Phase-field Modeling of Rate- and State-dependent Frictional Faults

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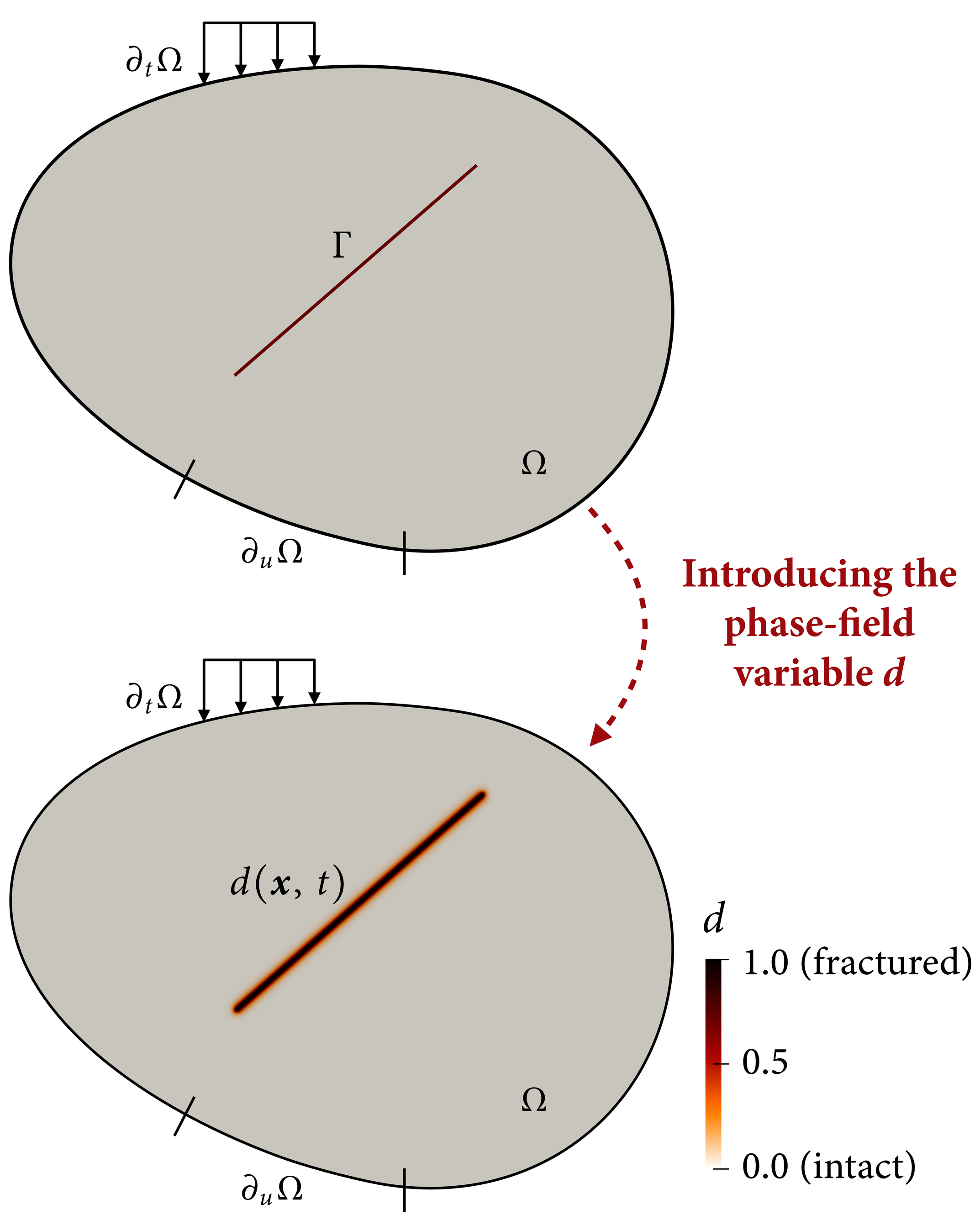
Introduction & Motivation

The branches of the San Andreas fault system (captured from https://pubs.usgs.gov/)

- Faults commonly have **complex geometries**, e.g. branches, kinks. See the left figure.
- Computational modeling of fault rupture using the discontinuous method (e.g. discontinuous finite element method) usually requires **complicated implementation** to track the complex geometries.
- This study presents a **continuous phase-field approach** to dynamic fault rupture simulation, due to its ability to handle arbitrary fault geometry **without using any tracking algorithms**.
- The formulation is built upon the previous method for the contact with constant friction [1], and further incorporates the **rate- and state-dependent friction** with **radiation damping**.

Phase-field Approximation



- Fracture is modeled by solving the partial different equation ⇒ **no need of tracking algorithms**.

$$\underbrace{-g'(d)}_{\text{Degradation function}}\underbrace{\mathcal{H}^+}_{\text{Crack driving force}} + \underbrace{\mathcal{G}_c}_{\text{Fracture energy}}\underbrace{\delta\Gamma_d(d, \nabla d)}_{\text{Crack density function}} = 0$$

- The width of the phase-field diffusive region is governed by the **phase-field length parameter, L**.
- **Γ-convergence**: phase-field solution converges to the solution of the original sharp interface as L decreases.

Stress Formulation

- Phase-field contact formulation [1].

$$\boldsymbol{\sigma} = g(d)\underbrace{\boldsymbol{\sigma}_m}_{\text{Stress in the intact material}} + [1 - g(d)]\underbrace{\boldsymbol{\sigma}_f}_{\text{Stress in the fractured material}}$$

- Stress in the **intact material** (linear elasticity assumed)

$$\boldsymbol{\sigma}_m = \mathbb{C}^e : \boldsymbol{\varepsilon}$$

- Stress in the **fractured material**

$$\boldsymbol{\sigma}_f = \mathbb{C}^e : \left[\boldsymbol{\varepsilon} - \frac{1}{2}(\boldsymbol{n} \otimes \boldsymbol{m} + \boldsymbol{m} \otimes \boldsymbol{n})v_f\mathcal{L}^{-1}\right]$$

  - v_f is fault slip
  - $\mathcal{L}^{-1} = \Gamma_d$
  - Calculate the stress and slip by satisfying the yield function

$$F(\boldsymbol{\sigma}_f, v_f) = \tau + p_N f(V, \theta) - \underbrace{\eta V}_{\text{Radiation damping}} \leq 0$$

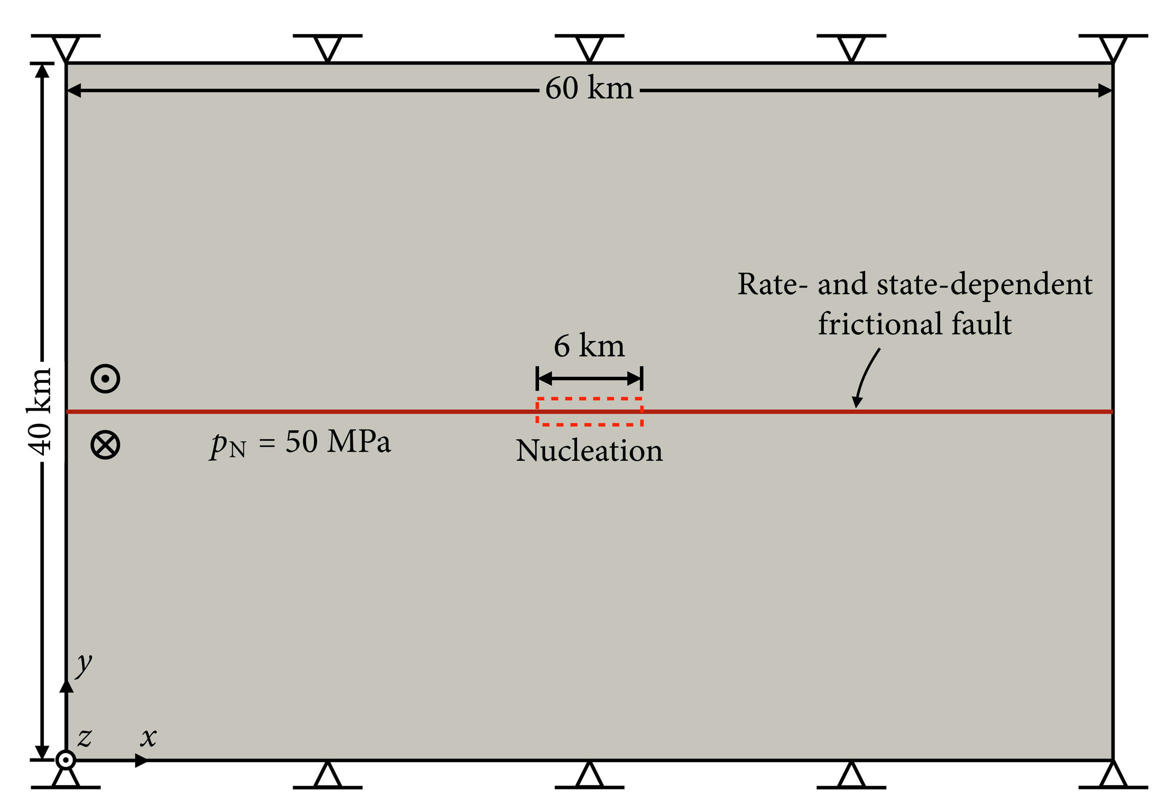
where

$$f(V, \theta) = A\sinh^{-1}\left\{\frac{V}{2V_0}\exp\left[\frac{f_0 + B\ln(\theta V_0/D_c)}{A}\right]\right\}$$

Regularized version of rate- and state-dependent friction

Verification Example

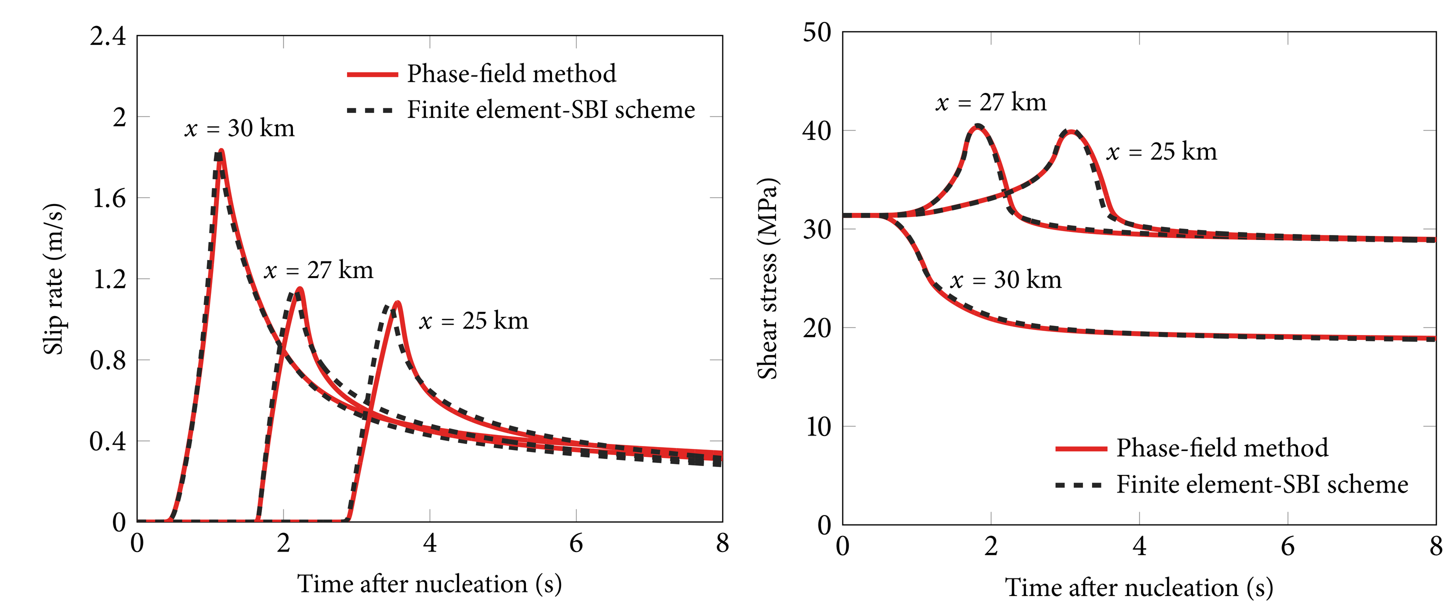
- An anti-plane fault rupture problem



- The fault has $A < B$ ⇒ a **velocity-weakening** fault.
- **Gradual reduction of reference friction** in the nucleation zone (27 km < x < 33 km) to initiate rupture.
- Phase-field results are compared with those from a discontinuous method—**the finite element-spectral boundary integral (SBI) scheme** [2].

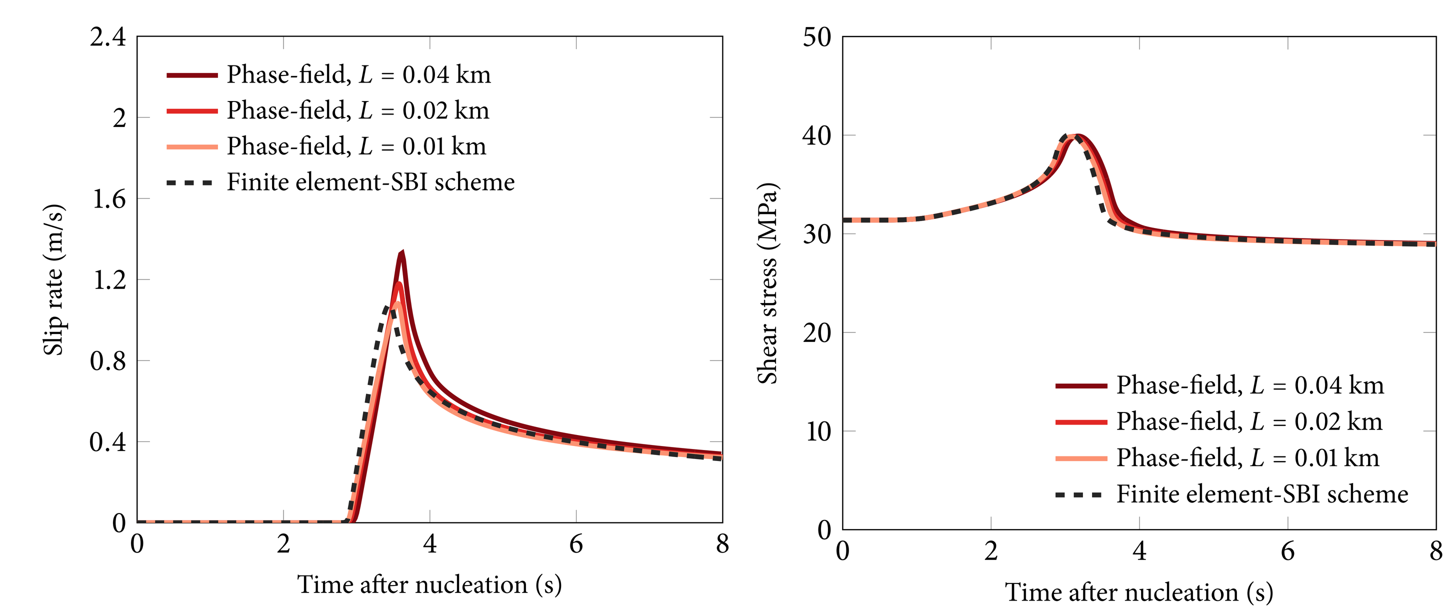
Results

- Evolutions of slip rate and shear stress at various locations



Well matched!

- Results at x = 25 km with different phase-field length parameter



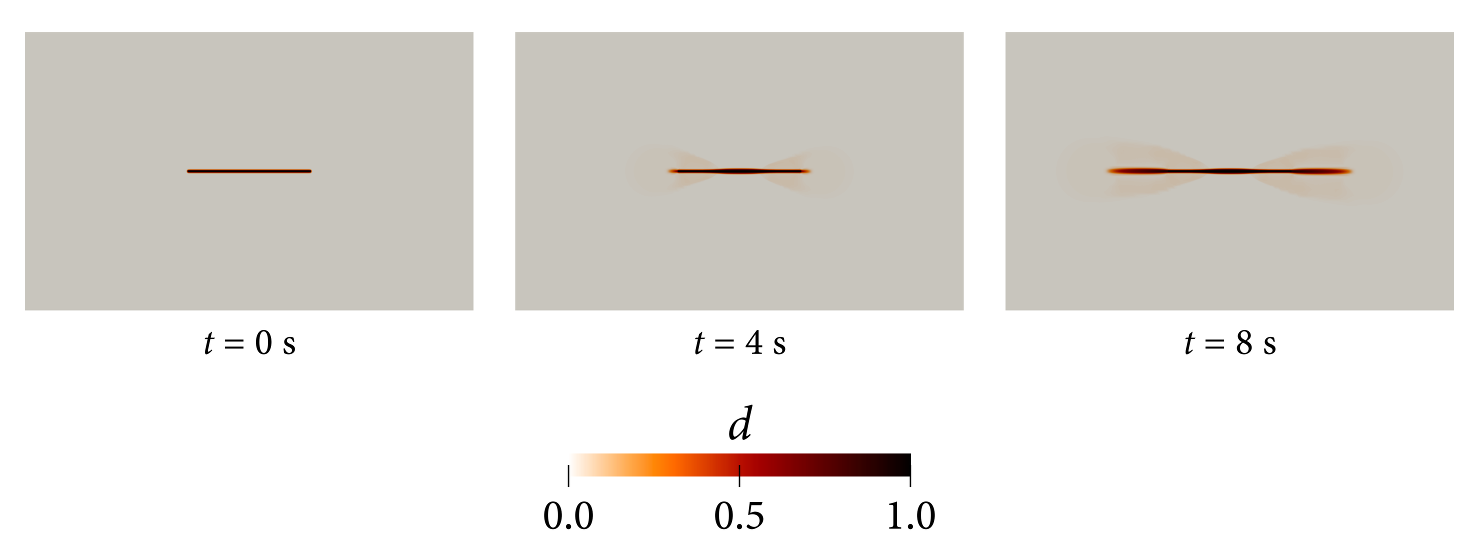
Results converge with decreasing L!

Summary

- We **verified** the phase-field approach to dynamic fault rupture with the results from the finite element-SBI scheme.
- The phase-field results converge to those from the discontinuous method as the length parameter decreases ⇒ **formulation is mathematically correct**.

Future Work

- To take the advantage of the phase-field method, we can extend the formulation to the **modeling of fault propagation**. See preliminary results below.



References

[1] Fei, F. and Choo, J. (2020). A phase-field method for modeling cracks with frictional contact. *International Journal for Numerical Methods in Engineering*, 121(4), 740-762.

[2] Ma, X., Hajarolasvadi, S., Albertini, G., Kammer, D.S., and Elbanna, A.E. (2019). A hybrid finite element-spectral boundary integral approach: Applications to dynamic rupture modeling in unbounded domains. *International Journal for Numerical and Analytical Methods in Geomechanics*, 43(1), 317-338.