Exploring the Effects of a Sedimentary Basin on the Earthquake Cycle using a Non-Stiff Finite Difference Method for Elastodynamics

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Abstract

Geodetic observations of long term slip on faults cutting through sedimentary basins often do not match geologic estimates. These discrepancies might be partially resolved by the heterogeneous structure of sedimentary basins, which, in addition to spatial variations in fault friction, affect the pattern of coseismic slip and interseismic creep. One hypothesis is that the drop in shear modulus of sedimentary basins inhibits surface breaking ruptures. Previously this hypothesis was tested with a series of quasi-dynamic cycle simulations of sedimentary basins with anti-plane strike-slip faults governed by a rate-and-state friction law which generated a periodic rupture cycle with both buried and surface breaking ruptures.

In quasi-dynamic modeling, inertial effects are approximated with radiation damping and it has been shown that dynamic effects can result in rupture penetration into fault regions that are not favored to slip (either due to frictional properties or the state of stress). Thus, in this work, we revisit the previous basin simulations using a newly developed method that uses a quasi-dynamic description in the interseismic phase and the fully dynamic model during coseismic slip.

To do this we have developed a high-order finite difference scheme that is provably stable (robust) in both phases of the cycle. In the interseismic phase, large linear systems of equations must be solved to allow for long time steps. Previously, in the coseismic phase, enforcing a rate-state friction law resulted in a stiff system of equations that could be solved with a second-order semi-implicit method. We use a newly developed, non-stiff method that is compatible with the interseismic method and allows for the use of a generic explicit time-stepping method. We suspect that with the addition of dynamic effects surface rupture will occur with higher frequency.

Simulation Results

We compare quasi-dynamic simulations with fully-dynamic simulations using cumulative slip plots. A sedimentary basin of 4 km depth inhibits surface rupture every other cycle in quasi-dynamic simulations, but rupture reaches the surface every cycle in the dynamic simulations. The rupture appears to slow when it hits the basin, but eventually reaches the surface. Plots of displacement and velocity in the medium show the impact of the sedimentary basin on the wavefield and rupture.

Figure 5. Slip contours on the fault are plotted every 10 years in blue, and every 1 second in red.

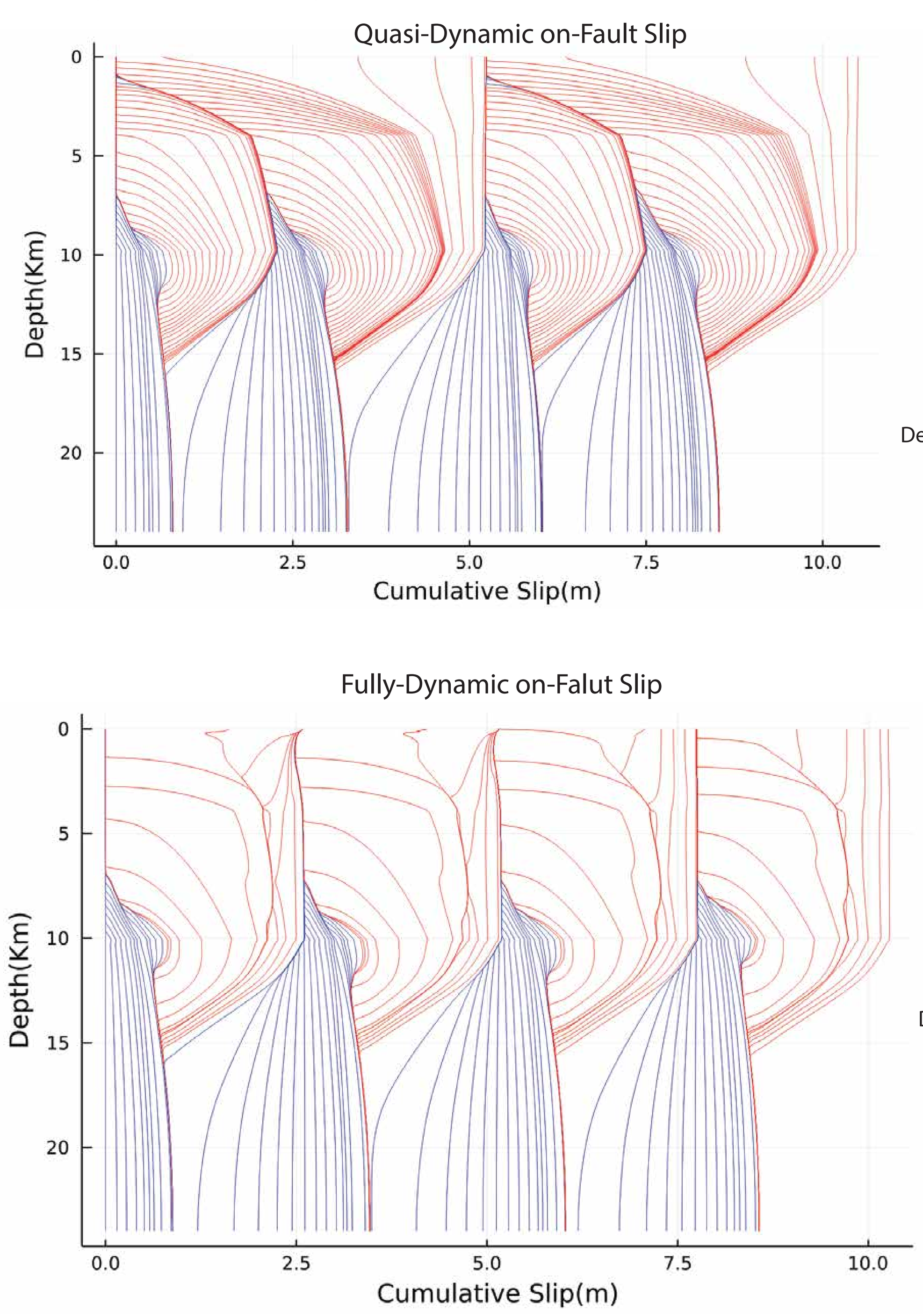
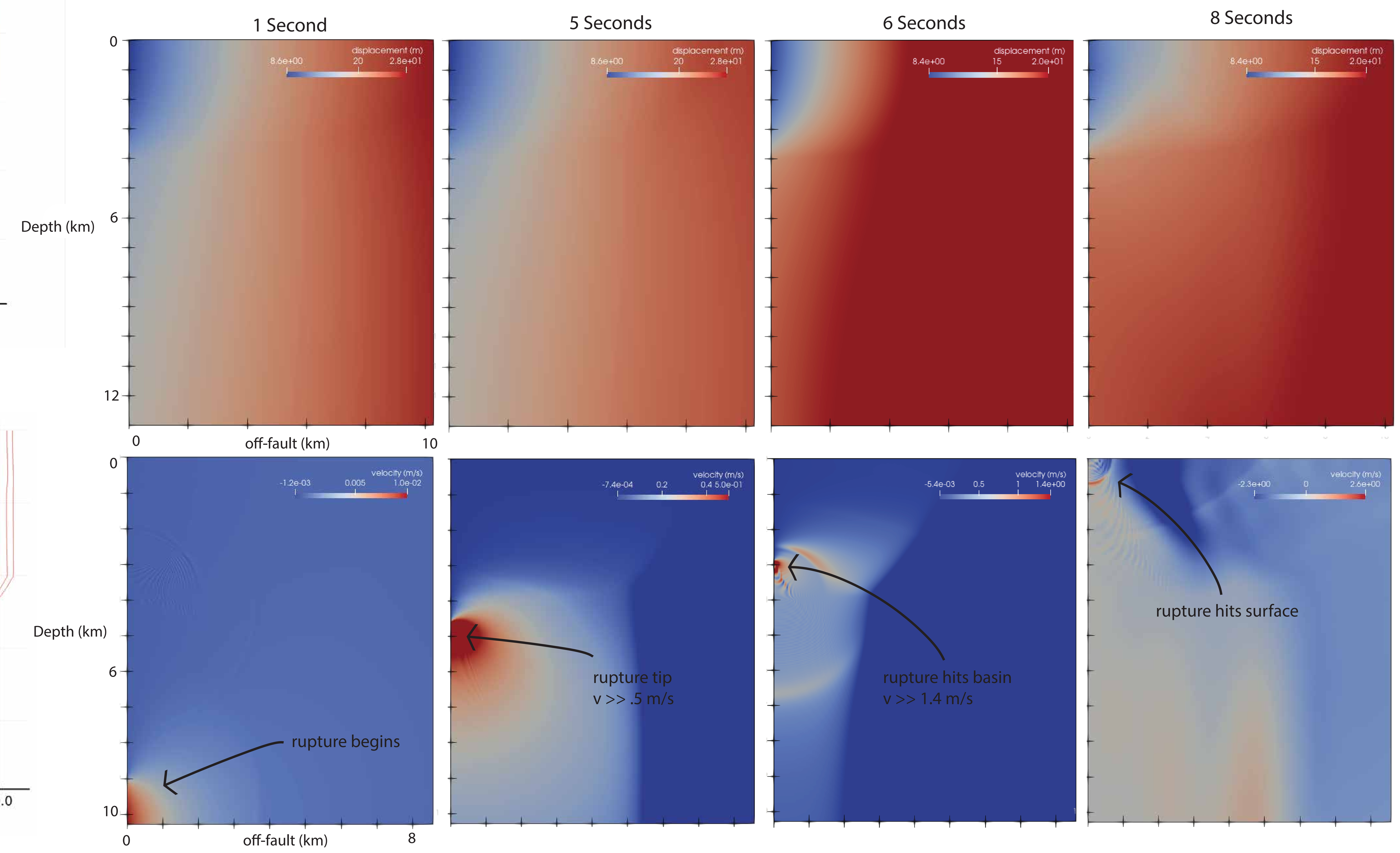


Figure 6. particle displacements (top) and velocity (bottom) plotted within the volume during a single coseismic phase at 1, 5, 6, and 8 seconds.



Model and Numerical Method

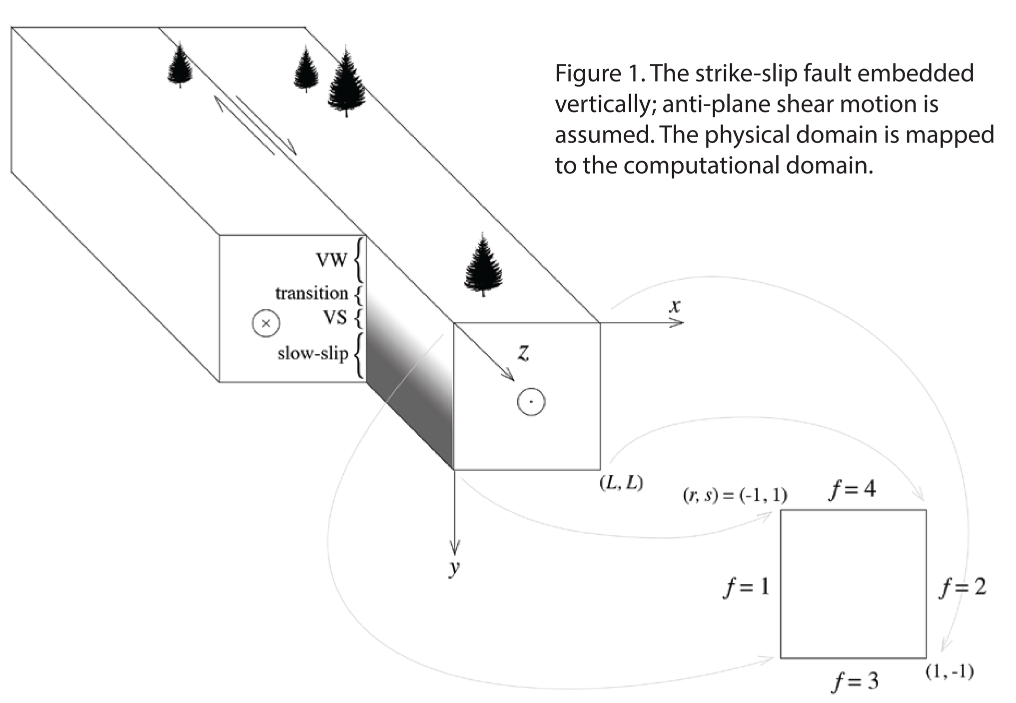


Figure 1. The strike-slip fault embedded vertically; anti-plane shear motion is assumed. The physical domain is mapped to the computational domain.

During the dynamic phase, the medium is modeled as a linear elastic half-space with a fault embedded at x=0 which is governed by rate-and-state friction:

$$\rho \ddot{u} = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} C \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix}, \qquad (x, y) \text{ in } (\infty, \infty) \times [0, \infty)$$

$$\tau = F(V, \psi), \qquad x = 0$$

$$\dot{\psi} = G(V, \psi), \qquad x = 0$$

$$\tau = 0, \qquad y = 0$$

Variables:

u	particle displacement (m)
ρ	density (kg/m^3)
C	matrix valued function containing shear modulus and metric terms
τ	shear stress (MPa)
V	slip rate (m/s)
ψ	state variable

The function F determines a non-linear equation that relates fault strength to slip rate and the state variable which evolves according to the state evolution law G. In this work we use the regularized form of an aging law:

$$F(V, \psi) = \sigma_n a \sinh^{-1}\left(\frac{V}{2V_0} e^{\frac{\psi}{a}}\right)$$

$$G(V, \psi) = \frac{bV_0}{D_c} e^{\frac{f_0 - \psi}{b} - \frac{|V|}{V_0}}$$

The parameter a-b determines if the fault is velocity strengthing (a-b >0) or velocity weaking (a-b < 0). The sedimentary basin is modeled by varying shear modulus μ and density ρ within the volume.

Figure 2. μ and ρ values defining the basin

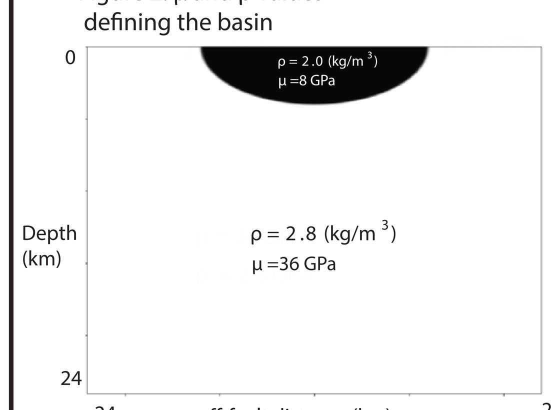
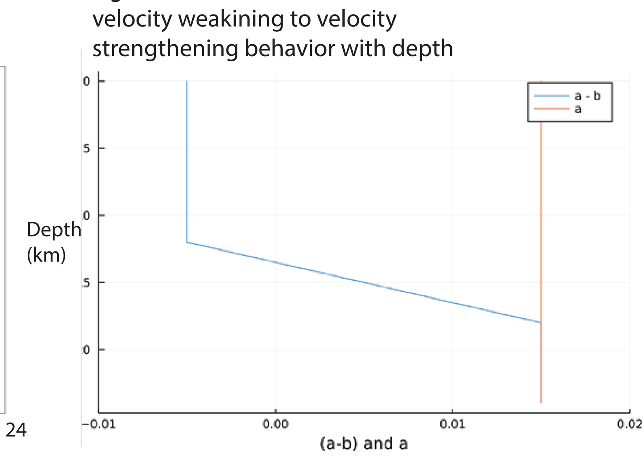


Figure 3. The fault moves from velocity weakining to velocity strengthening behavior with depth



Interseismic phase:

- Neglect acceleration (assume $\ddot{u} = 0$) and approximate inertial effects with radiation damping
- The spatial discretization is the hybridized finite difference method of Kozdon et al. (2021)
- Time-stepping using the approach proposed by Erickson and Dunham (2014)

Coseismic phase:

- Impose boundary conditions in a characteristic fashion:

$$Z\dot{u} + \tau = R(Z\dot{u} - \tau)$$

Where Z is the shear impedance. Notice R = 1 is a Neumann condition, R = -1 is a Dirichlet condition, and R = 0 is a non-reflecting condition.

Coseismic phase:

- Enforce boundary conditions weakly through fluxes u^* and τ^*.
- On the outer boundaries we define the fluxes so that:

$$Z\dot{u}^* + \tau^* = R(Z\dot{u} - \tau)$$

$$Z\dot{u}^* - \tau^* = Z\dot{u} - \tau$$

i.e. that the fluxes satisfy the boundary condition and the outgoing characteristic is preserved

- On the fault the fluxes are defined so that they satisfy the friction law

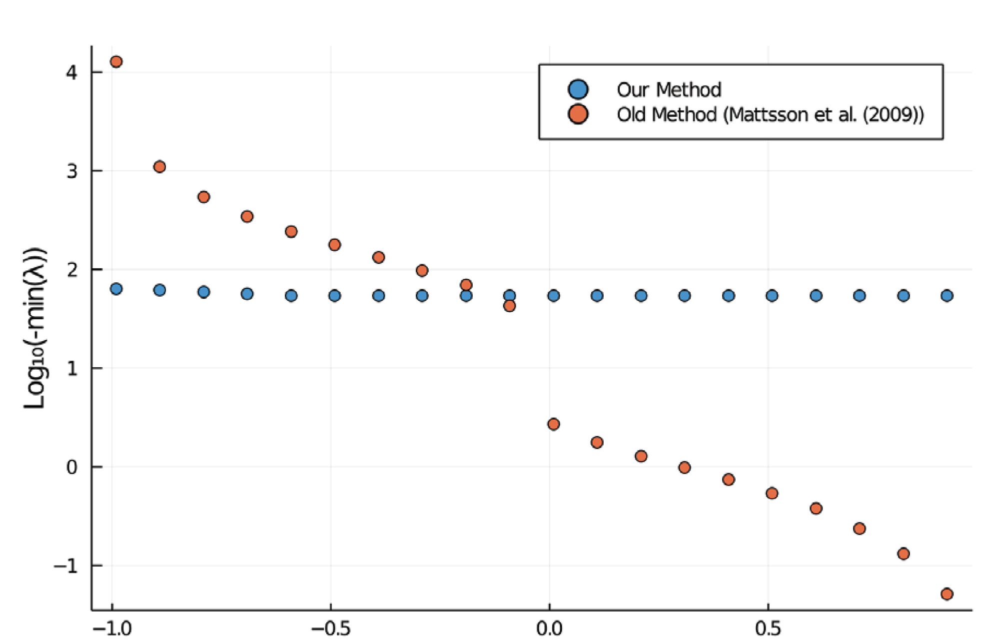
$$\tau^* = F(2\dot{u}^*, \psi)$$

$$Z\dot{u}^* - \tau^* = Z\dot{u} - \tau$$

$$\psi = G(2\dot{u}^*, \psi)$$

- $\dot{u}^*$ is solved for at every time step
- Integrate the non-stiff new SBP-SAT method and $\dot{u}^*$ at every time step with an explict time-stepper

Figure 4. Previous finite difference formulations of the 2nd order wave equation did not allow for Runge-Kutta time-stepping due to the numerical stiffness produced by the addition of the friction law. By using the characteristic formulation, the spectrum of the discrete operators shrink so that time steps are selected purely based on wave speeds.



Verification

The method of manufactured solutions was used to verify the coseismic solver. Below convergence rates for the method are shown for operators with theoretical orders p = 2 and p = 4, on a 2D domain with N+1 nodes per dimension.

$p = 2$

N	error	rate
16	7.5e-07	
32	1.8e-07	2.02
64	4.5e-08	2.02
128	1.1e-08	2.01

$p = 4$

N	error	rate
16	6.8e-09	
32	4.2e-10	4.01
64	2.6e-11	4.01
128	1.6e-12	4.00

Discussion

- What parameters control surface breaking rupture when dynamic effects are accounted for? For instance, basin width and depth, material parameters?
- What physical explanations can be offered to understand the results we have seen?
- What impact would moving to a 3D domain have?
- How robust are the results to the domain size and model resolution?

References

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