A Frequency-Dependent Spatial Correlation Model of Within-Event Residuals for Fourier Amplitude Spectra

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Objectives

Ground motions recorded at stations separated by up to a few tens of kilometers are found to be correlated. We analyze the frequency-dependent spatial correlation of ground motions for:

1) generating a frequency-dependent spatial correlation model of the effective Fourier amplitude spectra, and
2) incorporating the spatial correlation model into broadband stochastic ground motion simulations.

Background

Effective Fourier Amplitude Spectrum (EAS)

$$EAS(f) = \sqrt{\frac{1}{2}\left[FAS_{HC1}^2(f) + FAS_{HC2}^2(f)\right]}$$

Orientation independent — Fourier Amplitude Spectrum (FAS) of the two as-recorded horizontal components

Within-Event Residual

The within-event residual (δW_{es}) depicts the misfit between an individual observation at station s during earthquake e from the earthquake-specific mean prediction.

$$\ln EAS_{es}(f) = \mu_{es}(f) + \delta B_e(f) + \delta W_{es}(f)$$

Observation — Prediction — Between-event Residual — Within-event Residual

$$\varepsilon(f) = \frac{\delta W_{es}(f)}{\varphi(f)}$$

Normalized Within-event Residual $\sim \mathcal{N}(0,1)$ — Standard Deviation

Spatial correlation model

Frequency-dependent spatial correlation model from semivariogram regression

A semivariogram characterizes the strength of statistical dissimilarity as a function of distance and is often used to describe spatially distributed random variables. The semivariogram is defined as:

$$\gamma(h) = \frac{1}{2}\,\mathrm{E}\left[\left(Z(s_x) - Z(s_y)\right)^2\right],$$

where $E[]$ denotes the expectation, $Z(s_x)$ and $Z(s_y)$ are random variables at locations s_x and s_y, and h is the distance between the two locations.

The empirical semivariograms and covariances at each frequency pair (f_i, f_j) can be summarized by an isotropic semivariogram matrix (Γ) and isotropic covariance matrix (C) as a function of h:

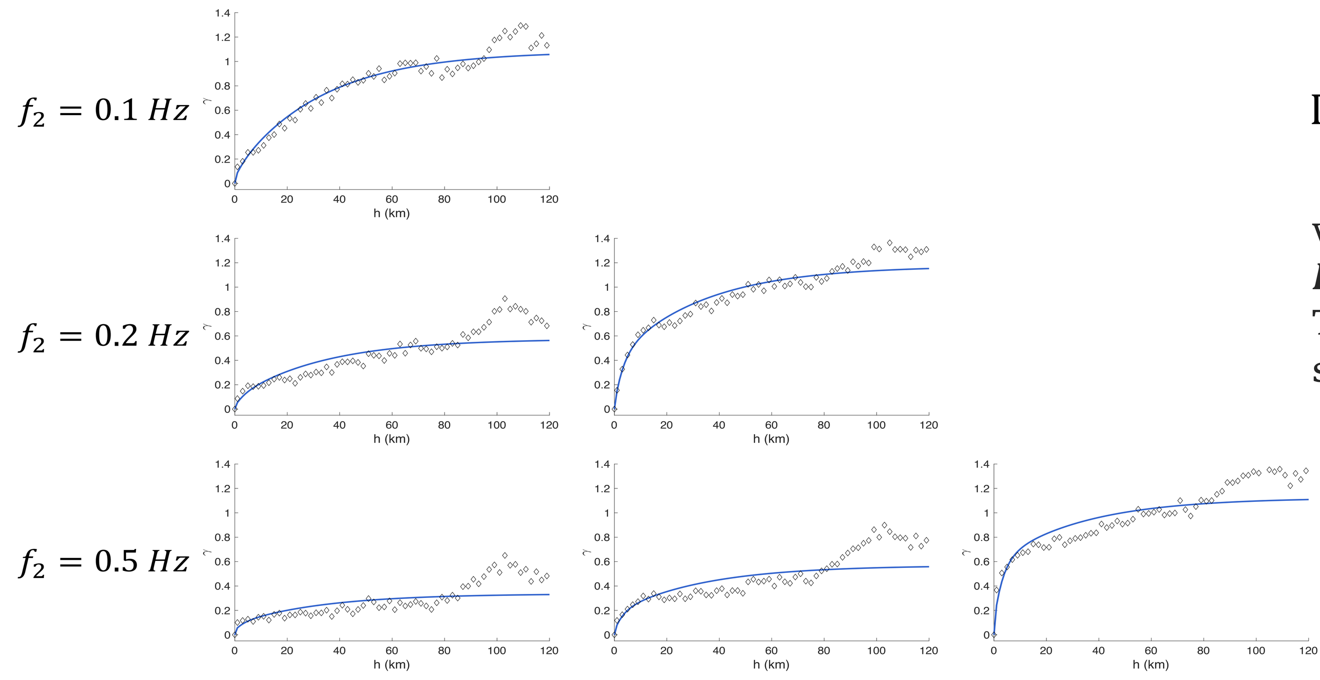
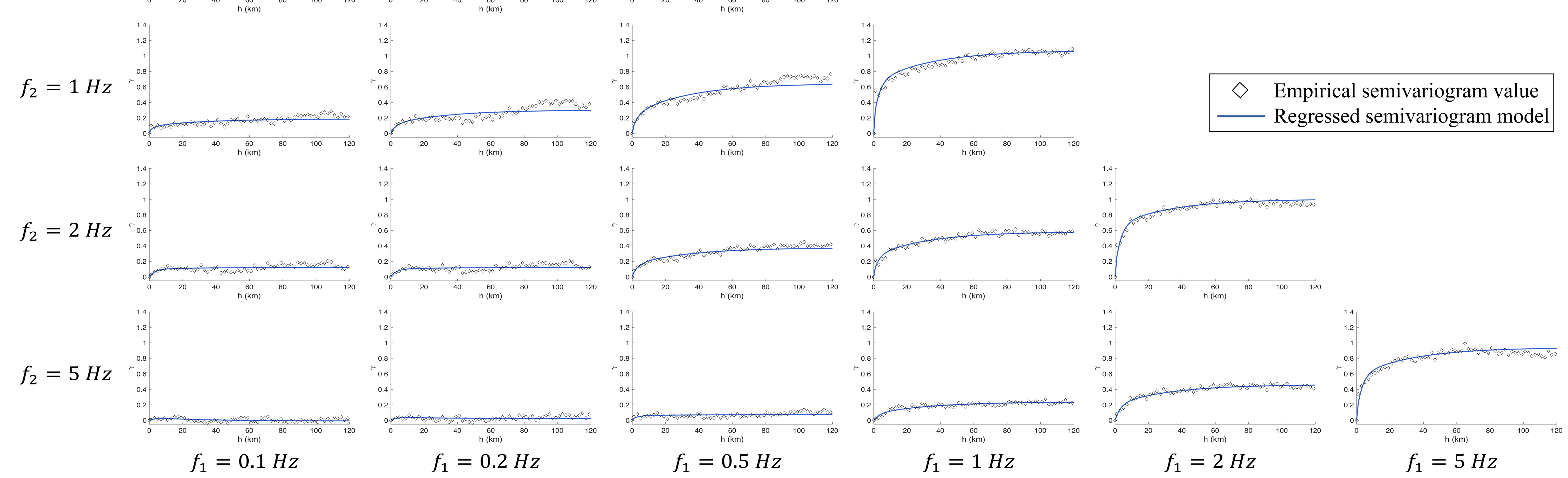
$$\Gamma(h) = \gamma_{f_i f_j}(h) = \begin{bmatrix} \gamma_{f_1,f_1}(h) & \cdots & \gamma_{f_1,f_n}(h) \\ \vdots & \ddots & \vdots \\ \gamma_{f_n,f_1}(h) & \cdots & \gamma_{f_n,f_n}(h) \end{bmatrix} \qquad C(h) = c_{f_i f_j}(h) = \begin{bmatrix} c_{f_1,f_1}(h) & \cdots & c_{f_1,f_n}(h) \\ \vdots & \ddots & \vdots \\ c_{f_n,f_1}(h) & \cdots & c_{f_n,f_n}(h) \end{bmatrix},$$

with the relationship: $C(h) = C(0) - \Gamma(h).$

We compute the empirical semivariogram of normalized within-event residuals of *EAS* at each station and each frequency using the Pacific Earthquake Engineering Research Center (PEER) Next Generation Attenuation (NGA) West2 database and the Bayless and Abrahamson (2018) *EAS* ground motion model. A predictive spatial correlation model is regressed using the Goulard-Voltz algorithm (Goulard and Volta, 1992) from the empirical semivariogram with a linear coregionalization model (linear combination of isotropic exponential functions).

$$\Gamma(h) = \boldsymbol{P^1}\left(1 - \exp\left(-\frac{3h}{R_1}\right)\right) + \boldsymbol{P^2}\left(1 - \exp\left(-\frac{3h}{R_2}\right)\right) + \boldsymbol{P^3}$$

where $R_1 = 10$ and $R_2 = 100$ are chosen for fitting, and $\boldsymbol{P^1}$, $\boldsymbol{P^2}$, $\boldsymbol{P^3}$ are the coregionalization matrices being regressed. The empirical (diamonds) and modeled semivariogram at selected frequencies are shown in following figure.

Model implementation

Incorporating spatial correlation into ground motion simulations

The San Diego State University (SDSU) Broadband Ground-Motion Generation Module is a hybrid method that merges deterministic low-frequency synthetics and high-frequency scatterograms, implemented on the SCEC Broadband Platform (BBP). The current SDSU Module has been validated for median ground motion metrics, however, it was not designed to provide satisfactory fits to data for correlations of ground motions. Here we implement the spatial correlation into the simulations making use of the predictive spatial correlation model.

$$C(h) = \boldsymbol{P^1}\exp\left(-\frac{3h}{R_1}\right) + \boldsymbol{P^2}\exp\left(-\frac{3h}{R_2}\right) + \boldsymbol{P^3}\mathbb{1}_{\{h=0\}} \qquad \mathbb{1}_{\{h=0\}} = \begin{cases} 1, & h = 0 \\ 0, & h \neq 0 \end{cases}$$

Step 1. Calculate matrices $\mathbf{D^1}$ and $\mathbf{D^2}$ that represent the cross-correlation at different station pairs (x, y) corresponding to the distance factors $\exp\left(-\frac{3h}{R_1}\right)$ and $\exp\left(-\frac{3h}{R_2}\right)$ in model $C(h)$: $D_{xy}^l = \exp\left(-\frac{3h_{xy}}{R_l}\right)$.

Step 2. Apply the Cholesky decomposition to $\boldsymbol{P^1}$, $\boldsymbol{P^2}$, $\boldsymbol{P^3}$ to get lower triangular matrices $\mathbf{K^1}$, $\mathbf{K^2}$, $\mathbf{K^3}$, and to $\boldsymbol{D^1}$, $\boldsymbol{D^2}$ to get upper triangular matrices $\boldsymbol{L^1}$, $\boldsymbol{L^2}$.

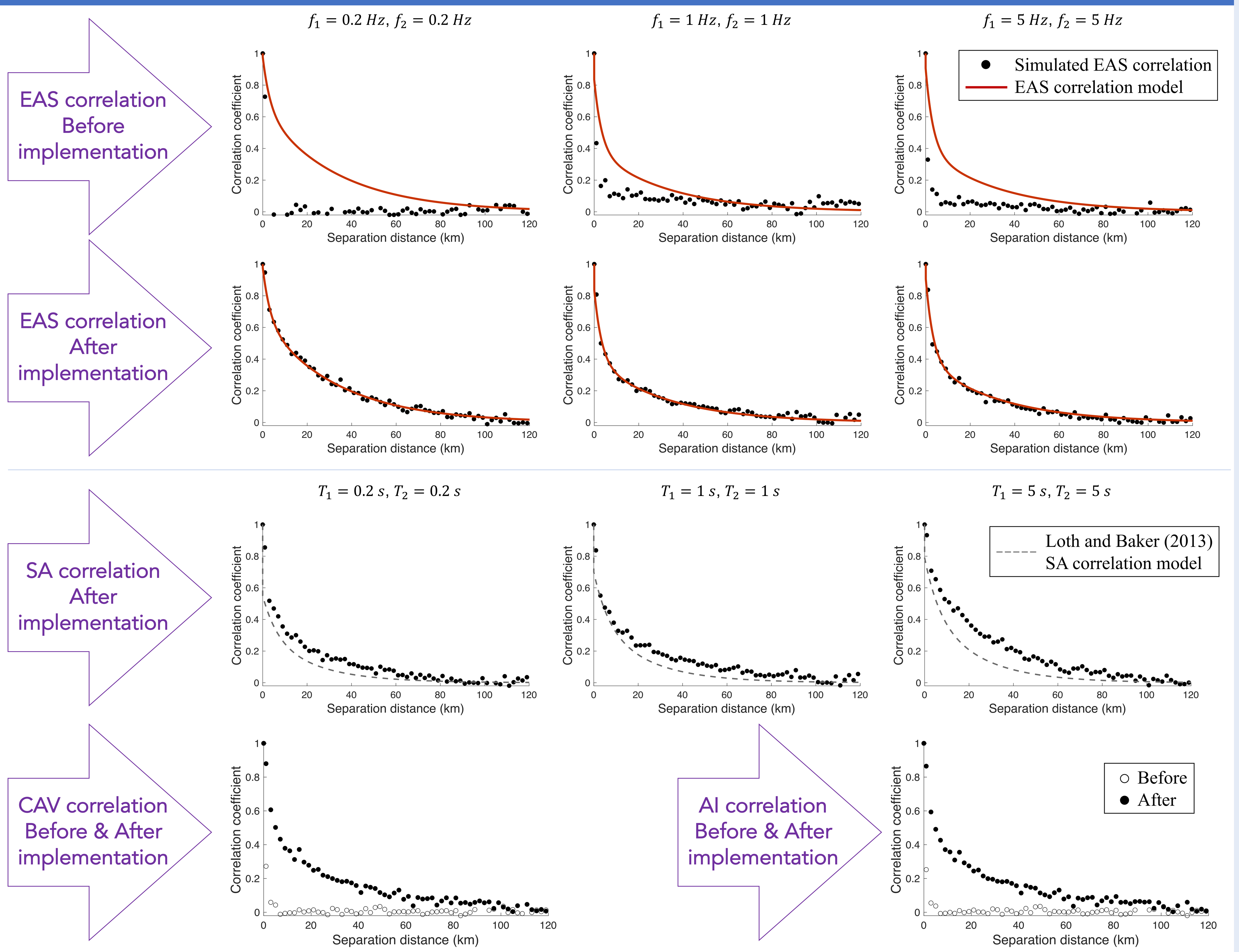
Step 3. Generate 3 independent two-dimensional standard normal random variables R^1, R^2, R^3 and compute

$$S = S^1 + S^2 + S^3 = \boldsymbol{K^1}R^1\boldsymbol{L^1} + \boldsymbol{K^2}R^2\boldsymbol{L^2} + \boldsymbol{K^3}R^3$$

such that S is a matrix of random variables with rows corresponding to different frequencies, columns corresponding to different stations, and elements in S have a covariance corresponding to model $C(h)$.

Step 4. Multiply the random variable S with the appropriate standard deviation and take the exponential of it, then multiply with the FAS. The ground motion time series with frequency-dependent spatial correlations are then generated by an inverse Fourier transform.

The following figure shows the resulting spatial correlation coefficients of *EAS* from 50 realizations of the M6.9 Loma Prieta, CA, event at example frequency pairs from the SDSU broadband synthetics using the correlation method compared with that from the original simulation without correlation implemented. Spatial correlation for spectral accelerations (SA), cumulative absolute velocities (CAV), and Arias intensities (AI) are also computed from the simulations. The results suggest that our approach successfully incorporates frequency-dependent spatial correlation into the SDSU broadband simulation for EAS as well as other ground motion metrics.



Conclusions

1) Ground motion data show frequency-dependent spatial correlation between stations.
2) Quantifying correlation is done with regression using linear coregionalization modeling of empirical semivariograms.
3) The frequency-dependent correlation model has been implemented into the SDSU Broadband Platform module, with particular benefits to the stochastic high-frequency component.
4) The implementation of EAS correlation also significantly improves the correlation of spectral acceleration, cumulative absolute velocities, and Arias Intensities, as compared to that produced by the original broadband module.

References

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