functions from a user-specified probability measure, e.g., Gaussian random field (GRF), and the generator outputs are synthetic data functions.

2. Model Validation
We trained our GANO model using 50K 3-component 100Hz seismograms from shallow crustal and subduction events in Japan, recorded by the seismogram network KIK-Net.

We validated against the empirical data by comparing normalized residuals of FAS at frequencies 0.1-20Hz; durations D5-75 and D5-95 and Arias intensity:

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e = \frac{\text{ln(observed IM)} - \text{mean(ln(GANO IM))}}{\text{std(ln(GANO IM))}}$$

3. Work in Progress -- Future Research

3.1. Normalized residuals of model predictions: D 0.05-0.75 vs magnitude (left) and rupture distance (Rrup) (right)

Figure 3. Normalized residuals of model predictions: D 0.05-0.75 vs magnitude (left) and rupture distance (Rrup) (right).

3.2. Scoring relationships of GANO-generated synthetics for shallow crustal events: mean horizontal Fourier Power Spectral ordinates at 5Hz (left) and 20Hz (right) vs magnitude (top); rupture distance (Rrup) (middle) and Vs30 (bottom).

Figure 4. Scoring relationships of GANO-generated synthetics for shallow crustal events: mean horizontal Fourier Power Spectral ordinates at 5Hz (left) and 20Hz (right) vs magnitude (top); rupture distance (Rrup) (middle) and Vs30 (bottom).

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