ETAS v STEP
OEF testing strategies

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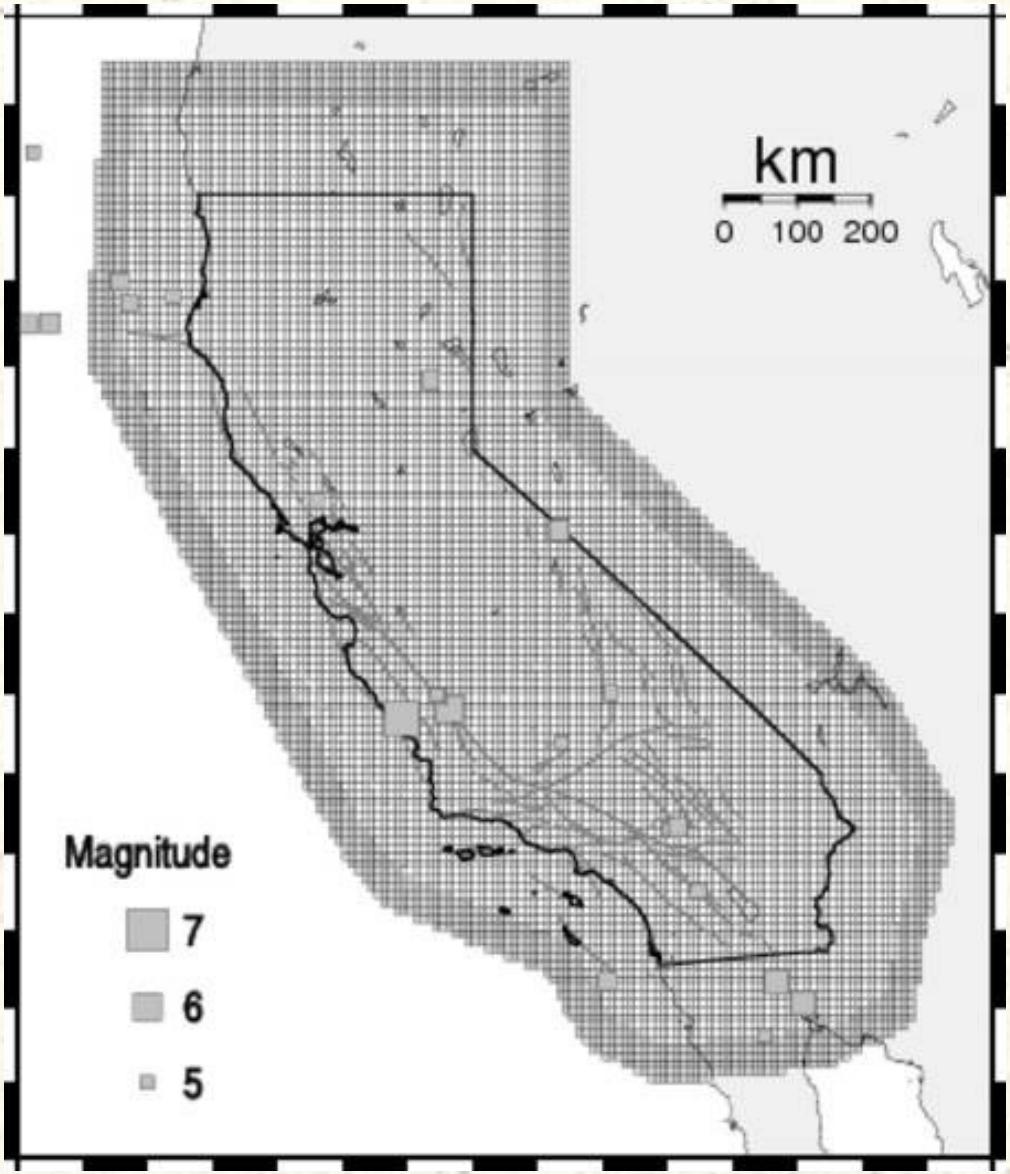
Daily forecast experiment in California

Target eqks with $M_{ANSS} \geq 3.95$ in California for $\Delta t = 1$ day

Lat/lon/mag bins are 0.1° x 0.1° x 0.1.

270 observed target eqks earthquakes between Sept. 2007 and August 2010.

2 participating models: ETAS (implemented by Zhuang) and STEP (implemented by Gerstenberger)



Zechar et al. in prep.

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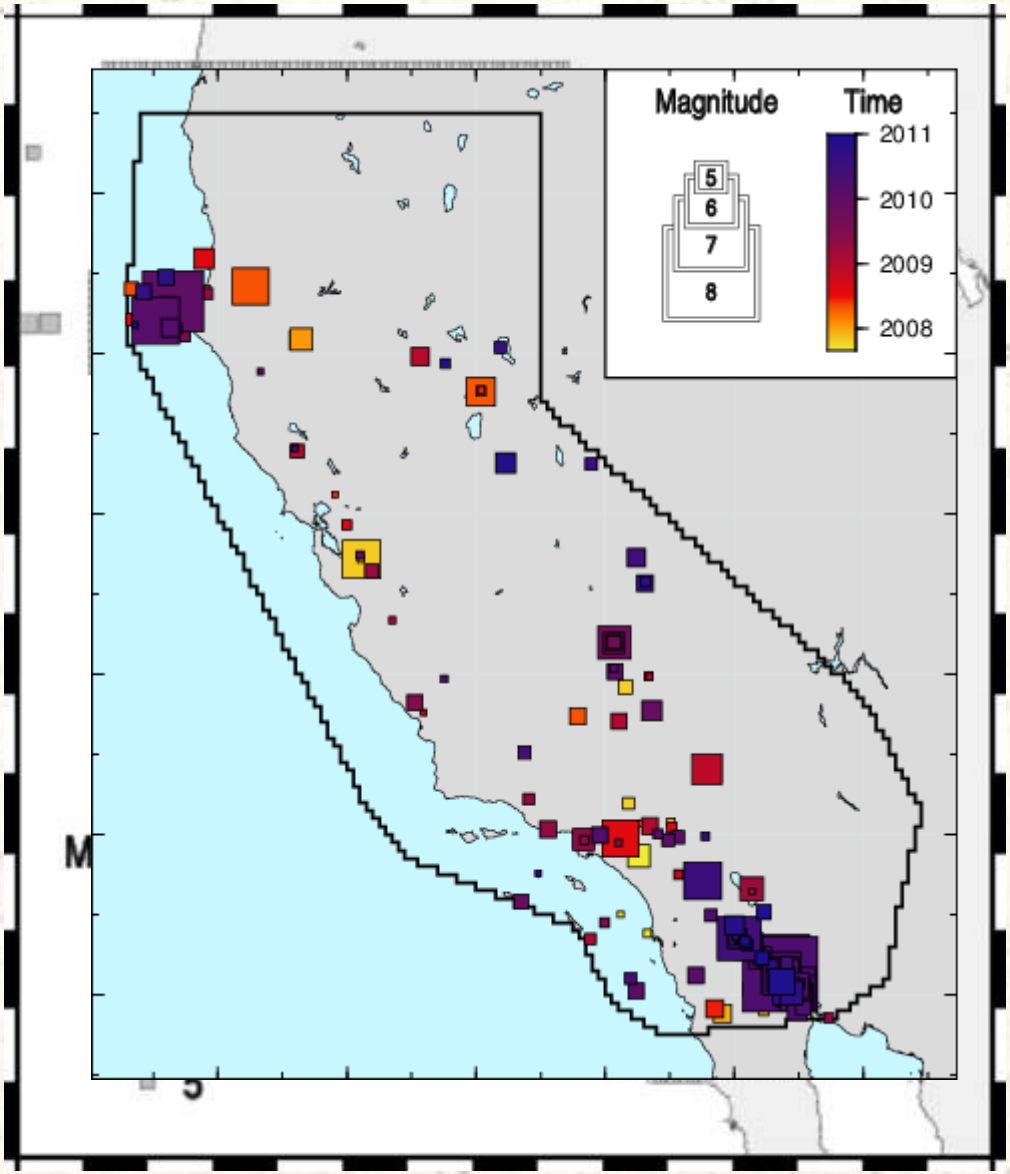

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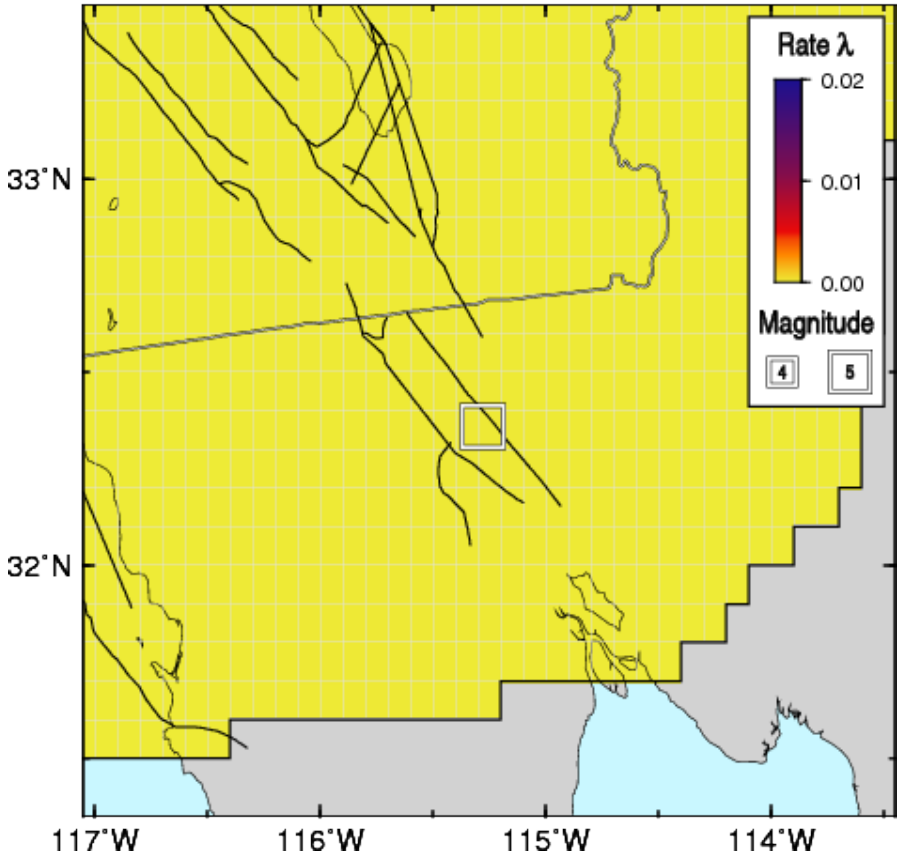
Testing:
Feb. 2008 swarm in Baja California

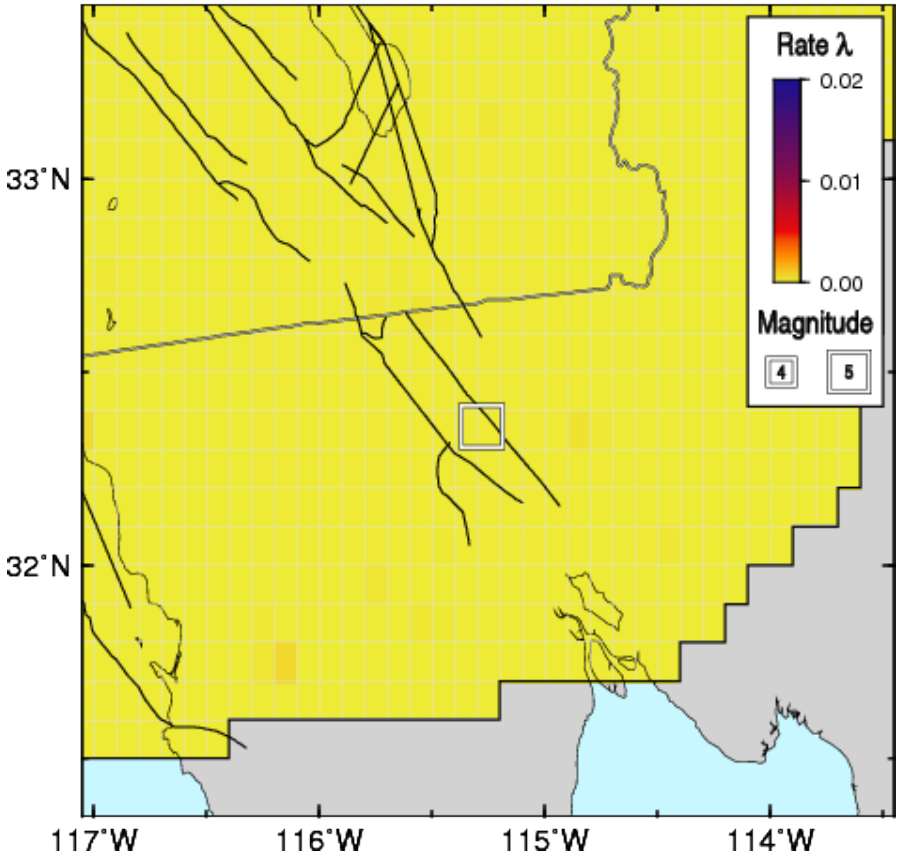
9 Feb. 2008

(background)

ETAS



STEP



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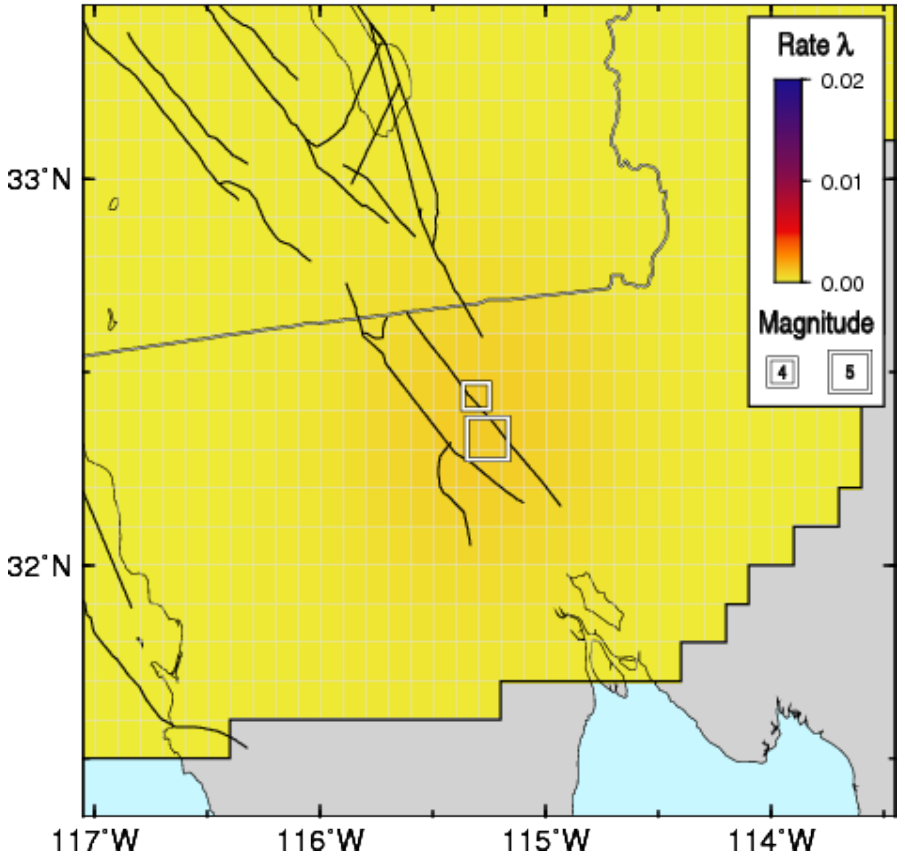
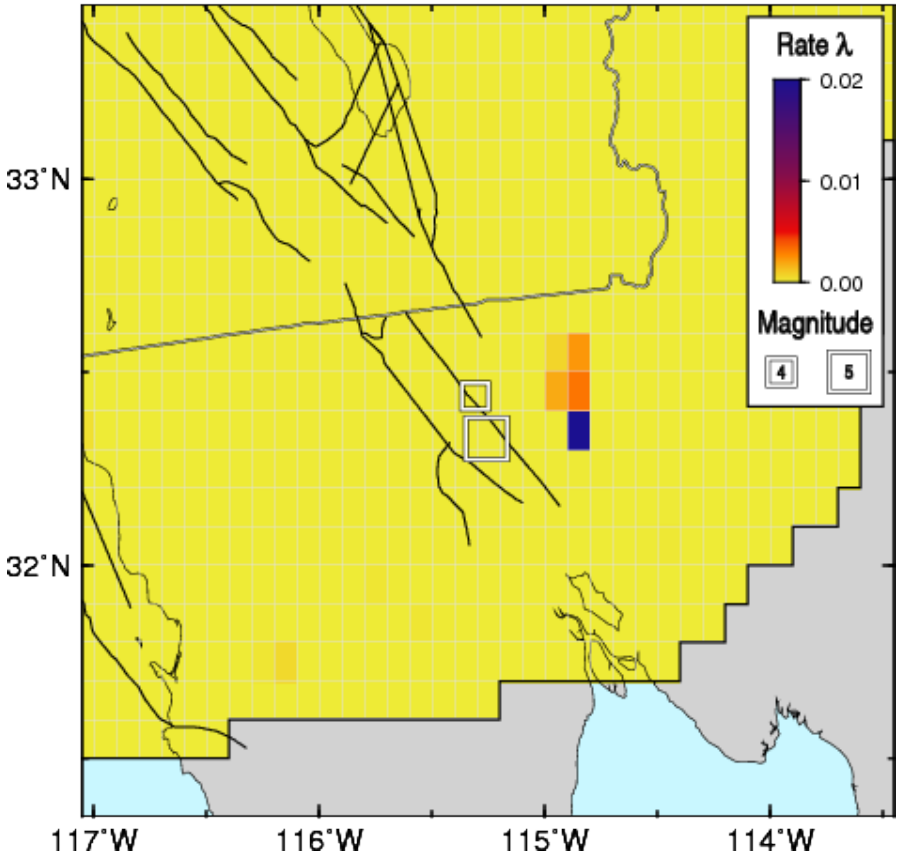
ETAS



STEP



Testing:
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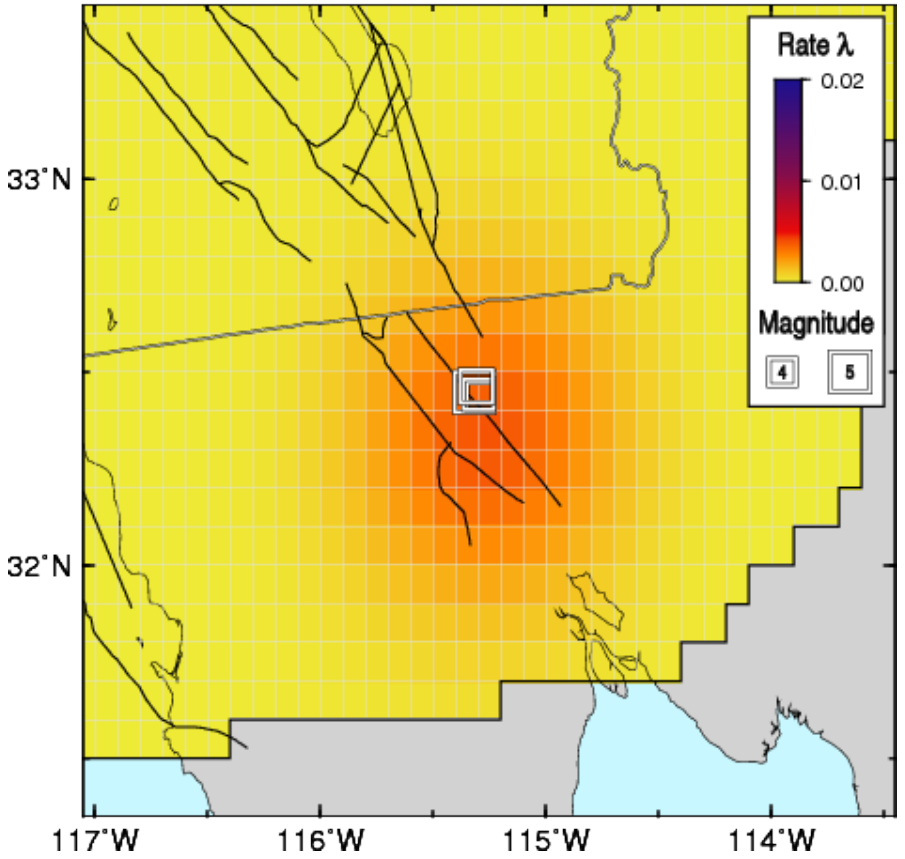
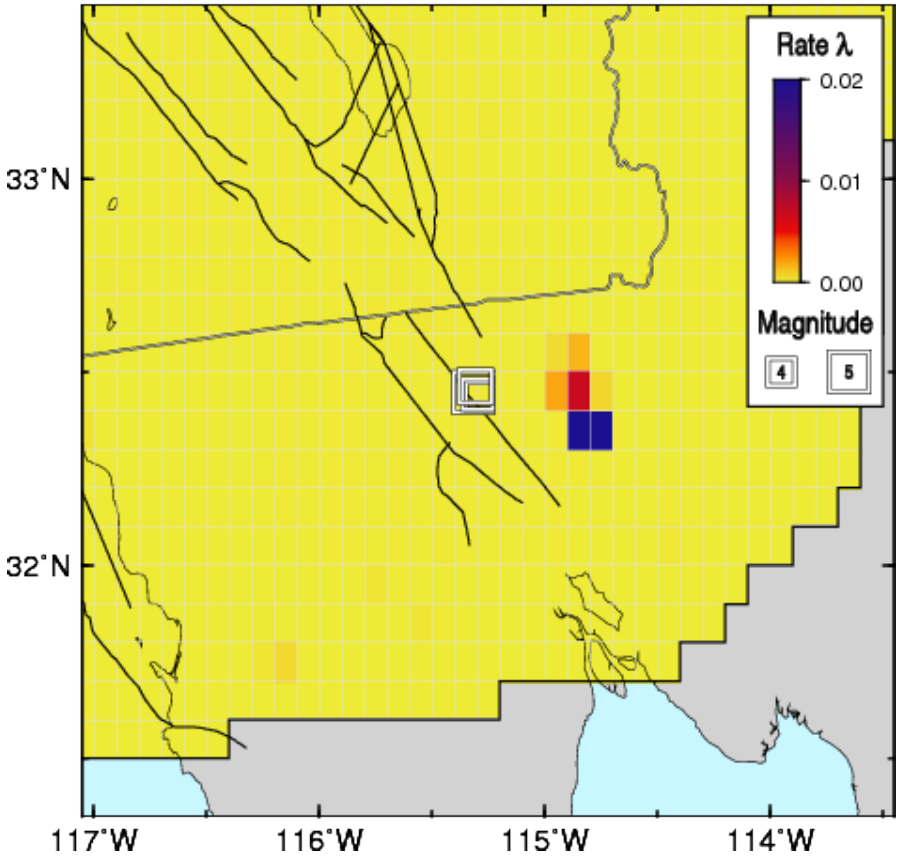
12 Feb. 2008

ETAS



STEP



Oops!

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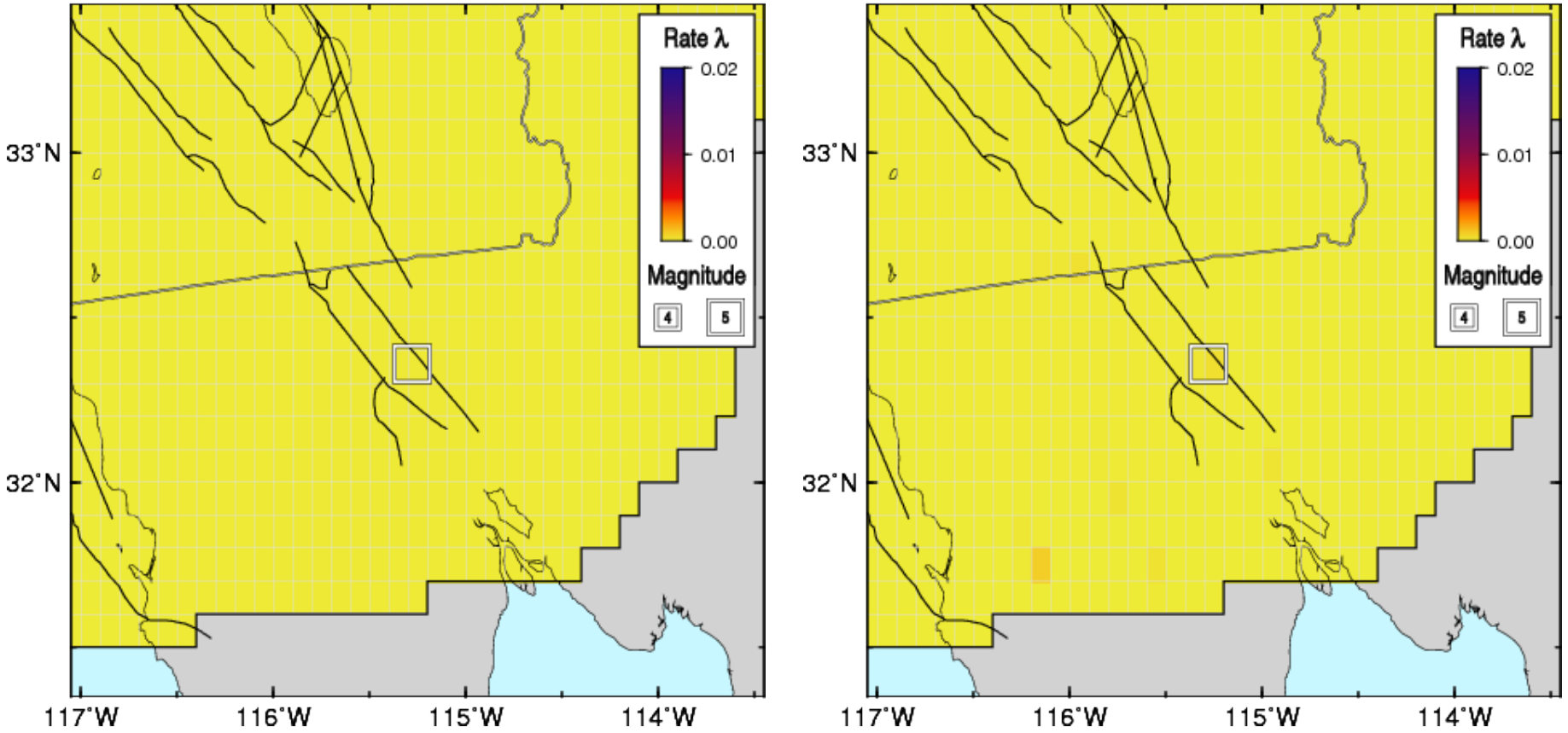
9 Feb. 2008

(background)

ETAS

STEP, corrected



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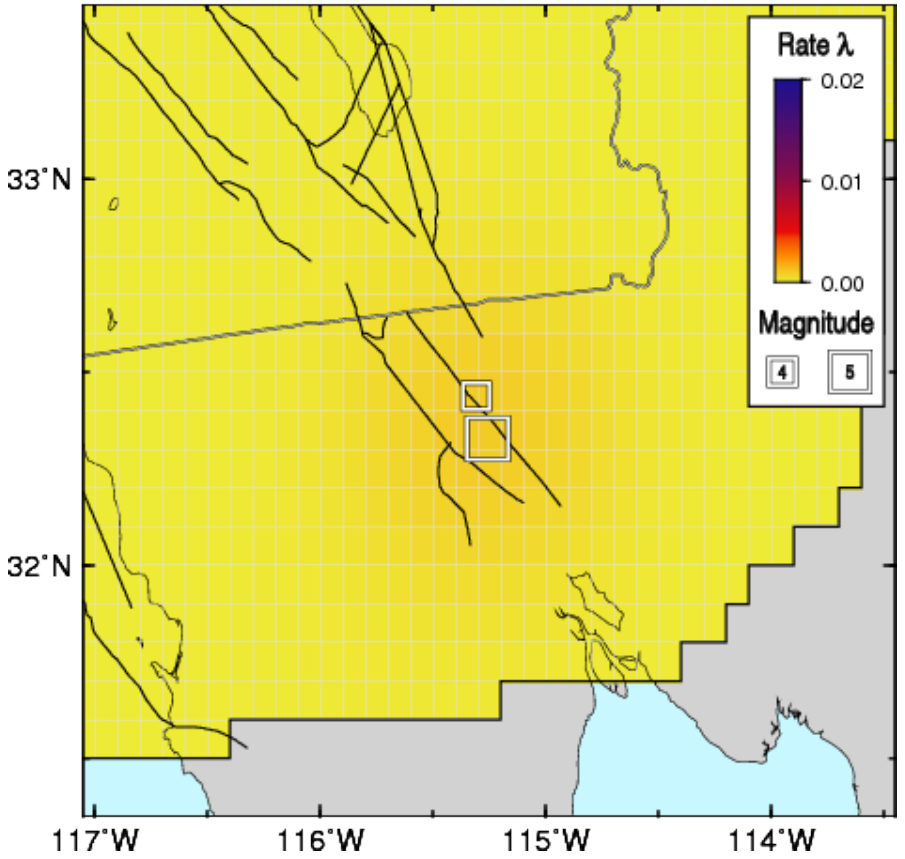

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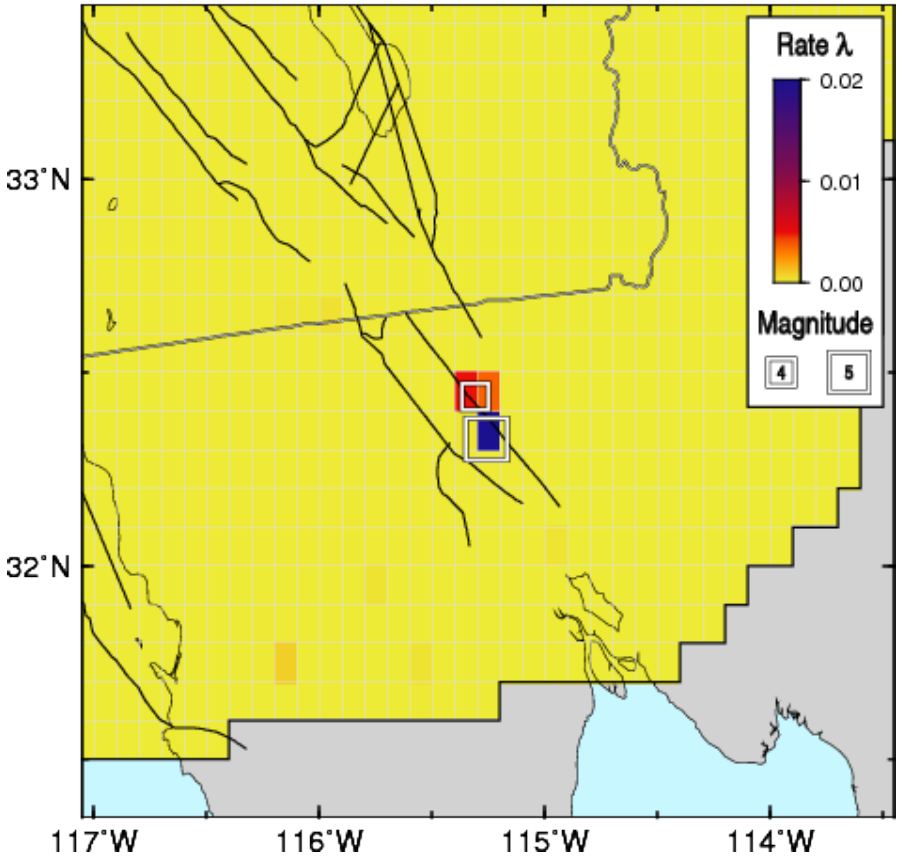

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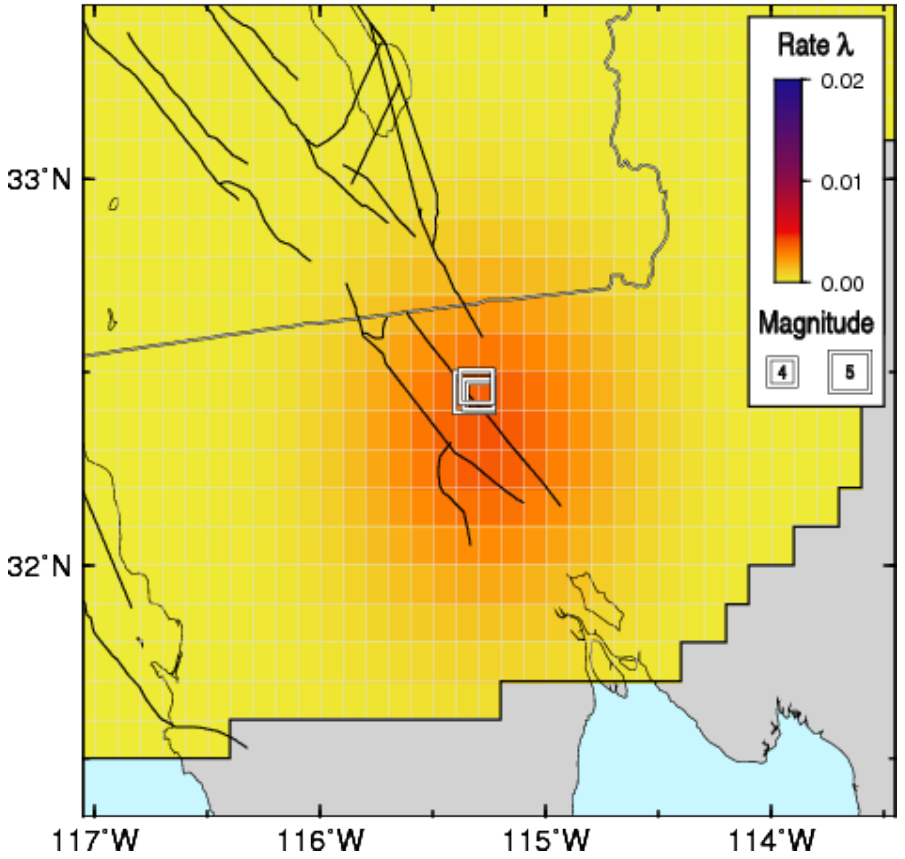

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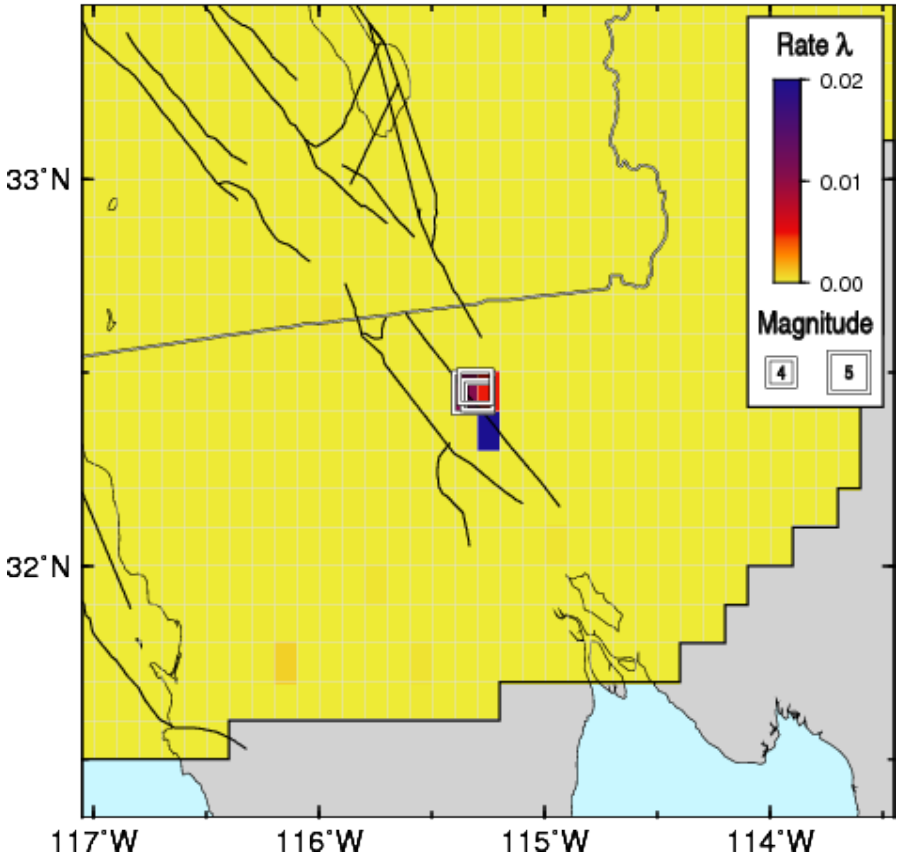


Evaluation – consistency

The forecasts tell us the probability of any possible observation...

- How many earthquakes were expected, and how many were observed?
- What is the likelihood of the observation?

Evaluation – consistency

Number of earthquakes forecast

$$N_\Lambda = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \lambda_{ijk}$$

Number of earthquakes observed

$$N = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \omega_{ijk}$$

Log-likelihood of observation given forecast

$$L(\Omega|\Lambda) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \left(\omega_{ijk} \ln \lambda_{ijk} - \lambda_{ijk} - \ln(\omega_{ijk}!) \right)$$

For each experiment, we obtain one value for each of these measures; how can we tell if it's "good"?

Evaluation – consistency

To approximate the likelihood function of a forecast, produce many (1K-1M) stochastic catalogs consistent with the forecast, and compute the likelihood of each.

The percentile of the observed likelihood can then be thought of as a p-value.

We can do this for space-rate-magnitude forecast and each component (i.e., space, rate, and magnitude in isolation).

- Resulting tests are called L, M, N, and S.

Schorlemmer et al. 2007, 2010
Zechar et al. 2010
Werner et al. 2011

Evaluation – comparison

$$I_N(\boldsymbol{A},\boldsymbol{B}) = \frac{1}{N}\sum_{i=1}^{N}\left(\ln\frac{A_i}{B_i}\right) + \frac{N_{\boldsymbol{B}}-N_{\boldsymbol{A}}}{N}$$

I_N: rate-corrected average info. gain per eqk

$\boldsymbol{A}$, $\boldsymbol{B}$: forecasts to compare

N: number of target eqks

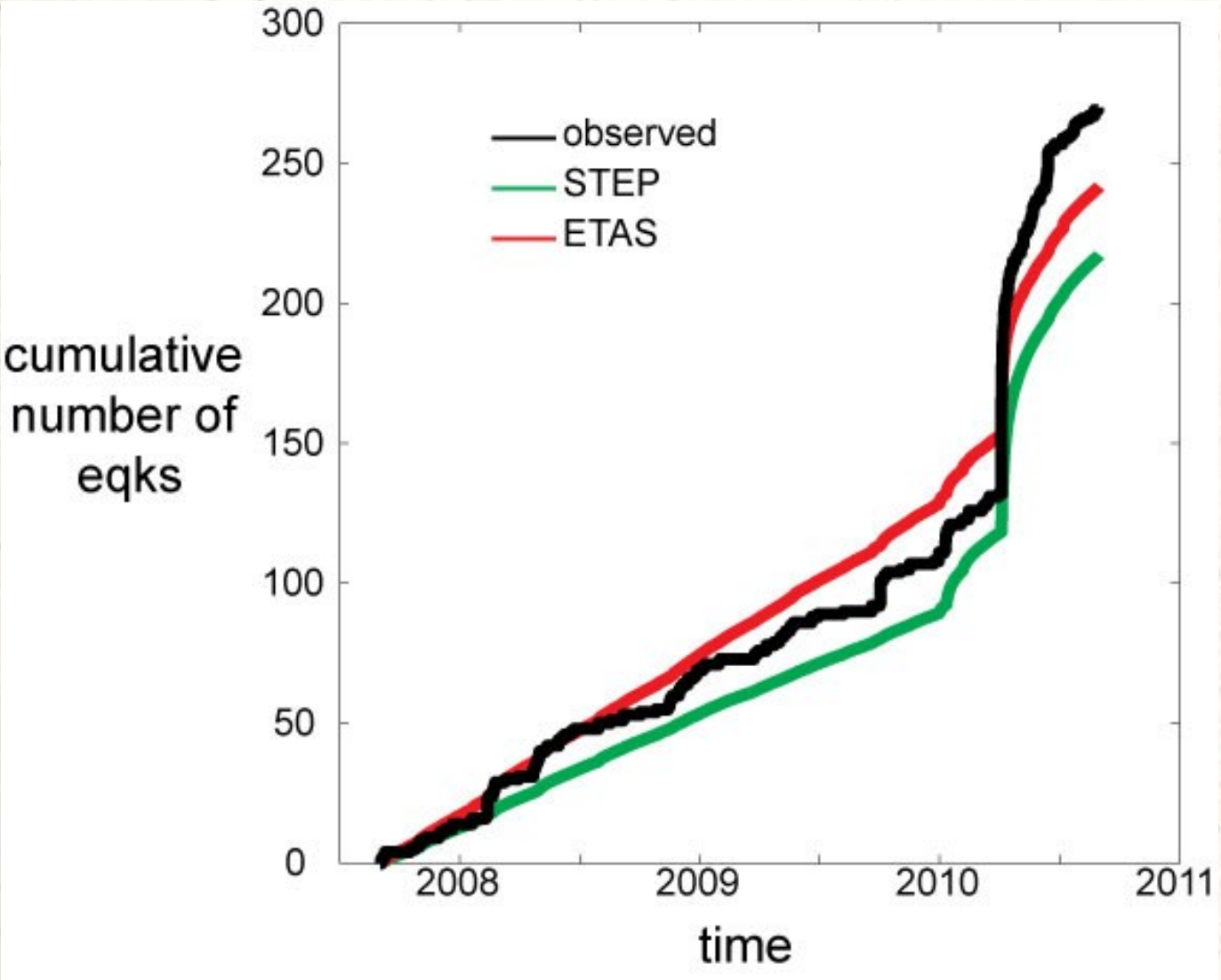
A_i: rate of forecast $\boldsymbol{A}$ in the bin of the i^{th} eqk

$N_{\boldsymbol{A}}$: total rate of forecast $\boldsymbol{A}$

Is I_N significantly different from zero? (Apply Student's paired t-test and Wilcoxon signed rank test to sample gains, resulting tests are called T and W.)

Rhoades et al. 2011

Evaluation – consistency
ETAS & STEP



STEP offers a better fit to individual numbers per day.

Overall, STEP significantly underestimates the total number of earthquakes.

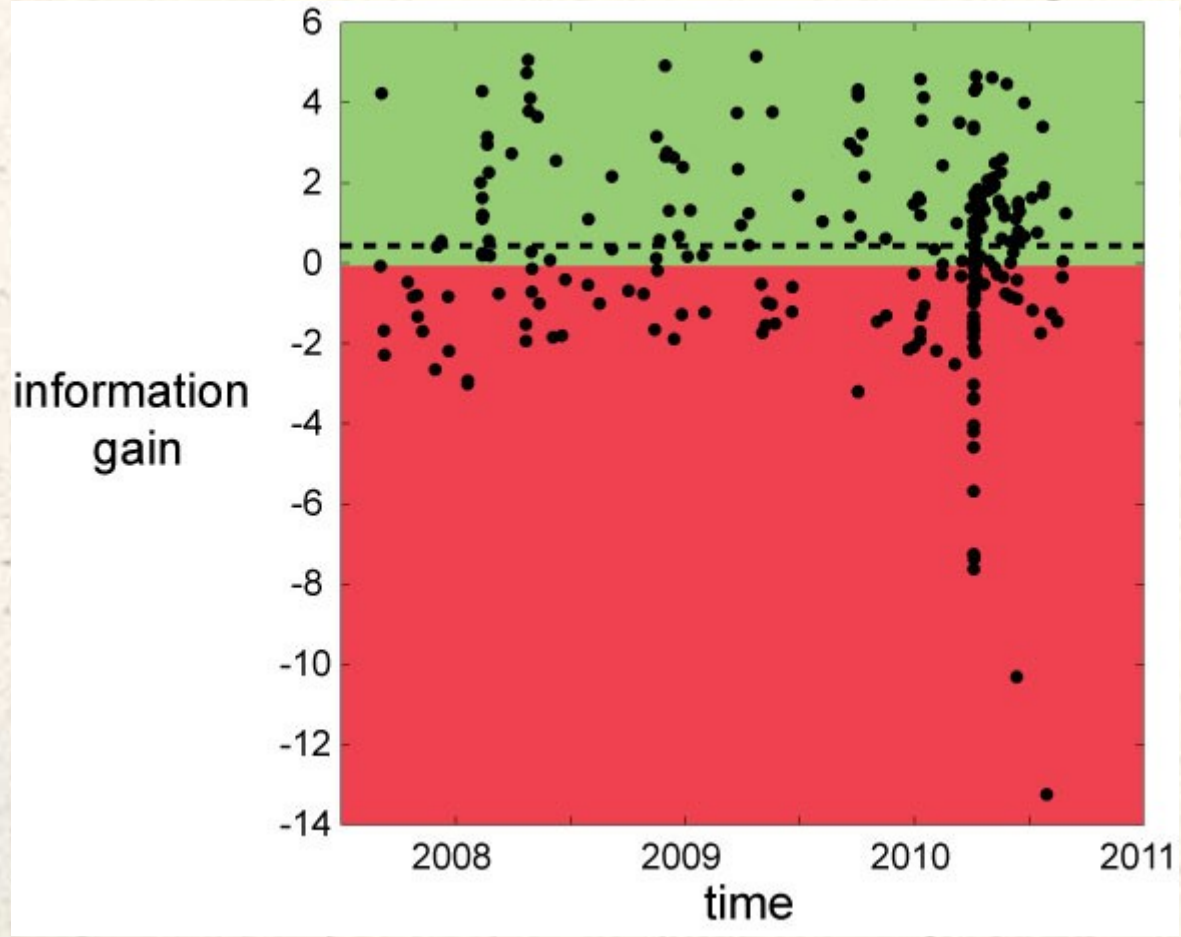
Should we “split” or should we “lump”?

Evaluation – consistency
ETAS & STEP

Evaluation Metric	Model	
	ETAS	STEP
Rate	18/1096 (1.000)	19/1096 (1.000)
Magnitude	5/161 (0.909)	2/161 (0.998)
Space	7/161 (0.699)	21/161 (0.000)
Space-Rate-Magnitude	47/1096 (0.877)	77/1096 (0.002)

- Table indicates frequency of inconsistency
- 95% confidence for each metric for each day

Evaluation – comparison ETAS & STEP



- I_N(STEP, ETAS) = 0.43
- With more than 99% confidence, we can say that STEP was better than ETAS.

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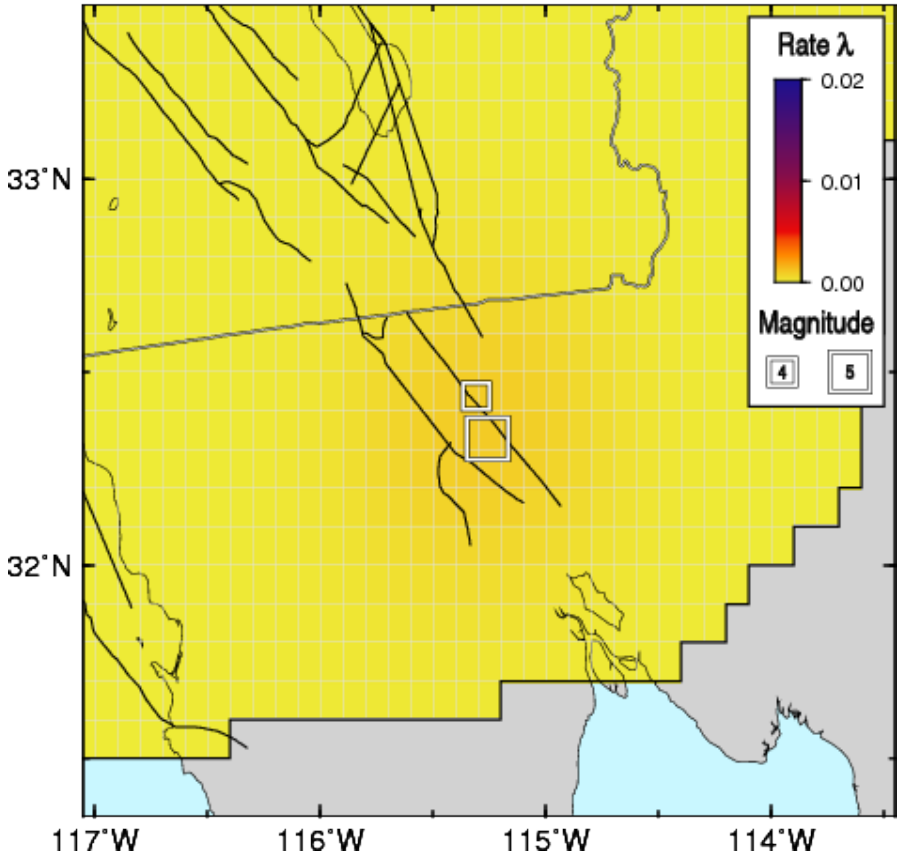

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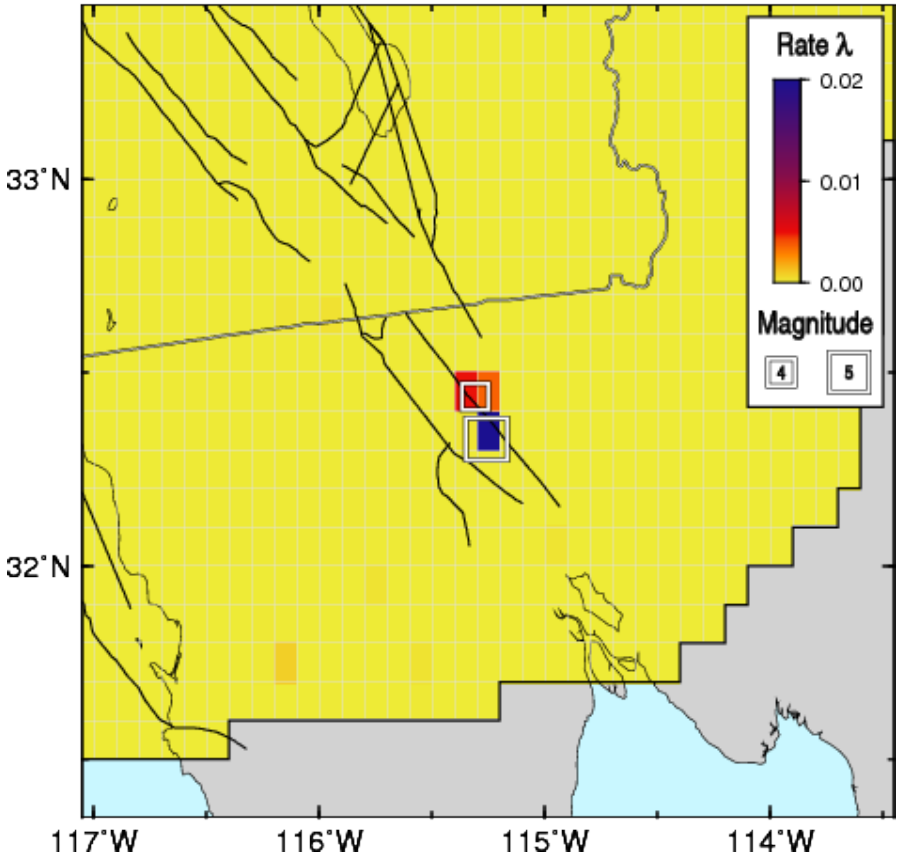

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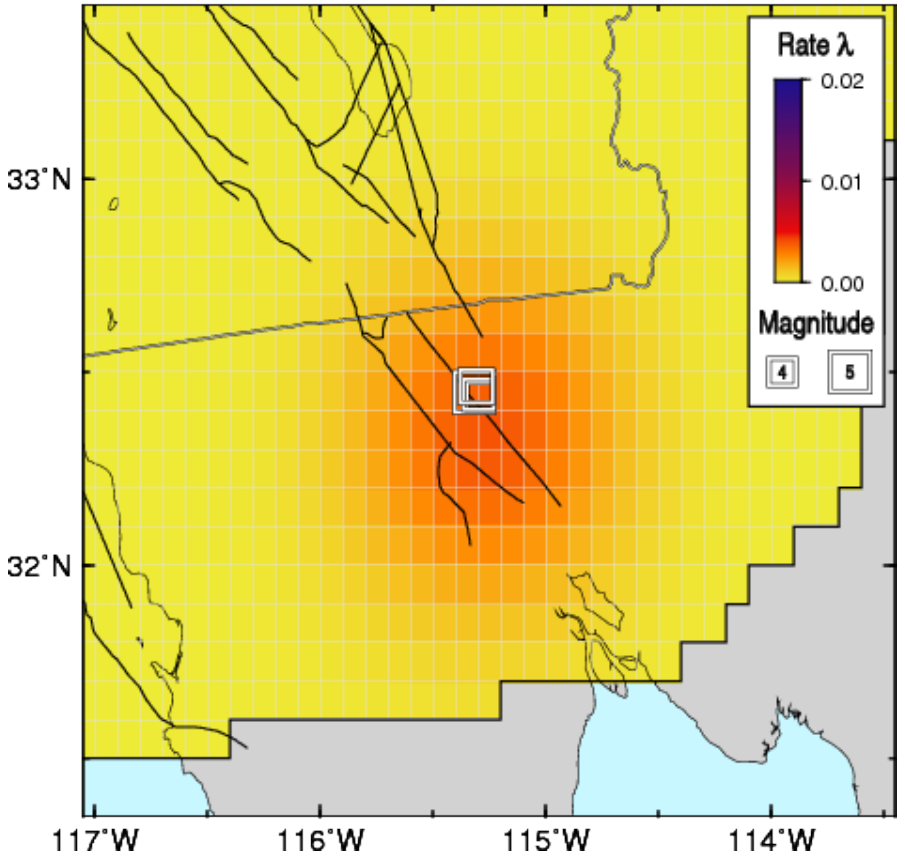

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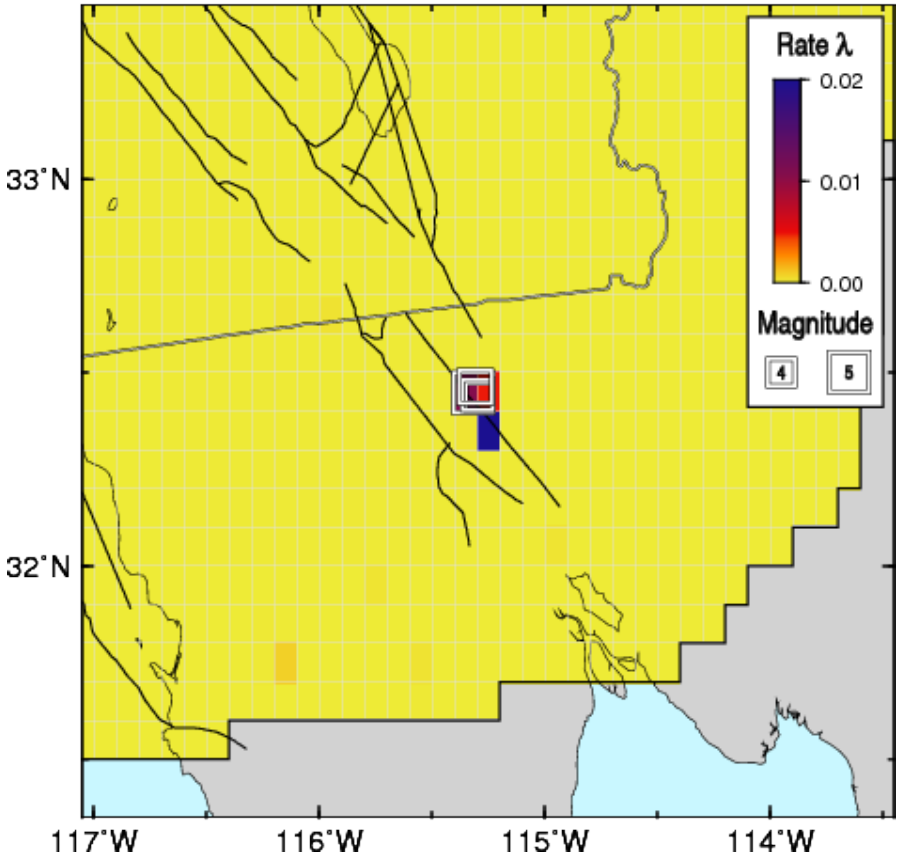
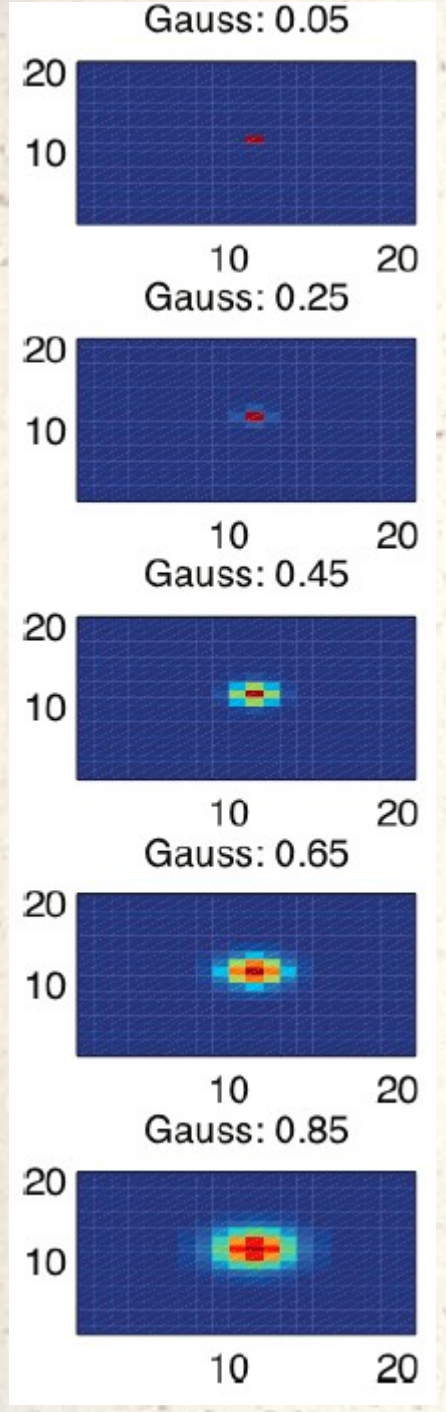
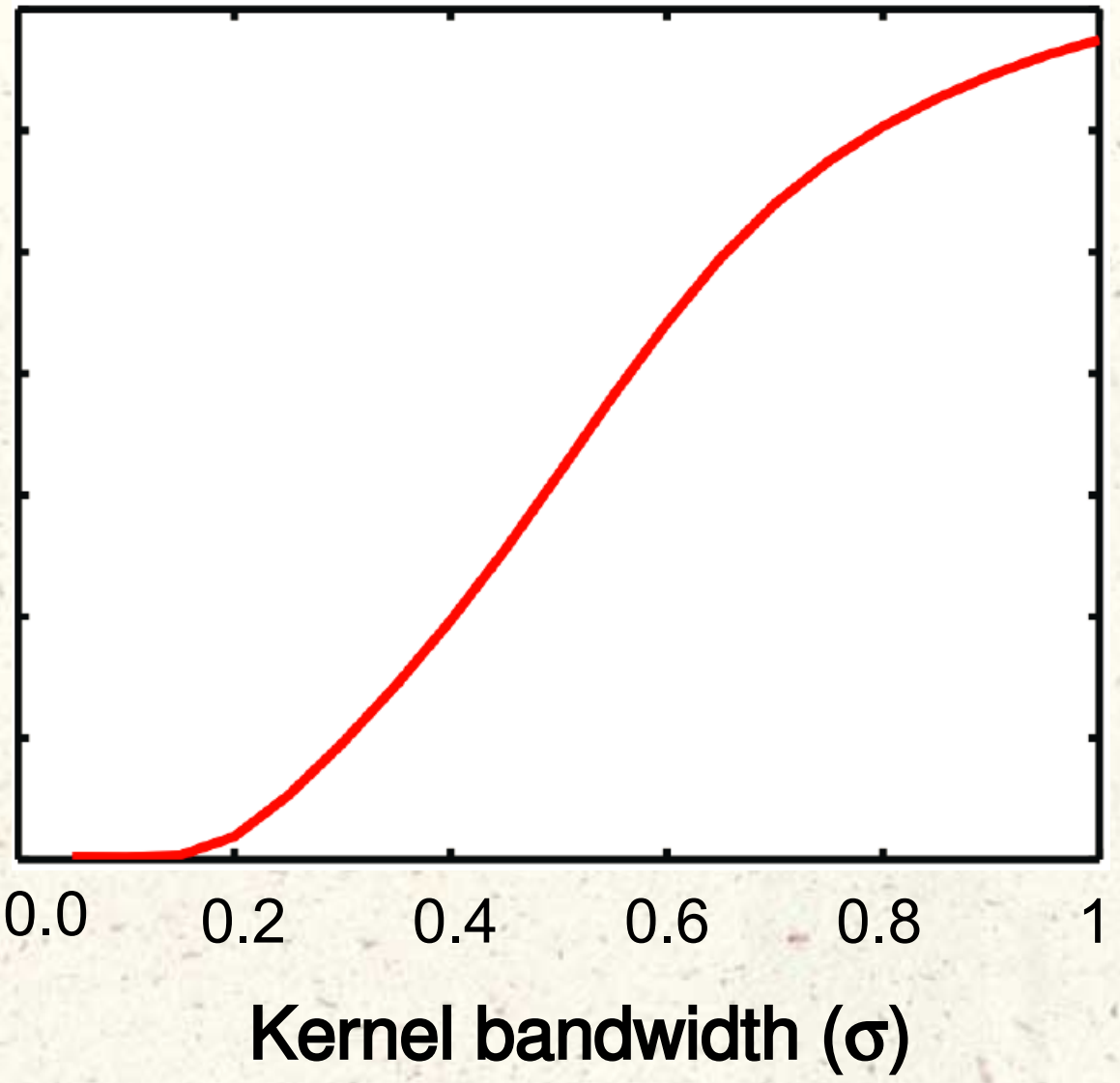


Is ETAS smoother than STEP?

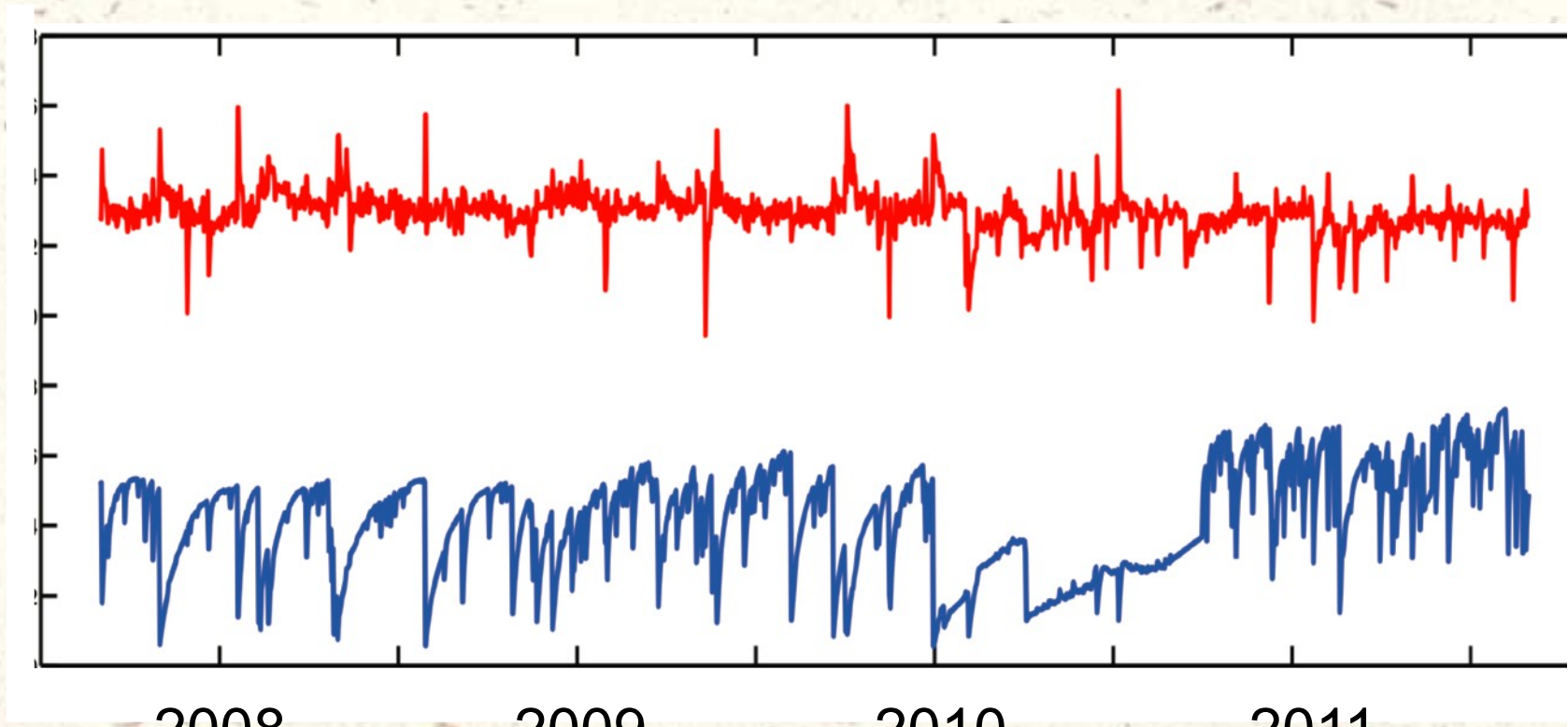
To characterize smoothness, compute 2D Fourier transform and subtract the mean value from the maximum...


Gauss: 0.05
20
10
10
20
Gauss: 0.25
20
10
10
20
Gauss: 0.45
20
10
10
20
Gauss: 0.65
20
10
10
20
Gauss: 0.85
20
10
10
20

smoothness


0.0
0.2
0.4
0.6
0.8
1
Kernel bandwidth (σ)

ETAS is much smoother than STEP



Smoothness is a double-edged sword

- STEP is “better” than ETAS because STEP is less smooth
- STEP fails S-test more frequently than ETAS because STEP is less smooth

Potential next steps

- See if complexities of STEP (sequence-specific parameterization, spatially-varying parameters) provide strong advantage
  - How often does STEP go into these modes?
- Re-analyze with updated data
- More splitting vs. lumping
- Bayesian analysis
- Construct ensemble forecasts