**Abstract**
The earthquake magnitude-frequency distribution is characterized by the b-value. Some observations suggest that the b-value for aftershocks is larger than for spontaneous events, and that exceptions to this rule occur exclusively in foreshock sequences. Unfortunately, measurements of b-value are biased during aftershock sequences by short-term incompleteness of the earthquake catalog. Here I develop a new estimator of the b-value that is insensitive to transient changes in catalog completeness and allows for the real-time, unbiased measurement of b-value during an ongoing sequence. The estimator is based on the differences in magnitude between successive earthquakes, which are described by a double-exponential distribution with the same b-value as the exponential magnitude distribution itself. The new estimator partially confirms the findings of Guilia and others [Nature, 2019; SRL 2020], showing a decrease in b-value immediately after the foreshock for several prominent foreshock sequences over the past decade – but the magnitude of the change is smaller than previously estimated and is not unambiguously linked to the times of the large earthquakes.

**The b-positive estimator**
Magnitudes $m$ are exponentially distributed (Gutenberg-Richter) above some magnitude $M_c$, with parameter $\beta$.

$$ f(m|M_c) = \beta e^{-\beta(m-M_c)} \quad m > M_c $$

(1)

Taking the difference of an exponentially distributed variable gives a Double Exponential or Laplace distribution. Magnitude differences $m' = m - m''$ are therefore also exponentially distributed with parameter $\beta$.

$$ f(m'|M_c) = \beta e^{-\beta(m'-M_c)} $$

(2)

The maximum likelihood estimates of $\beta$ are:

$$ \hat{\beta}_L = (m - M_c)^{-1} \quad \hat{\beta}_\text{Laplace} = (m' - M_c)^{-1} $$

(3)

(4)

Where $M_c$ is a minimum magnitude difference. (Incomplete detection of small magnitudes in the Gutenberg-Richter formulation translates to incomplete detection of small differences in the Laplace formulation). The Laplace estimator makes no reference to absolute completeness $M_c$, assuming it is the same for the two successive earthquakes. This is a much softer assumption than with the Gutenberg-Richter estimator, where $M_c$ is assumed constant for the entire population.

**Transient incompleteness bias**
The occurrence of a large earthquake temporarily obscures for the entire population. Gutenberg-Richter estimate, where earthquakes estimator makes no reference to absolute completeness differences in the Laplace formulation).

$$ (c > 0) $$

(5)

**Discussion and Conclusion**
**Confirmed b-value changes:**
- The unbiased b-positive estimator confirms a reduction in the b-value after the M6.4 foreshock and before the M7.1 mainshock, and an increase in b-value after the mainshock.

**Caveats:**
- The magnitude of the b-value drop barely exceeds historic fluctuations when transient incompleteness is avoided. This may complicate real time alerts.
- The b-value recovers ‘prematurely.’ In fact, the b-value has recovered to nearly a historic maximum at the moment immediately before the mainshock. This casts some doubt on the straightforward interpretation of b-value as a stress-meter.
- This pattern is repeated in other sequences (below): b-value does change, but changes do not greatly exceed historic fluctuations, and the mainshock typically occurs with one of the highest b-values of the intervening period.

**Application to other sequences**
- Sequences previously analyzed by Guilia and Wiemer [Nature 2019]