Earthquake source spectra can provide crucial observational constraints for understanding the physics of earthquake rupture. While there exist a number of approaches to extract key earthquake source parameters like corner frequency and stress drop from waveform spectra, the most widely-used workflows are based on simple optimization procedures that do not accurately characterize the inherent data and modeling uncertainties. Perhaps because of this, source parameter estimates exhibit a puzzling degree of scatter, and results from different methods often disagree even when using the same underlying datasets. Here we develop a Bayesian framework for source spectral analysis that allows for the encoding of prior physical knowledge to guide inversions while solving for the full posterior probability distribution that fully quantifies parameter tradeoffs and uncertainties. In these inversions, input data include spectral ratios between a target event and one or more nearby empirical Green’s function (EGF) events, and the Bayesian inversion solves for corner frequencies and spectral falloff rates for all events, as well as a database of 1929 regular grid points for each event relating to the target event. Initial application of this computationally intensive technique to large (M5 and greater events) during the July 2019 Ridgecrest earthquake sequence appears promising, with physically viable priors enabling robust corner frequency estimates for EGFs and target events alike. We anticipate that the work performed here will form the basis of a future, larger study applying the method at scale across California and beyond, likely with the help of high-performance computing resources.

Background

For more than 50 years, earthquake source spectra have been analyzed in the context of a generalized Brune (1970) or Boatwright (1980) model of the form:

\[ S(\Omega) = S_0 \left[ 1 + \left( \frac{\Omega}{f_c} \right)^{\gamma} \right]^m \]

Despite this apparent simplicity, source spectral modeling and interpretation are surprisingly tricky. Often, the falloff rate \( \gamma \) and sharpness \( \Omega \) are fixed a-priori, which simplifies computations but ignores the inherent tradeoff with the full posterior probability distribution that fully quantifies parameter tradeoffs and uncertainties. In these inversions, input data include spectral ratios between a target event and one or more nearby empirical Green’s function (EGF) events, and the Bayesian inversion solves for corner frequencies and spectral falloff rates for all events, as well as a database of 1929 regular grid points for each event relating to the target event. Initial application of this computationally intensive technique to large (M5 and greater events) during the July 2019 Ridgecrest earthquake sequence appears promising, with physically viable priors enabling robust corner frequency estimates for EGFs and target events alike. We anticipate that the work performed here will form the basis of a future, larger study applying the method at scale across California and beyond, likely with the help of high-performance computing resources.

Prominent SoCal Earthquakes (starting w/ Ridgecrest)

Bayesian Inference: Turing.jl

Example of spectral ratio data from one target and EGF event. Ratios from an arbitrary number of target/EGF pairs can be input into Turing.jl for source parameter inference.

Fig. 2: Model setup using Turing.jl, a Julia package for Bayesian parameter estimation. Here the model is setup to solve for spectral moment, corner frequency, and high-frequency falloff rate for each earthquake (EGFs and Mainshock). For the mainshock, which has a much larger sample size of data and better azimuthal coverage, the model also tries to solve for a cosine-like directivity function. Spectral ratio data is input into the model, and the Bayesian inference process relies on weakly informative priors for moment (based on an earthquake catalog), corner frequency (a reasonable stress drop), falloff rate (something near 2.0), and directivity (nodal planes give an initial guess at strike). The likelihood of the data, given the model, assumes Laplacian (rather than Gaussian) errors to mitigate outliers in the spectral ratio data.

Example Results: M5.51 Mainshock + 19 EGFs

Fig. 3: Posterior estimates for a M5.51 mainshock and 19 EGFs (M3-4) occurring within ~3 km hypocentral distance of the mainshock. Each panel corresponds to a different event, with the mainshock at the top left. The panels display posterior samples that show the tradeoff between the corner frequency (x-axis) and the high-frequency falloff rate (y-axis). EGF events with broad (round) posterior distributions have only a small number of spectral ratios and thus more closely resemble the assumed prior.

Mainshock Comparison: M5.51 and M5.37

Fig. 4: Comparison of source spectral estimates for two different target events: a M5.37 (left) and a M5.51 (right). The top panels show posterior estimates for corner frequency and falloff rate. The middle panels show the inferred directivity functions. The bottom panels show the inferred strike direction. The M5.37 (left) has a moderate stress drop and S-wave spectra without strong unilateral directivity. The M5.51 (right) has a high stress drop and more prominent directivity toward the NE.

Discussion and Next Steps

The work presented here demonstrates initial promise in using Bayesian techniques to analyze source spectra. Although this work differs in its implementation, other studies have also achieved solid results by applying Bayesian methods to source spectral estimation (e.g. Van Houette & Denolle, 2018; Supino et al. 2019; Törnman et al. 2021). These methods are appealing because they (i) quantify parameter tradeoffs and uncertainties and (ii) allow us to encode physical knowledge of the problem through the use of priors. This is of particular importance for EGF events subject to bandwidth or otherwise limited recordings. Despite this promise, Bayesian methods are not a panacea to all of the inherent issues with source spectral analysis. For example, the M6.4 and M7.1 mainshocks have corner frequencies that push the lower bandwidth of broadband sensors and may not be reliably estimated. EGF events with high corner frequencies high have elevated uncertainties, even with firm prior constraints. And these methods are computationally expensive, with the results presented here requiring parallelized sampling on a local computing cluster. While the limitations are important to consider, there appears to be enough promise to extend this methodology to study other earthquake sequences in southern CA and beyond.

References


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