Scaling of solution in an elastic dynamic rupture problem: an analytical proof

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Argument to be proved

For a dynamic rupture model with a slip-weakening friction law, if we scale the initial stress and slip-weakening distance by a certain factor, the solution will be identical except being scaled by the same factor.

Motivation of this proof

Although such a scaling property is widely used in early dynamic rupture models, it is less obvious whether it also works for an arbitrarily complex model.

Analytical proof: 1. formulating the problem

- A dynamic rupture problem in a purely elastic medium can be formulated as the task of finding the solutions to a set of integral equations. The fault Γ can be treated as an internal surface embedded in the medium.

- Representation theorem suggests that, if we obtain the spatial-temporal evolution of slip $\Delta u_i(\boldsymbol{\xi}, t)$, and traction $T_i(\boldsymbol{\xi}, t)$ on fault ($\boldsymbol{\xi} \in \Gamma$), we obtain the solution of the problem. In total, there are **six** unknown variables.

- Representation theorem can provide three equations,

$$T_i(\boldsymbol{\xi},t) = T_i^0(\boldsymbol{\xi}) + \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}') \int_0^t \mathrm{d}\tau \hat{K}_{ij}(\boldsymbol{\xi}, t-\tau; \boldsymbol{\xi}', 0)\Delta\dot{u}_j(\boldsymbol{\xi},\tau) \quad (\boldsymbol{\xi},\ \boldsymbol{\xi}' \in \Gamma)$$

- Shear slip requirement gives one equation,

$$\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)\cdot\boldsymbol{n}(\boldsymbol{\xi}) = 0$$

- "Friction law" gives two equations,

$$\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_1(\boldsymbol{\xi}) = \left(\mu(\boldsymbol{\xi},t)\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_3(\boldsymbol{\xi})\right)\left(\frac{\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)}{|\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)|}\cdot\hat{\boldsymbol{n}}_1(\boldsymbol{\xi})\right)$$

$$\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_2(\boldsymbol{\xi}) = \left(\mu(\boldsymbol{\xi},t)\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_3(\boldsymbol{\xi})\right)\left(\frac{\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)}{|\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)|}\cdot\hat{\boldsymbol{n}}_2(\boldsymbol{\xi})\right)$$

  We introduce an additional variable, "friction coefficient" $\mu(\boldsymbol{\xi}, t)$, and it is described with an additional equation,

$$\mu(\boldsymbol{\xi},t) = \begin{cases} \mu_y\left(1 - \frac{\int_0^t |\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},\tau)|\,\mathrm{d}\tau}{D_0}\right) & \int_0^t |\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},\tau)|\,\mathrm{d}\tau < D_0 \\ 0 & \int_0^t |\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},\tau)|\,\mathrm{d}\tau \geq D_0 \end{cases} \quad (\boldsymbol{\xi}\in\Gamma),$$

- The seven variables $\Delta u_i(\boldsymbol{\xi}, t)$, $T_i(\boldsymbol{\xi}, t)$, and $\mu(\boldsymbol{\xi}, t)$ are constrained with seven independent equations.

Analytical proof: 2. the scaling relation

- For a new problem, where $T_i^0(\boldsymbol{\xi})$ and D_0 are scaled by a factor of C, and μ_y unchanged. We want to see if $C\Delta u_i(\boldsymbol{\xi}, t)$, $CT_i(\boldsymbol{\xi}, t)$, and $\mu(\boldsymbol{\xi}, t)$ are the solutions to the new set of equations,

$$CT_i(\boldsymbol{\xi},t) = CT_i^0(\boldsymbol{\xi}) + \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau \hat{K}_{ij}(\boldsymbol{\xi}, t-\tau; \boldsymbol{\xi}', 0)C\Delta\dot{u}_j(\boldsymbol{\xi}',\tau) \quad (\boldsymbol{\xi}, \boldsymbol{\xi}' \in \Gamma)$$

$$C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)\cdot\boldsymbol{n}(\boldsymbol{\xi}) = 0$$

$$C\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_1(\boldsymbol{\xi}) = \left(\mu(\boldsymbol{\xi},t)C\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_3(\boldsymbol{\xi})\right)\left(\frac{C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)}{|C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)|}\cdot\hat{\boldsymbol{n}}_1(\boldsymbol{\xi})\right)$$

$$C\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_2(\boldsymbol{\xi}) = \left(\mu(\boldsymbol{\xi},t)C\boldsymbol{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{n}}_3(\boldsymbol{\xi})\right)\left(\frac{C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)}{|C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},t)|}\cdot\hat{\boldsymbol{n}}_2(\boldsymbol{\xi})\right)$$

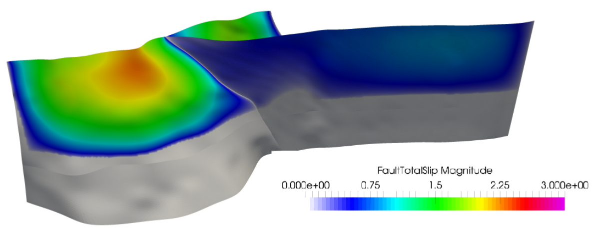
$$\mu(\boldsymbol{\xi},t) = \begin{cases} \mu_y\left(1 - \frac{\int_0^t |C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},\tau)|\,\mathrm{d}\tau}{CD_0}\right) & \int_0^t |C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},\tau)|\,\mathrm{d}\tau < CD_0 \\ 0 & \int_0^t |C\Delta\dot{\boldsymbol{u}}(\boldsymbol{\xi},\tau)|\,\mathrm{d}\tau \geq CD_0 \end{cases} \quad (\boldsymbol{\xi}\in\Gamma),$$

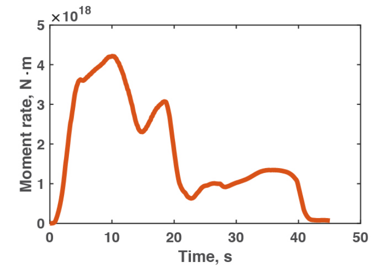
- All C factors cancel in the equations; scaling proved. A similar approach can be applied to other friction laws to find the scaling property (e.g., time-weakening, rate-state).

A numerical example

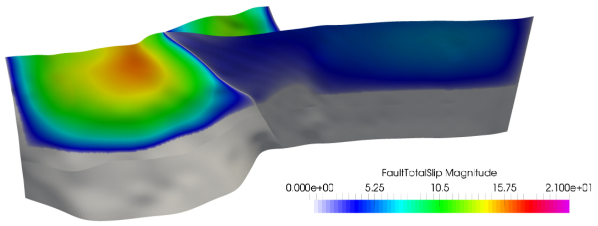
Talk to Baoning for details!

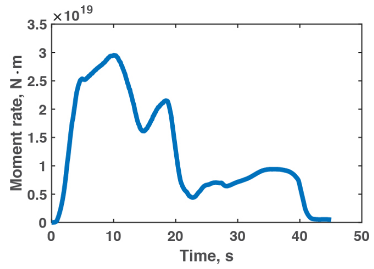
Case 1: C=1 max slip = 2.28296 m max MR = 4.2159e18 Nm/s





Case 2: C=7 max slip = 15.9807 m max MR = 2.9511e19 Nm/s





A complex dynamic rupture model for San Gorgonio Pass (Jennifer Tarnowski, PhD dissertation)