B-positive: Measuring robust *b*-value changes from magnitude differences

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Abstract

The earthquake magnitude-frequency distribution is characterized by the b-value. Some observations suggest that the b-value for aftershocks is larger than for spontaneous events, and that exceptions to this rule occur exclusively in foreshock sequences. Unfortunately, measurements of b-value are biased during aftershock sequences by short-term incompleteness of the earthquake catalog. **Here I develop a new estimator of the b-value that is insensitive to transient changes in catalog completeness and allows for the real-time, unbiased measurement of b-value during an ongoing sequence.** The estimator is based on the differences in magnitude between successive earthquakes, which are described by a double-exponential distribution with the same b-value as the exponential magnitude distribution itself. The new estimator partially confirms the findings of Gulia and others [*Nature, 2019; SRL 2020*], showing a decrease in *b*-value immediately after the foreshock for several prominent foreshock sequences over the past decade – but the magnitude of the change is smaller than previously estimated and is not unambiguously linked to the times of the large earthquakes.

The *b-positive* estimator

Magnitudes *m* are exponentially distributed (**Gutenberg-Richter**) above some magnitude M_c with parameter β.

$$f(m|\beta,M_c)=\begin{cases}\beta e^{-\beta(m-M_c)} & m\ge M_c\\ 0 & m<M_c\end{cases} \qquad (1)$$

Taking the difference of an exponentially distributed variable gives a Double Exponential or **Laplace** distribution. Magnitude differences *m'* are therefore also exponentially distributed with parameter β.

$$f(|m'|\,|\,\beta)=\beta e^{-\beta|m'|} \qquad (2)$$

The maximum likelihood estimates of β are:

Gutenberg-Richter

$$\hat{\beta}=\left(\bar{m}-M_c\right)^{-1} \qquad (3)$$

Laplace

$$\hat{\beta}'=\left(\overline{|m'|}-M_c'\right)^{-1} \qquad (4)$$

Where M_c' is a minimum magnitude *difference*. (Incomplete detection of small magnitudes in the Gutenberg-Richter formulation translates to incomplete detection of small differences in the Laplace formulation). **The Laplace estimator makes no reference to absolute completeness M_c, assuming it is the same for the two successive earthquakes.** This is a much softer assumption than with the Gutenberg-Richter estimate, where M_c is assumed constant for the entire population.

Transient incompleteness bias

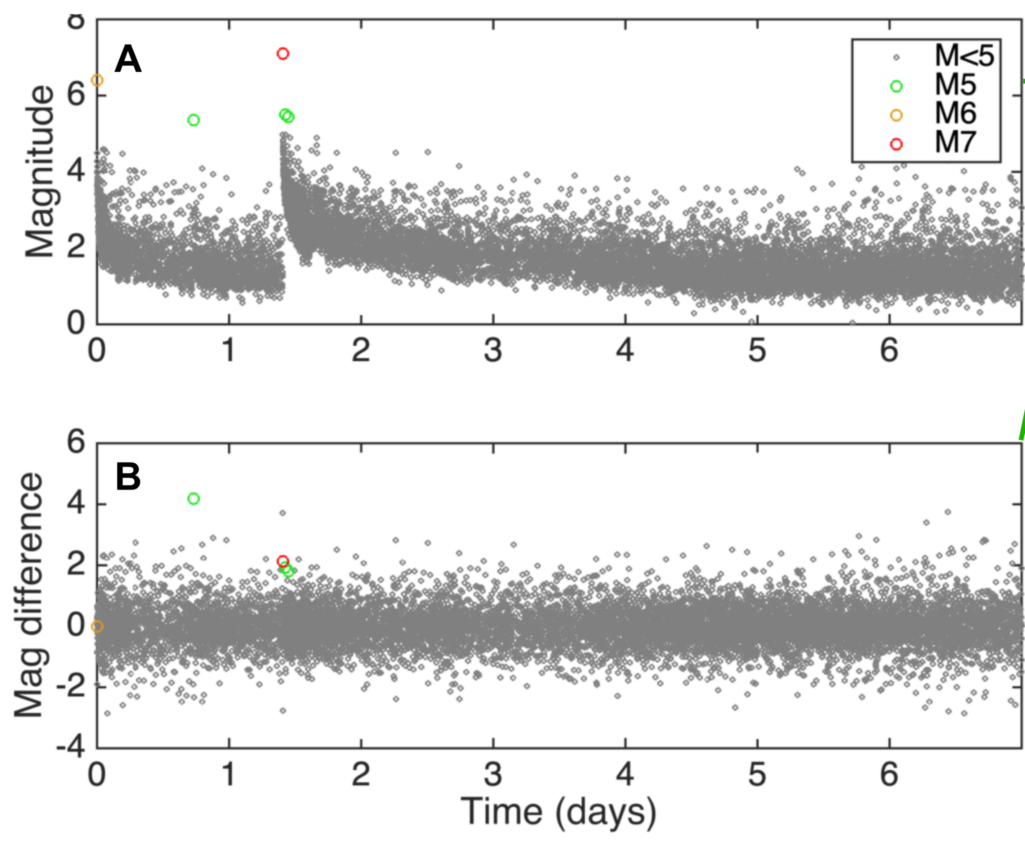
The occurrence of a large earthquake temporarily obscures the detection of smaller earthquakes (Helmstetter et al., 2006). **By considering only positive magnitude differences, where the second earthquake is larger than the first, we avoid transient incompleteness** and obtain the "*b-positive*" estimator.

b-positive

$$\hat{\beta}^{+}=\left(\overline{m'}-M_c'\right)^{-1} \qquad m'\ge M_c' \qquad (5)$$

Application to the 2019 Ridgecrest Sequence

The 2019 Ridgecrest sequence included a M6.4 foreshock followed by a M7.1 mainshock approximately 1.5 days later. For this sequence, Gulia and others [*SRL* 2020] report a reduced *b*-value between the foreshock and mainshock. This observation is likely biased by transient incompleteness, and needs to be confirmed by the *b-positive* estimator.

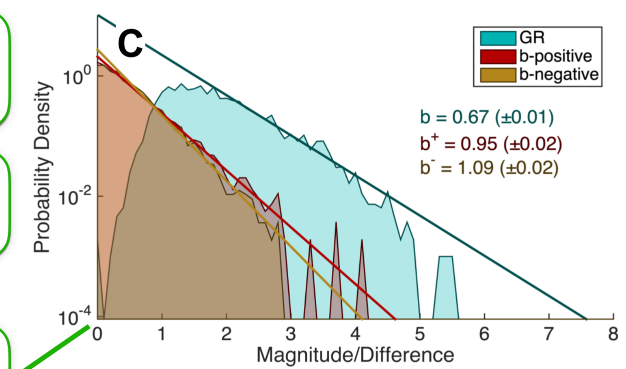


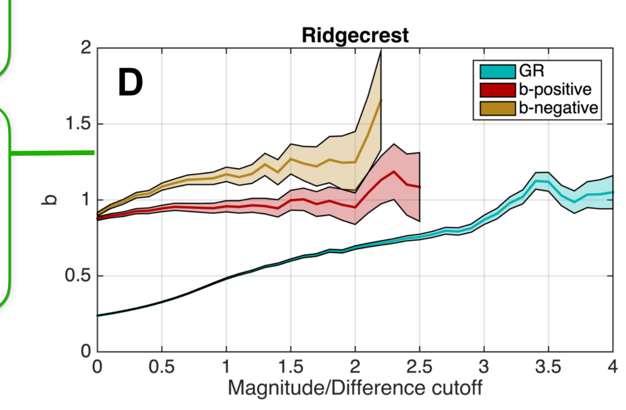
A. The 2019 Ridgecrest sequence, with its time-varying completeness level.

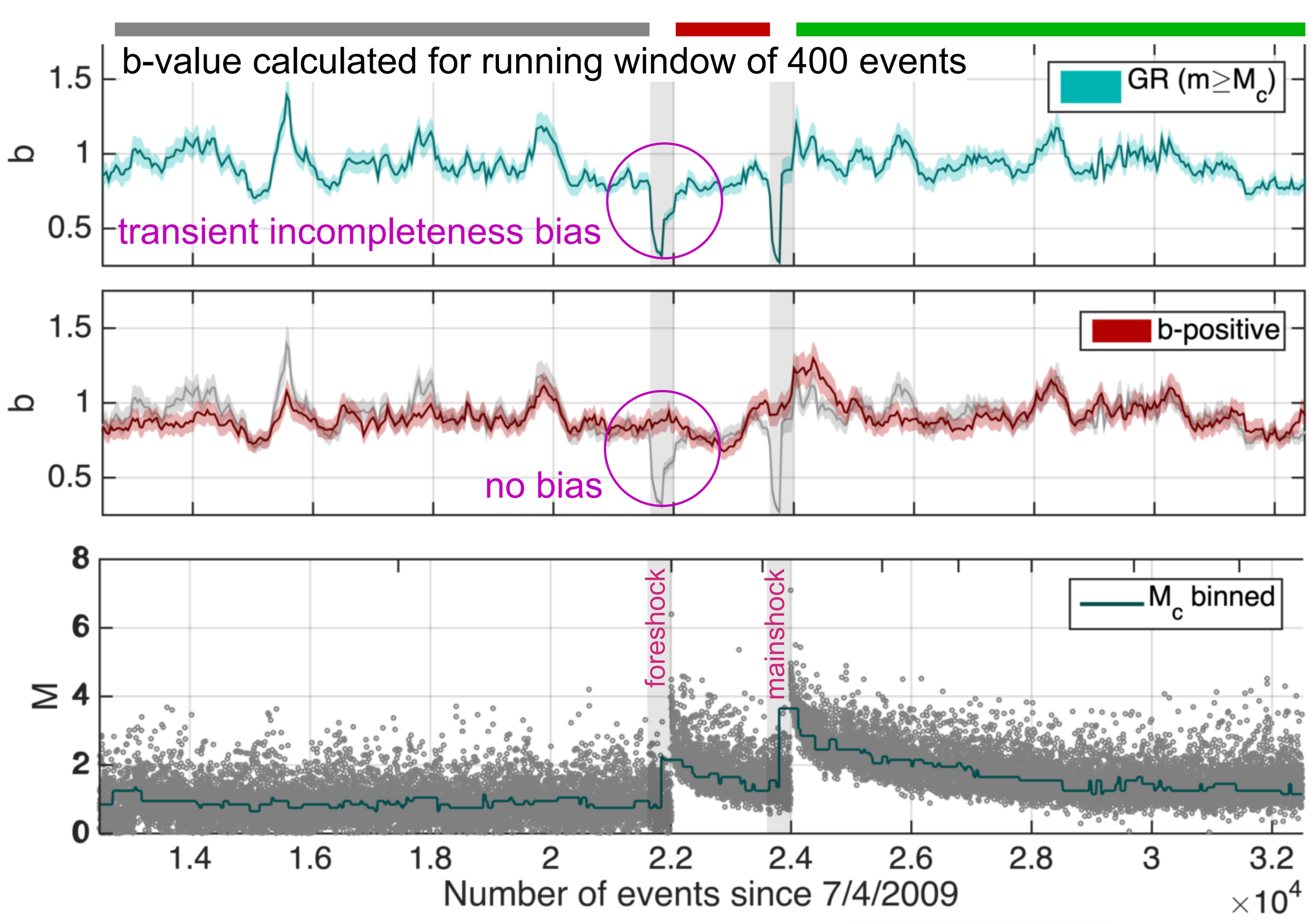
B. Time series of magnitude differences, which are far less affected by transient incompleteness.

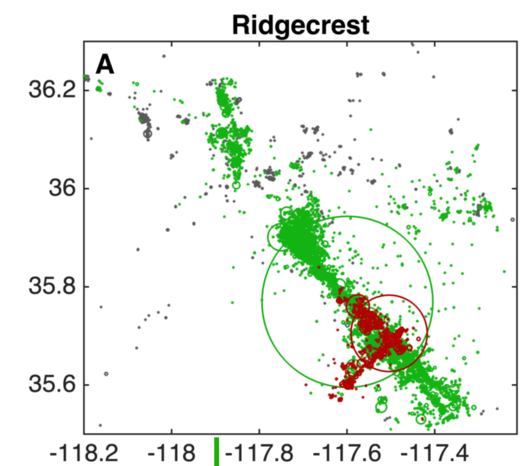
C. Distribution of magnitude and magnitude differences (separated into b-positive and b-negative).

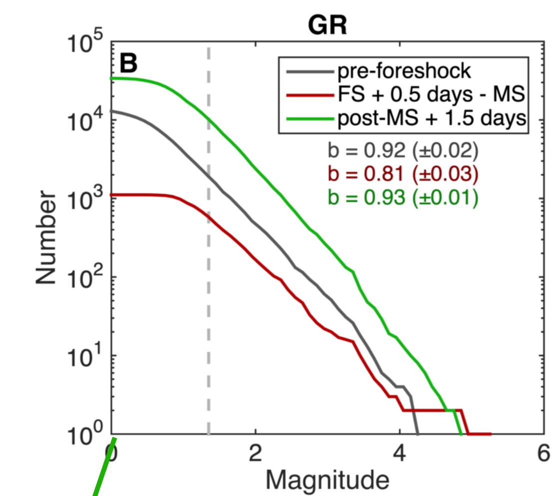
D. b-value as a function of the lower cutoff (with 1σ bootstrap uncertainty). ***The b-positive* estimate is stable over almost the entire range of the data.**







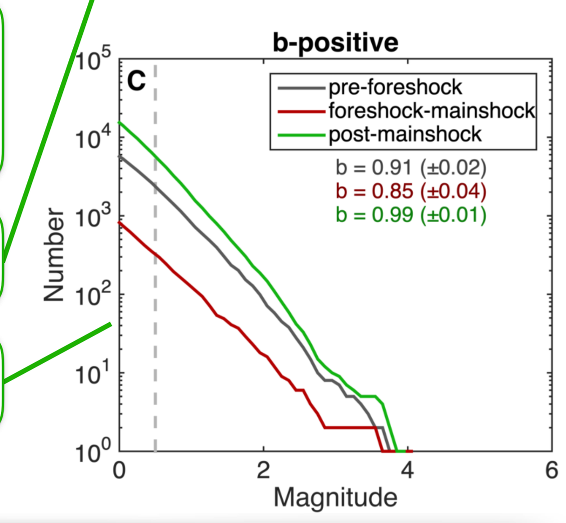


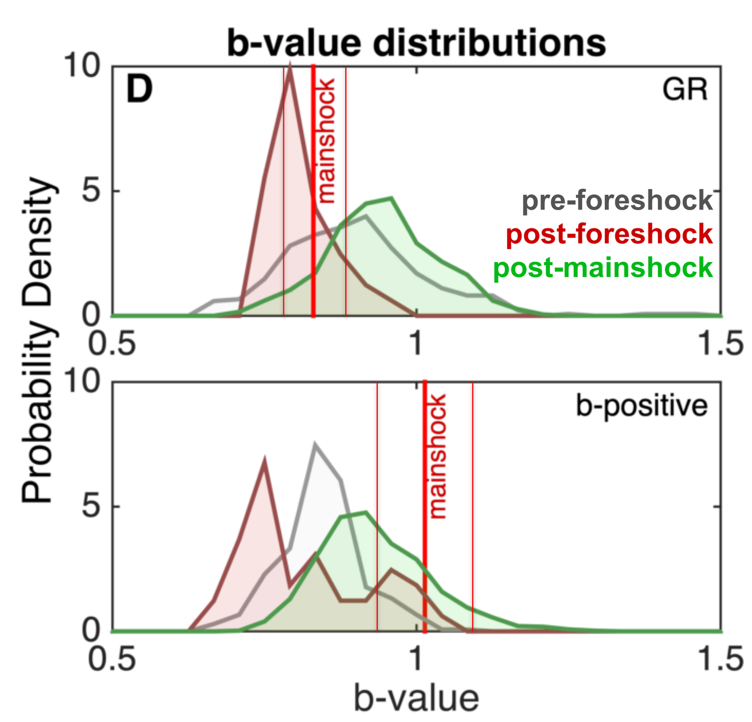


A. Ridgecrest sequence divided into **pre-foreshock**, **foreshock to main-shock** and **post-mainshock** time periods. Circles scaled to magnitude.

B. Magnitude distribution and GR b-value estimates.

C. Magnitude difference distribution and *b-positive* estimates.





Discussion and Conclusion

Confirmed b-value changes:

- The unbiased *b-positive* estimator confirms a reduction in the *b*-value after the M6.4 foreshock and before the M7.1 mainshock, and an increase in *b*-value after the mainshock.

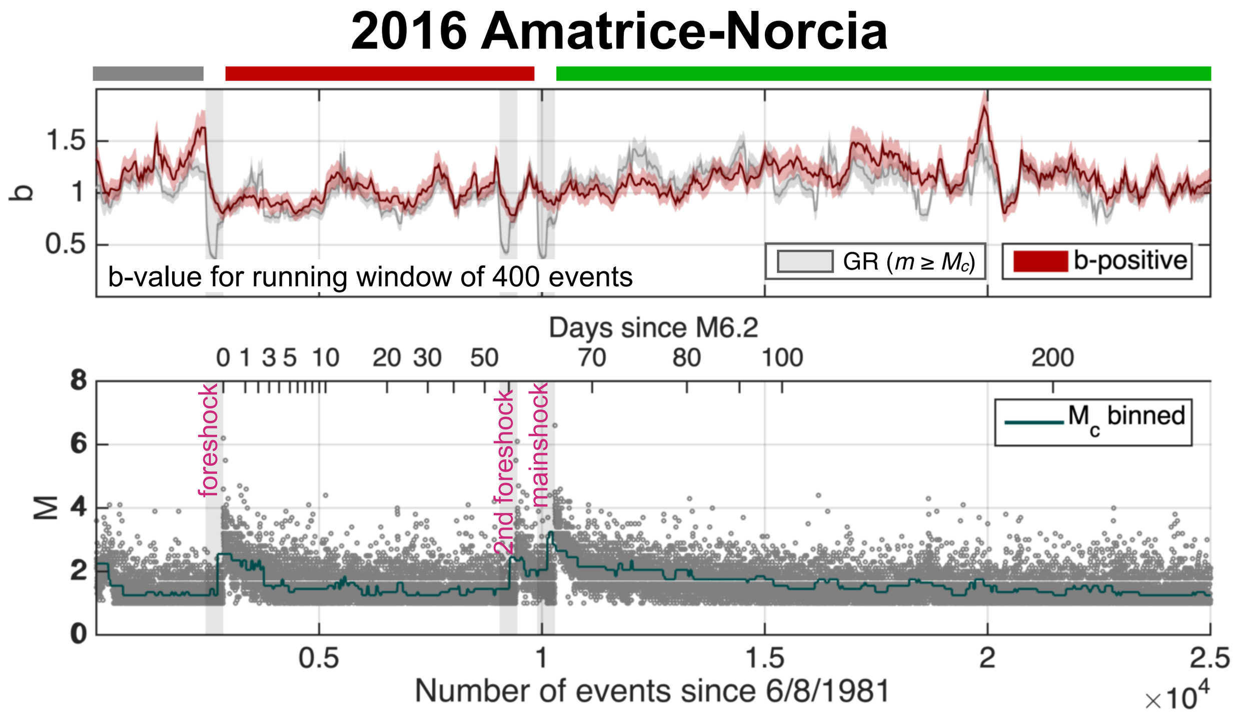
Caveats:

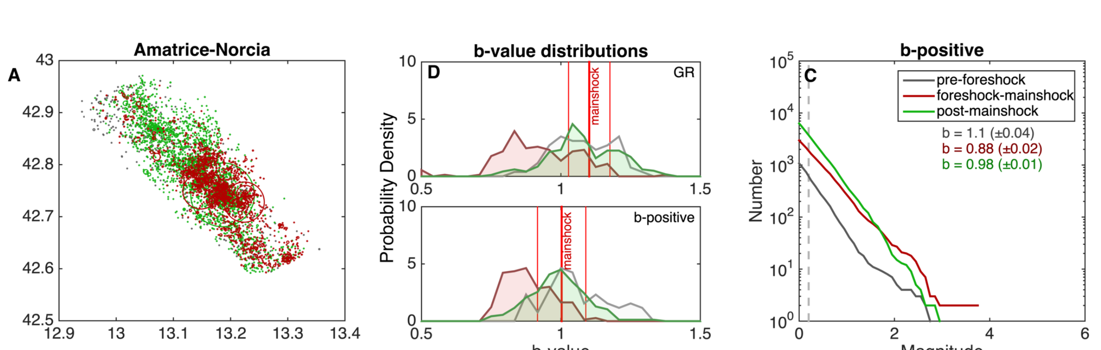
- The magnitude of the *b*-value drop barely exceeds historic fluctuations when transient incompleteness bias is avoided. This may complicate real time alerts.
- b-value recovers 'prematurely.' In fact, **the b-value has recovered to nearly a historic maximum at the moment immediately before the mainshock.** This casts some doubt on the straightforward interpretation of *b*-value as a stress-meter.
- This pattern is repeated in other sequences (below): *b*-value does change, but changes do not greatly exceed historic fluctuations, and the mainshock typically occurs with one of the highest *b*-values of the intervening period.

Application to other sequences*

* Sequences previously analyzed by Gulia and Wiemer [*Nature* 2019]

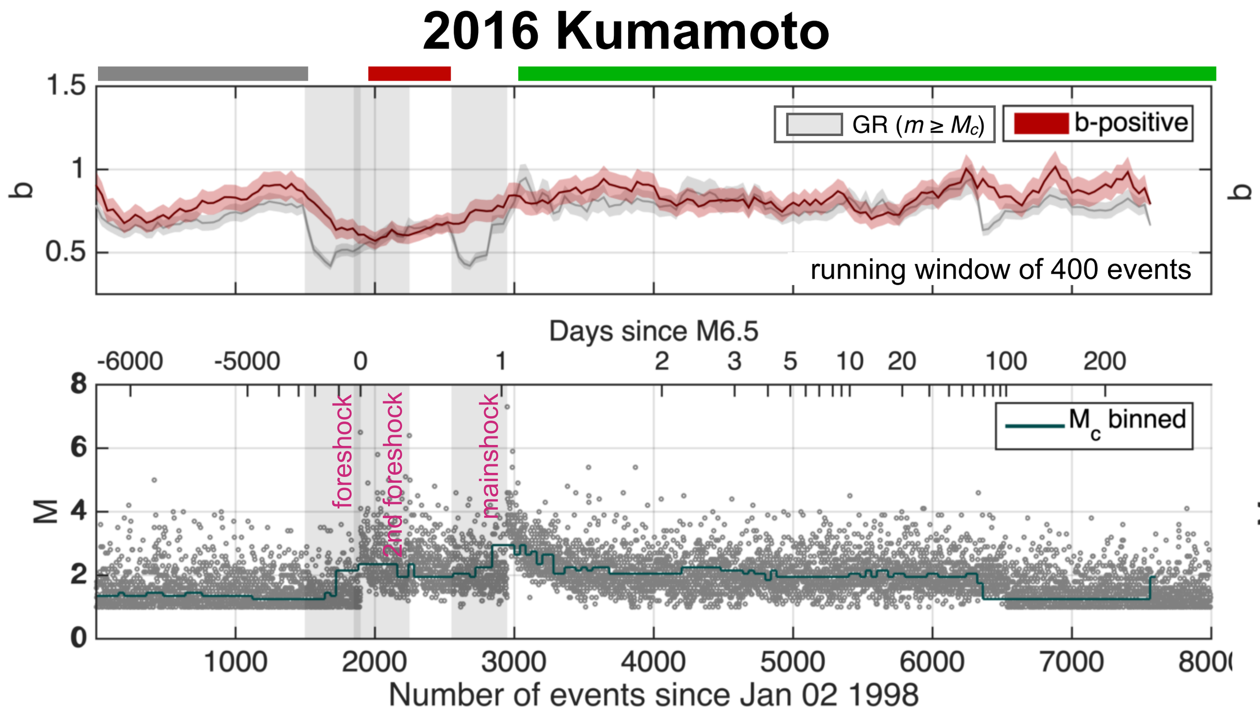
2016 Amatrice-Norcia

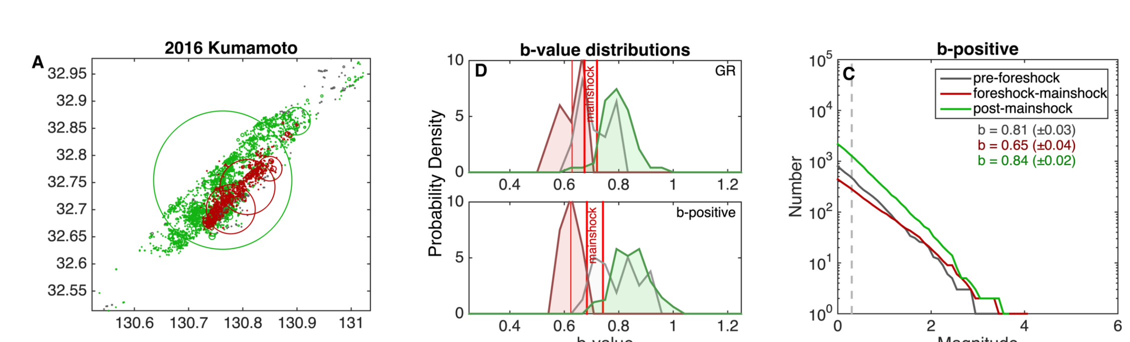




- ✔ b-value drop at time of foreshock.
- ✘ No increase at time of mainshock (gradual recovery only).
- ✘ Mainshock occurs when b-value is close to average level.

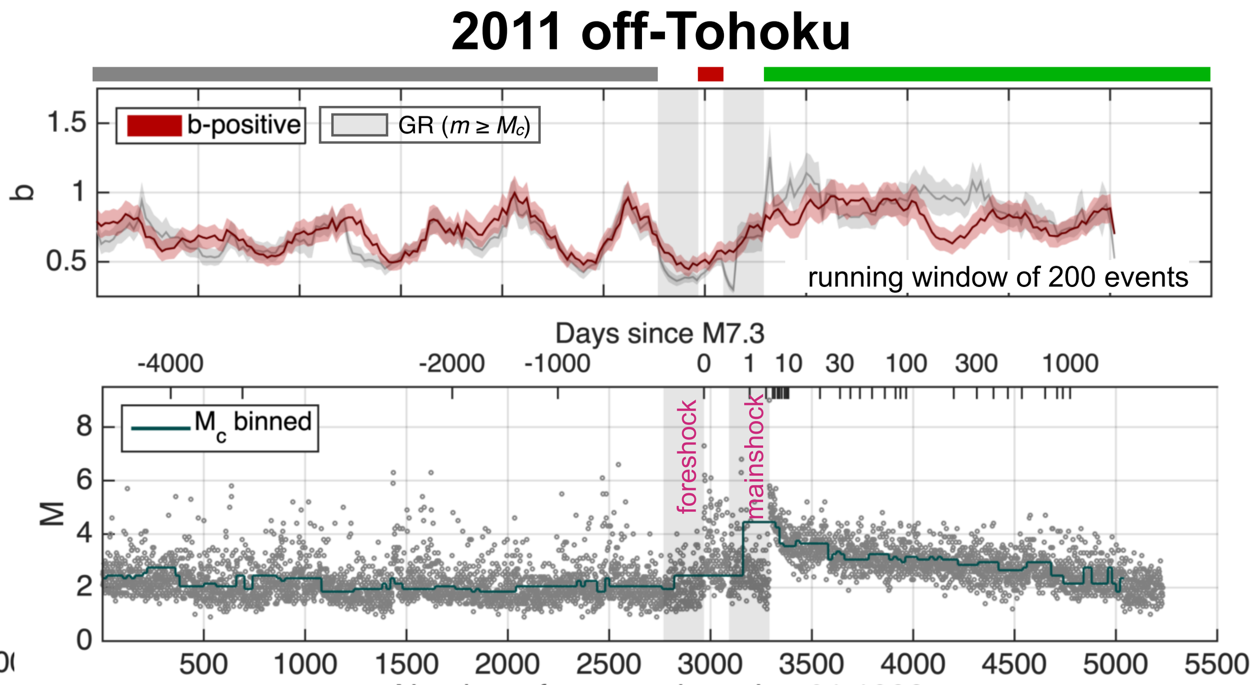
2016 Kumamoto

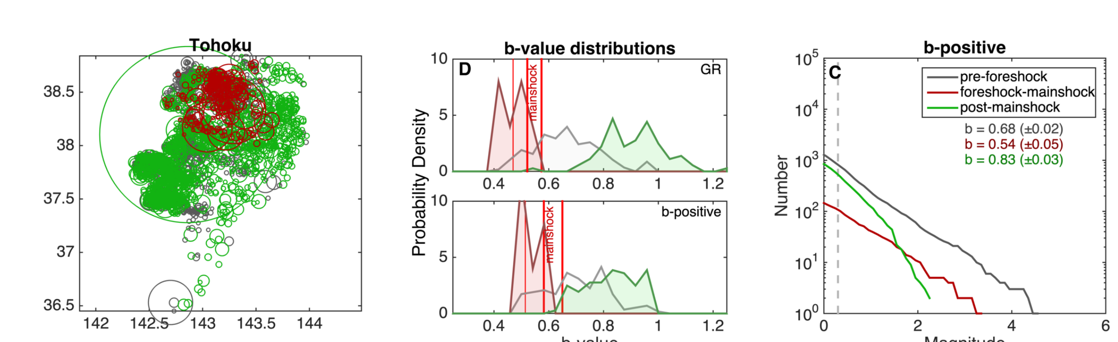




- ✔ b-value drop at time of foreshock.
- ✔ b-value increase at time of mainshock
- ✔ Mainshock occurs while b-value is still low.

2011 off-Tohoku





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