Probabilities from Precursors:

What do we Need?

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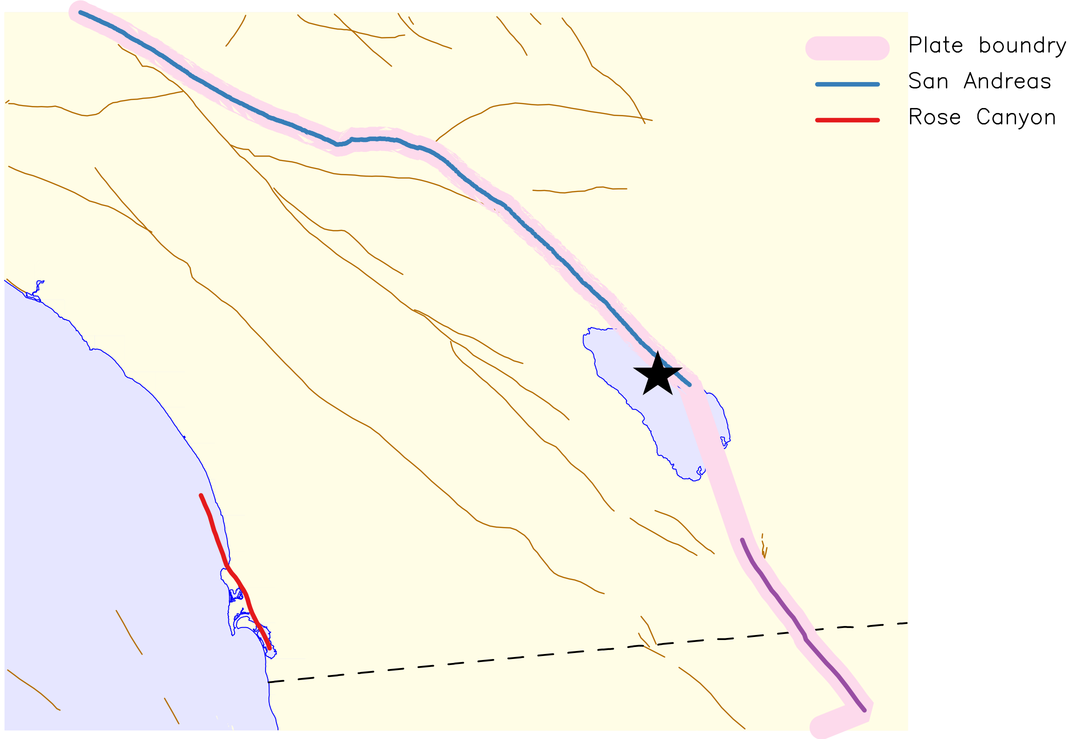
The Problem: Was That (Maybe) a Foreshock?

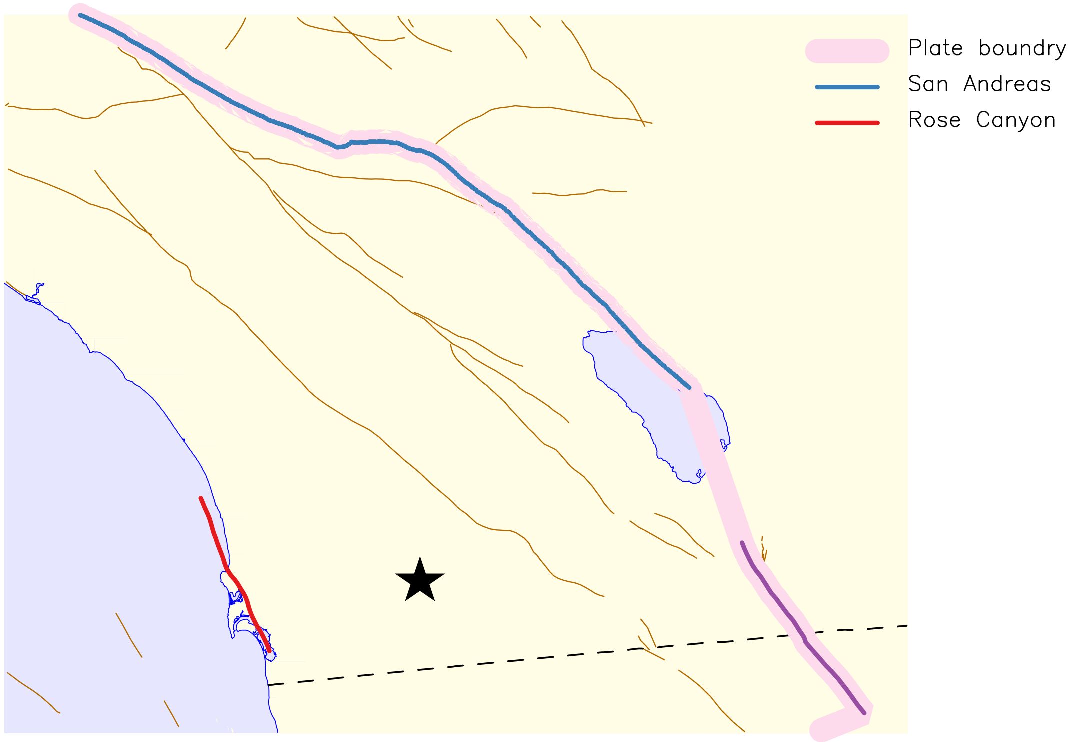
We have plenty of data that show foreshocks to bigger earthquakes: perhaps the only unquestioned precursor,

So, a small earthquake (possible foreshock) close to an active fault is cause for concern.



Was That (Maybe) a Foreshock?

But a small earthquake far from an active fault is not.



How do we quantify this? Assign a probability

A Simple Model (I)

A "zero-dimensional" version: just time, and three kinds of earthquakes:

- Big earthquakes (rare), which is what we try to find the probability of.
- Little earthquakes (common)
- Foreshocks, which look like Little earthquakes, but always have a Big earthquake within the next 3.65 days (0.01 year).

A Simple Model (II)

We observe an event which is either Little, or a Foreshock. What, given this, is the probability of a Big earthquake?

Formula from Bayes' theorem:

$$Pr(B|F \cap L) = \frac{Pr(F}{Pr(F) + Pr(L)} = \frac{Pr(F|B)Pr(B)}{Pr(F|B)Pr(B) + Pr(L)}$$

- $Pr(B)$: probability (over some time) of there being a Big earthquake, foreshock or not.
- $Pr(L)$: probability of there being a Little earthquake
- $Pr(F|B)$: probability of there being a Foreshock given a Big earthquake

An Even Simpler Approach

- There is a B (on average) every 100 years (say).
- There are 10 L's per year.
- We observe an L that we (later) identify as an F for 50% of the B's.

In 1000 years we have

- 10 B's, and hence 5 F's
- 10,000 L's

That is 10,005 L's and F's together, 5 of these are F's. So the probability that an L is really an F is $\frac{5}{10,005}$

The (possible) foreshock increases the probability by a factor of five (but it is still small).

The Multidimensional Case

Allowing for space, time, and magnitude, the probability of a foreshock is a messy integral:

$$P(F) = \int_{t}^{t+\delta_0} dt \int_{t+\Delta}^{t+\Delta+\delta_1} dt' \, \Phi_t(t, t') \int_{M-\mu}^{M+\mu} dM \int_{M_C}^{M_C+\mu_C} dM' \, \Phi_m(M, M') \, e^{-\beta' M'}$$

$$\cdot \int_{x_0-e_0}^{x_0+e_0} dx \int_{y_0-e_0}^{y_0+e_0} dy \int\int_{A_C} dx'dy' \, \Phi_s(x, y, x', y') \Omega_s(x', y')$$

Functions in red give precursor probability density before mainshocks as a function of

- space
- time
- magnitude (of both events)

Conclusions

The mathematics and conclusions apply to any precursor:

- A prediction needs to have a probability.

- To create such a probability, we need to know, for any precursor:

  - The rate of occurrence of precursor-like things throughout the space-time region of interest.

  - The probability of a precursor, given a mainshock, as a function of
    - Time difference
    - Spatial separation
    - Magnitude (of the mainshock at least)