Non-Linear Radiation Damping:
A New Method for Dissipating Energy in Dynamic Earthquake Rupture Simulations

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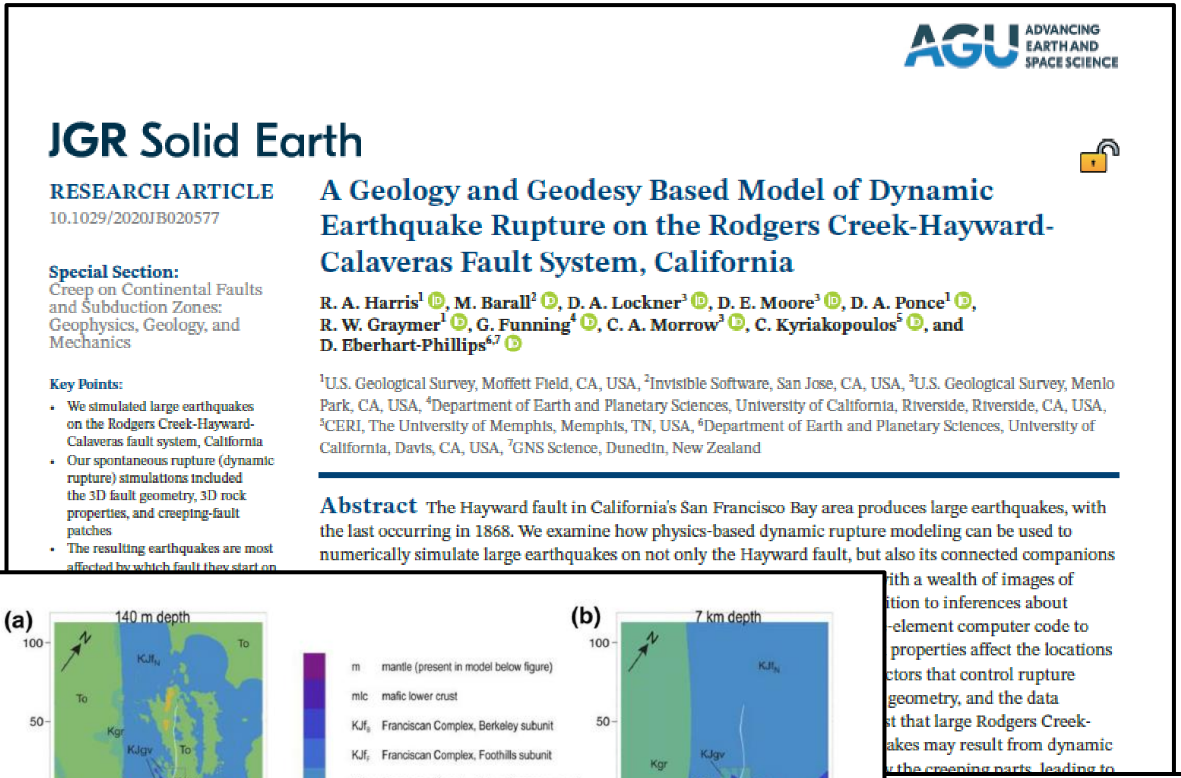
Our Hayward dynamic rupture simulations used linear elastic material properties.

It's easy to set up initial conditions in elastic models:

- Can assign arbitrary shear and normal tractions on the fault.
- No need to know absolute stress tensor in the model volume.
- No need for gravity or fluid pressure.

<u>But</u>: Elastic models can produce unrealistically high slip rates and ground motions.



ADVANCING EARTH AND SPACE SCIENCE

JGR Solid Earth

RESEARCH ARTICLE

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Special Section:
Creep on Continental Faults and Subduction Zones: Geophysics, Geology, and Mechanics

Key Points:

- We stimulated large earthquakes on the Rodgers Creek-Hayward-Calaveras fault system, California
- Our spontaneous rupture (dynamic rupture) simulations included the 3D fault geometry, 3D rock properties, and creeping-fault patches
- The resulting earthquakes are most affected by which fault they start on

A Geology and Geodesy Based Model of Dynamic Earthquake Rupture on the Rodgers Creek-Hayward-Calaveras Fault System, California

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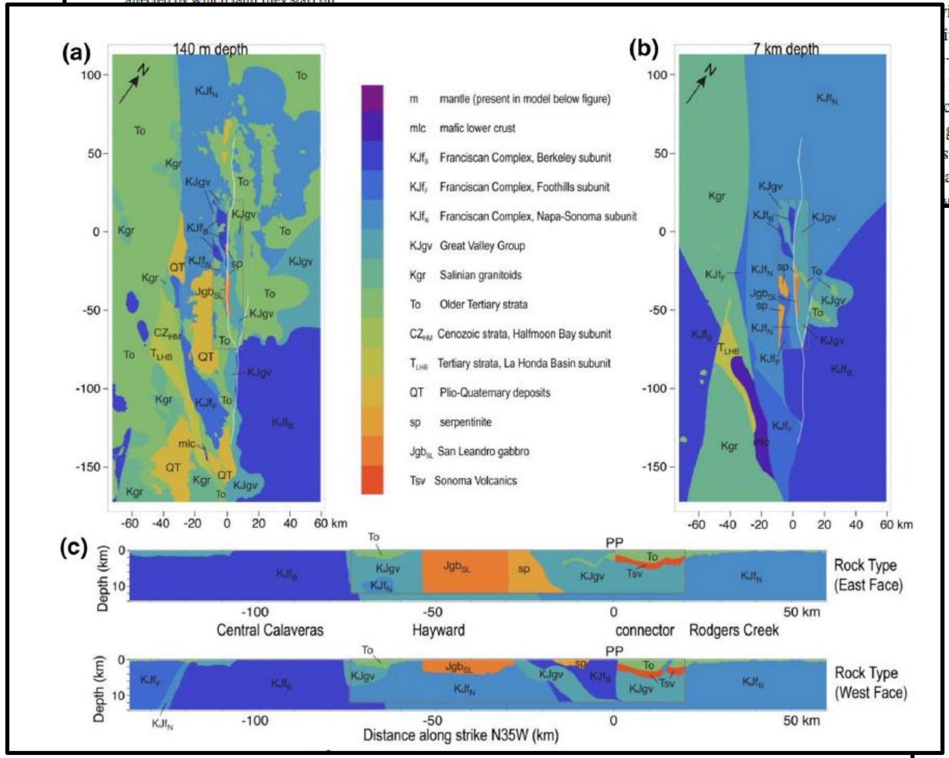
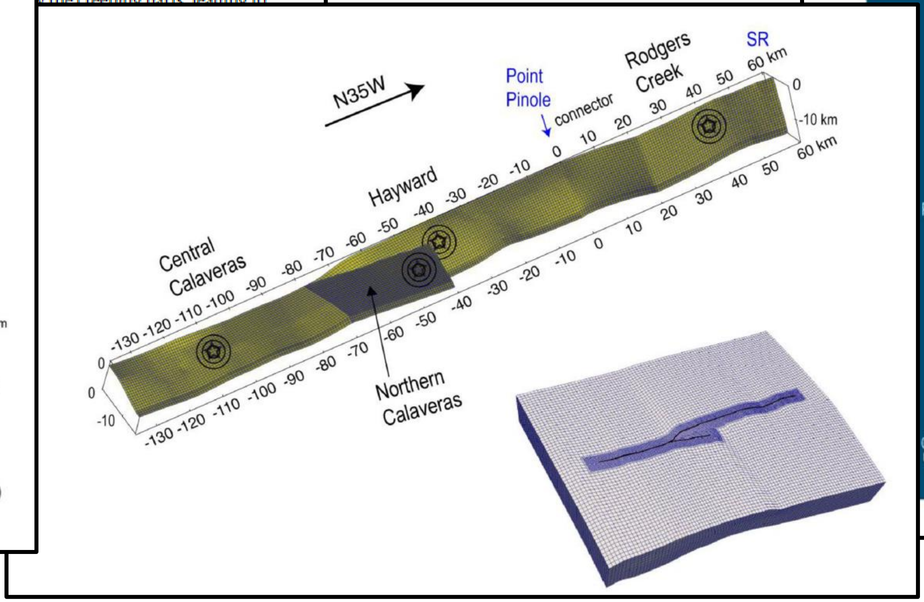
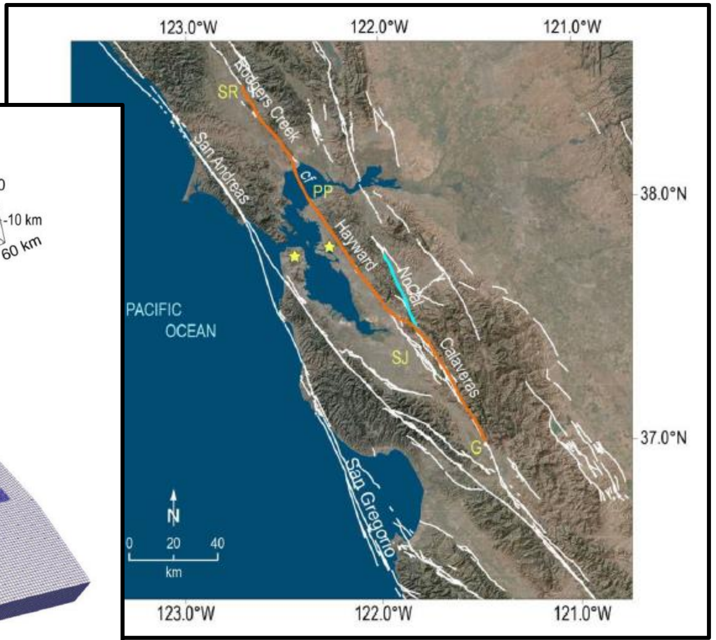
Abstract The Hayward fault in California's San Francisco Bay area produces large earthquakes, with the last occurring in 1868. We examine how physics-based dynamic rupture modeling can be used to numerically simulate large earthquakes on not only the Hayward fault, but also its connected companions ... ith a wealth of images of ... ition to inferences about ... -element computer code to ... properties affect the locations ... ctors that control rupture ... geometry, and the data ... t that large Rodgers Creek- ... akes may result from dynamic ... the creeping parts, leading to

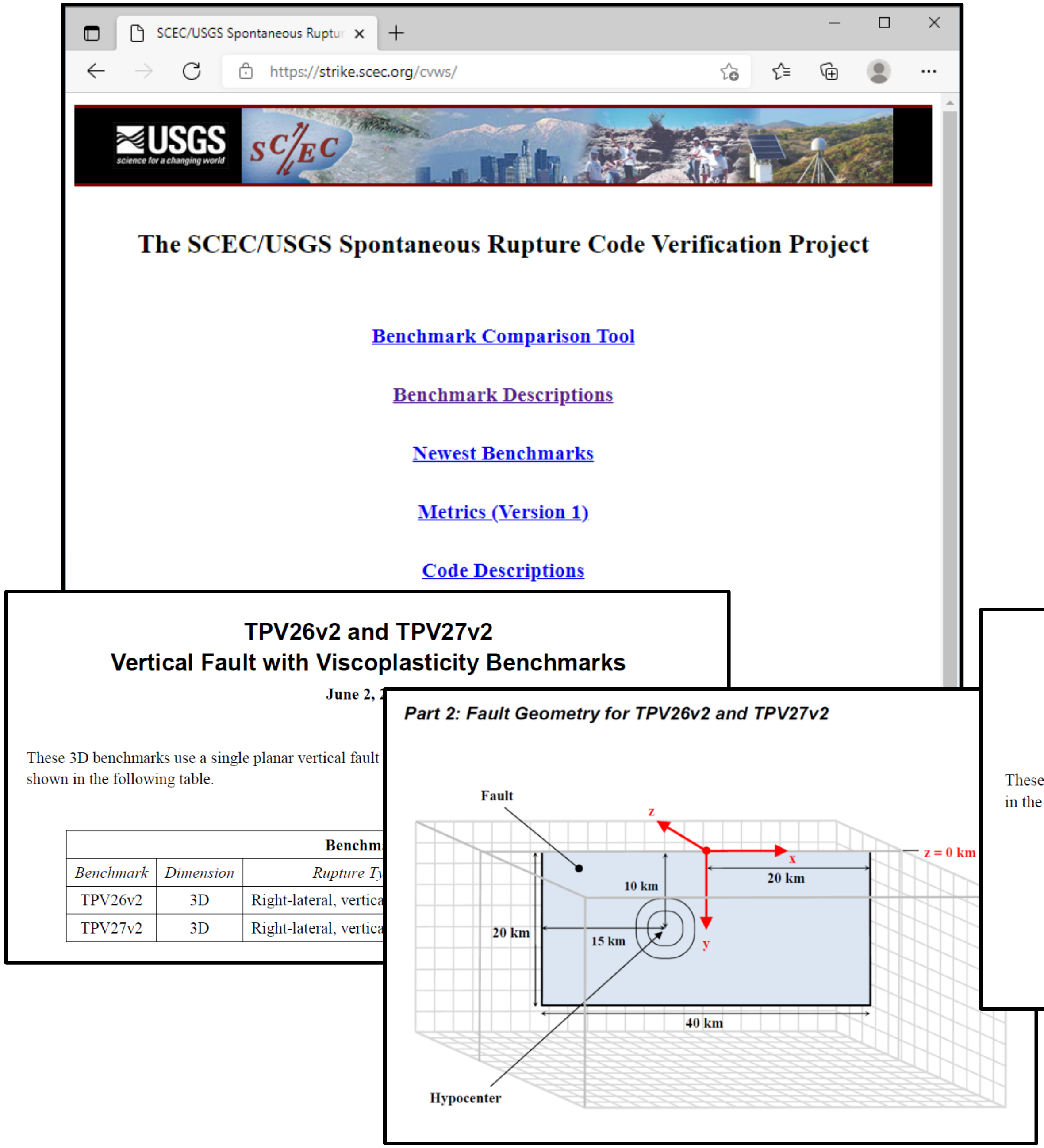


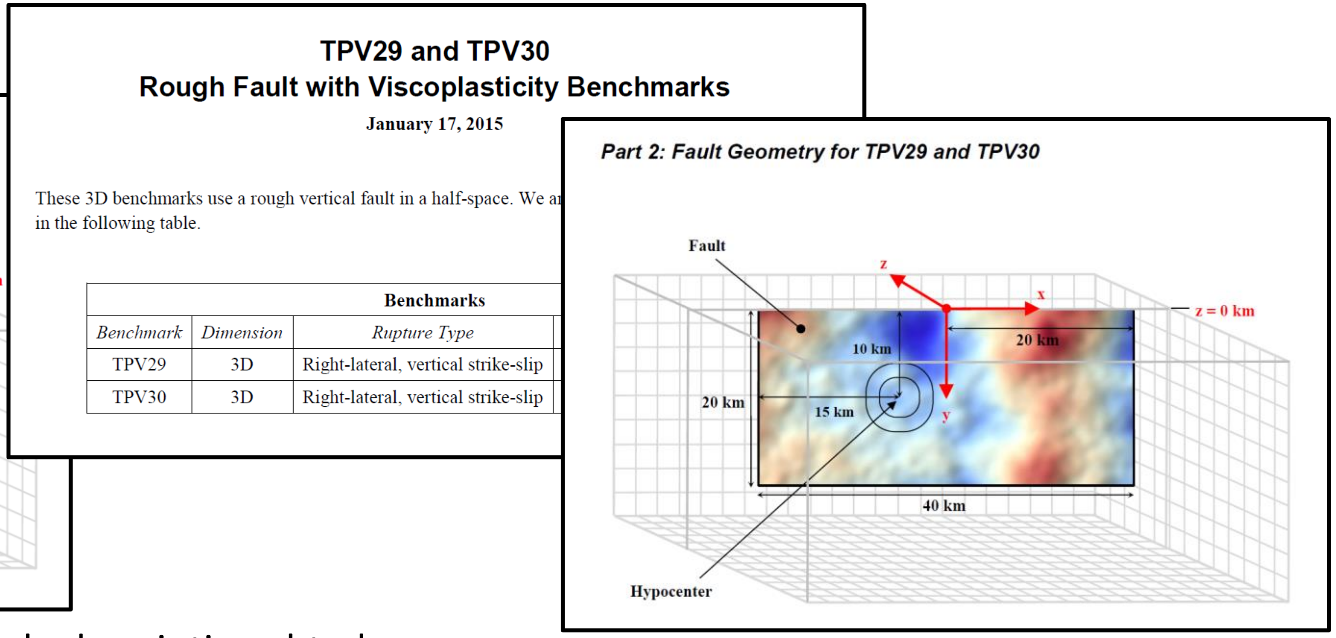




Traditional method for setting initial stress in viscoplastic models:

- Must specify absolute stress tensor throughout the model.
    - ➢ Stress tensor appears in viscoplastic constitutive law.
- Fault tractions are determined by stress tensor, and must be compatible with friction parameters.
- Initial stress tensor must be compatible with viscoplastic parameters.
- Must include gravity and fluid pressure.
- Initial stress must be in static equilibrium.



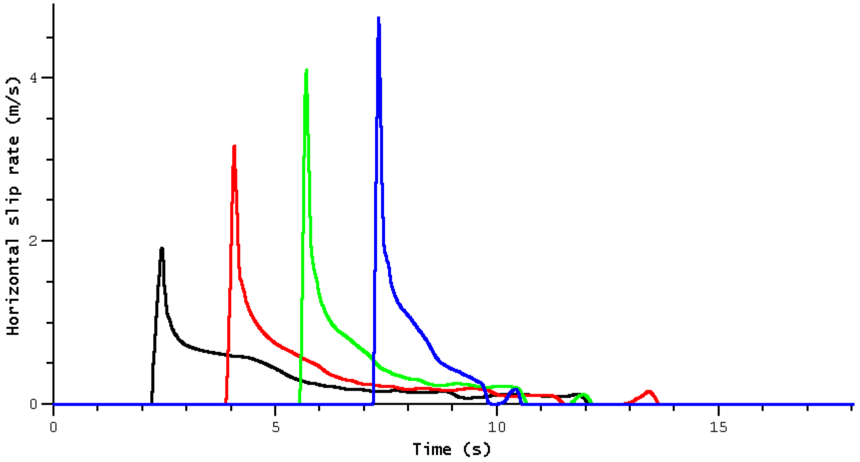


Benchmark descriptions: https://strike.scec.org/cvws/benchmark_descriptions.html

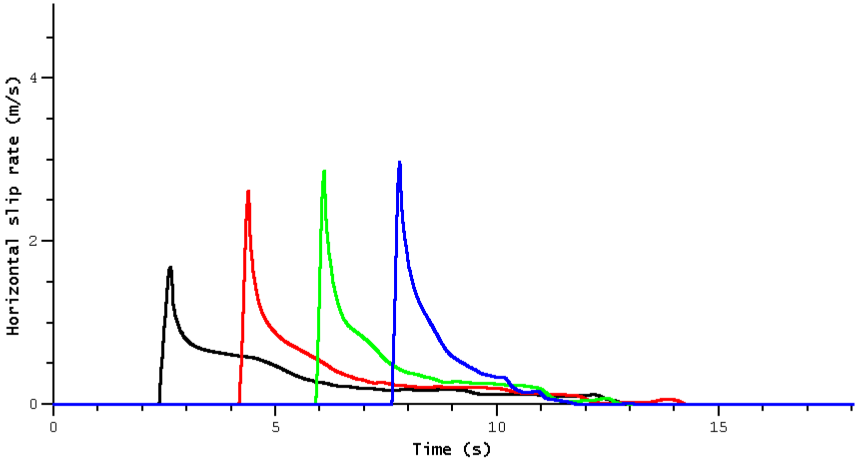
Example Effect of Viscoplasticity – Using SCEC/USGS Benchmarks TPV26v2 and TPV27v2

Plots show slip rate at distances of 5 km (black), 10 km (red), 15 km (green) and 20 km (blue) from the hypocenter.

TPV26v2 – Elastic

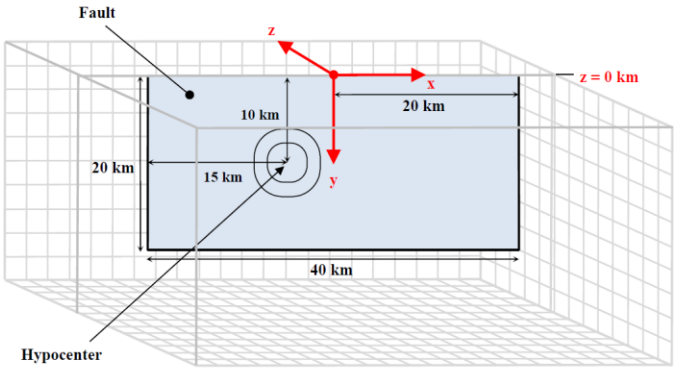


TPV27v2 – Viscoplastic



- In elastic case (TPV26v2), peak slip rate increases almost linearly with distance from hypocenter.
- In viscoplastic case (TPV27v2), slip rate is lower and tends toward leveling off as distance increases

Part 2: Fault Geometry for TPV26v2 and TPV27v2



TPV26v2 and TPV27v2 benchmark descriptions:
https://strike.scec.org/cvws/tpv26_27docs.html

Difficulties of Adding Viscoplasticity

Adding viscoplasticity to something like our Hayward model poses difficulties:

- 3D heterogeneous velocity and density structure, with gravity → It's hard to find an initial stress tensor in static equilibrium.
- 3D fault geometry → It's hard to find an initial stress tensor that produces acceptable tractions on the fault.
- Stress in the Earth's crust and viscoplastic parameters are poorly known.
- End result: A model with lots of free parameters and initial conditions, that are poorly constrained, and yet difficult to specify.
- Can increase the computational cost by as much as a factor of 3.

Our goal: Find a way to add the effects of viscoplasticity to our model, that avoids these difficulties, and retains the simplicity and efficiency of a linear elastic model.

Joe Andrews' Approach (JGR 2005) – "Velocity Toughening"

In a linear elastic model, impose a maximum slip rate:

$$V \leq V_{\text{max}}$$

Joe's implementation is to modify the friction law so that, when $V = V_{\text{max}}$, the friction law becomes (in 2D)

$$\tau_{\text{friction}} = \tau_{\text{elastic}}$$

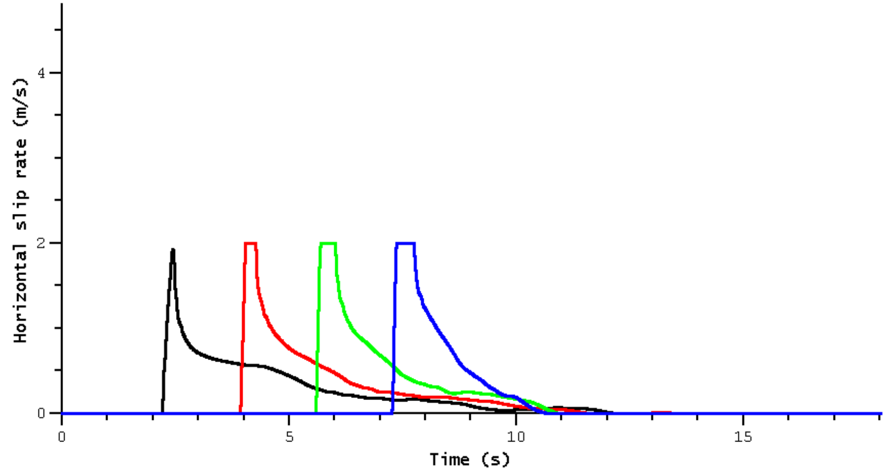
where τ_{elastic} is the shear stress induced by the elastic stress tensor (*not* including inertial forces). This reduces the acceleration to zero, leaving V constant. (In 3D there is an additional complication due to the possibility of rake rotation, but the concept is the same.)

Joe showed that velocity toughening could, in some ways, make a linear elastic model behave as if it had off-fault yielding. But there are several problems with this approach:

1. It produces very strange-looking slip histories, where the slip rate is constant from some period of time.

2. It is difficult to give a physical interpretation.

3. The separation of elastic and inertial forces is not how friction usually works. (Friction usually responds to the sum of elastic and inertial forces.)



Radiation Damping

Radiation damping is a standard technique in quasi-static earthquake simulators. It compensates for the lack of dynamics, by adding a term to the friction law:

$$\tau_{\text{damping}} = \frac{GV}{2\beta}$$

G = shear modulus

β = shear wave velocity

V = slip rate

Notice this is linear in V.

Physically, when fault slip occurs, the inertia of the surrounding rock produces a reaction force that opposes further slip. Quasi-static earthquake simulators don’t have inertia, and the absence of that reaction force produces slip rates that are too high.

The radiation damping term supplies the reaction force, which is otherwise not present in a quasi-static model. Including it allows a quasi-static model to behave, in some respects, as if the model contained dynamics.

Non-Linear Radiation Damping

In a viscoplastic model, off-fault yielding reduces the magnitude of the stress tensor, thereby reducing the shear traction acting on the fault. The effect is the same as if the off-fault yielding produced an additional reaction force that opposes further slip, above and beyond the reaction force of inertia.

Our idea is to take a linear elastic dynamic rupture model and add a *non-linear* radiation damping term to supply that additional reaction force. This allows a linear elastic model to behave, in some respects, as if the model contained viscoplastic yielding.

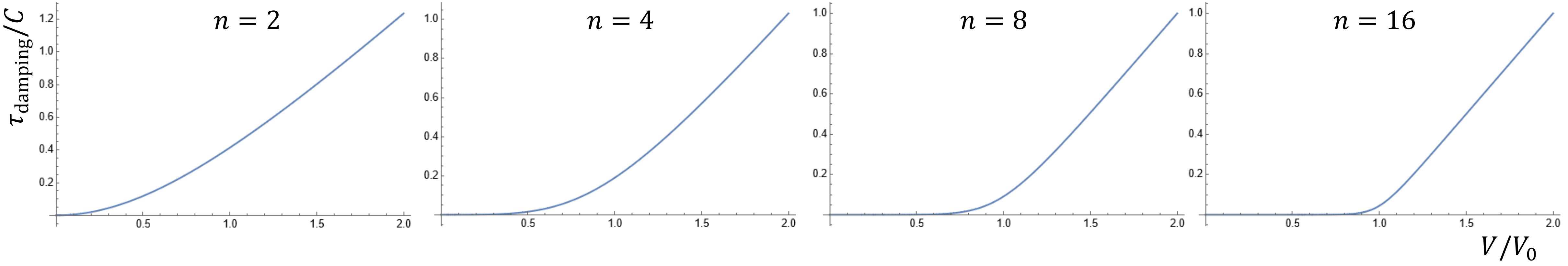
$$\tau_{\text{damping}} = C((1 + (V/V_0)^n)^{1/n} - 1)$$

C = Coefficient

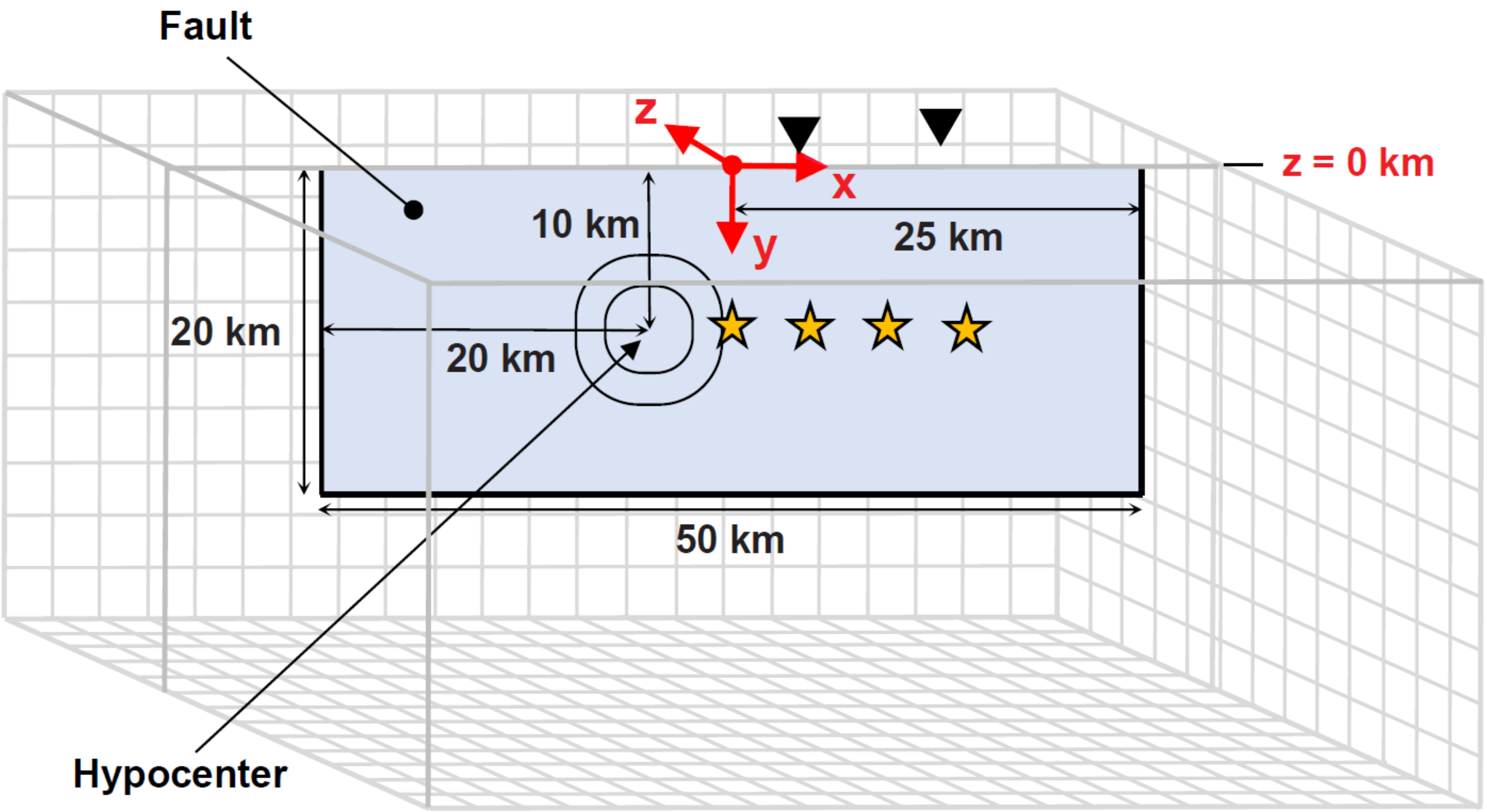
V_0 = Reference velocity

n = Transition exponent

There is low damping when $V < V_0$, transitioning to linear damping when $V > V_0$. The exponent n controls the transition.



Test Setup – Based on SCEC/USGS Benchmarks TPV26v2 and TPV27v2



- Linear slip-weakening friction.
- Vertical strike-slip fault.
- Depth-dependent stresses.
- Gravity and fluid pressure.
- TPV26v2: Linear elastic material.
- TPV27v2: Viscoplastic material.

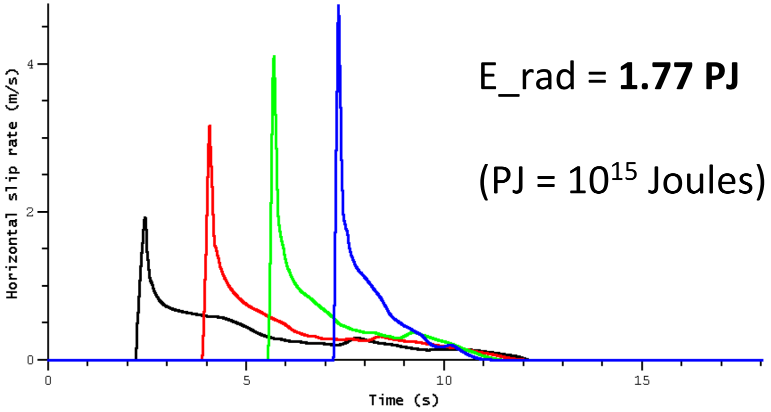
The setup is the same as the SCEC/USGS benchmarks, except that:

- We create “soft” boundaries at the lateral ends of the fault, so the rupture stops spontaneously before it reaches the ends of the fault. This is done by increasing the length of the fault from 40 to 50 km, and imposing increased frictional cohesion near the ends.
- We reduce the frictional cohesion near the surface so that the rupture can reach the surface in both elastic and viscoplastic cases.

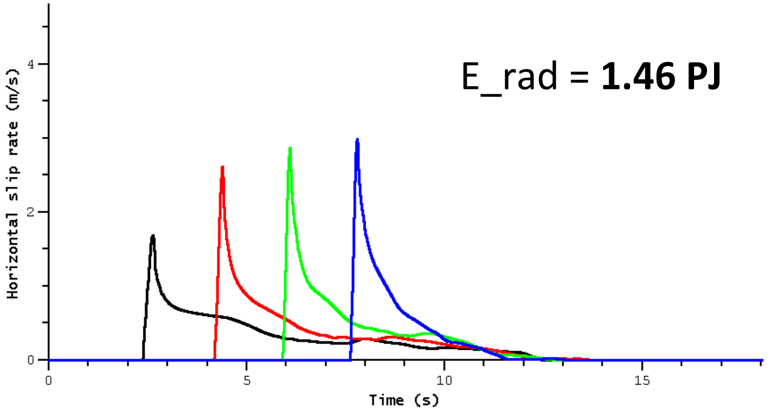
Sample Runs with Non-Linear Radiation Damping – Based on TPV26v2 and TPV27v2

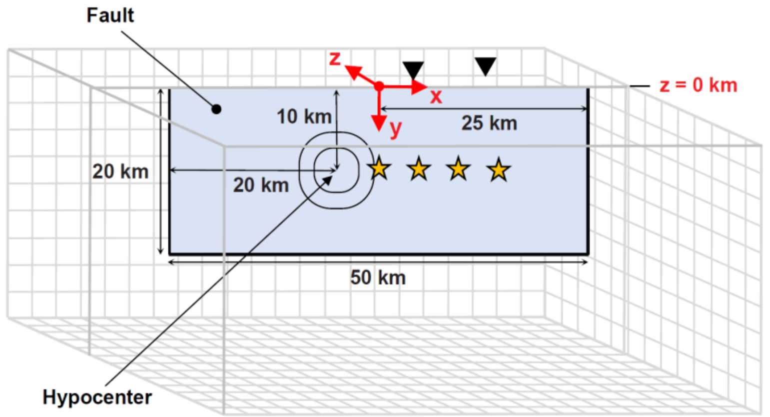
Plots show slip rate at distances of 5 km (black), 10 km (red), 15 km (green) and 20 km (blue) from the hypocenter.

Elastic (like TPV26v2)

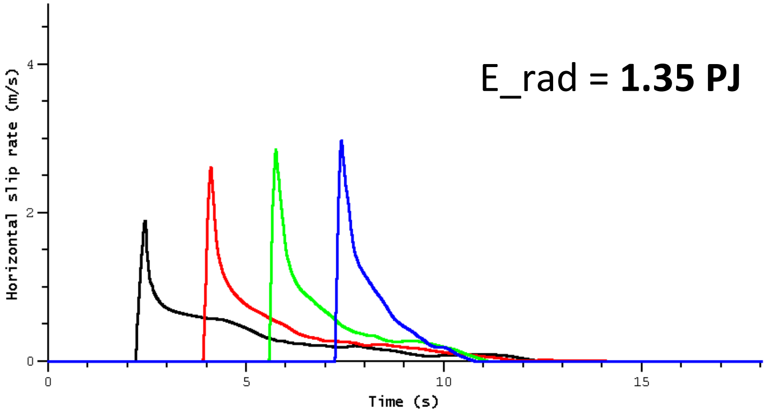


Viscoplastic (like TPV27v2)

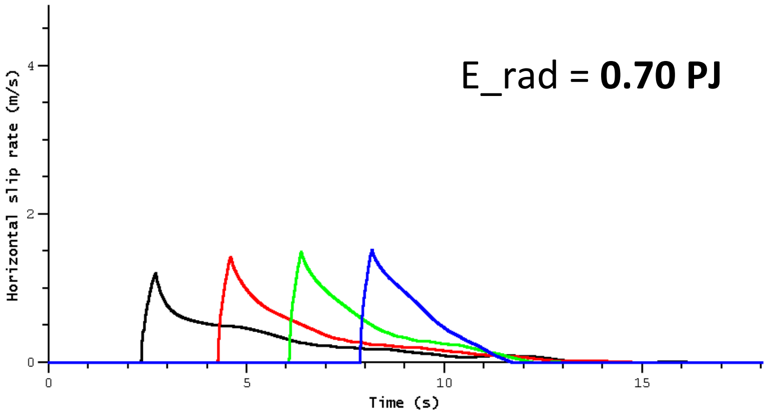




Elastic w/ nonlinear radiation damping
V_0 = 2.0 m/s, n = 16, C = 8.0 MPa

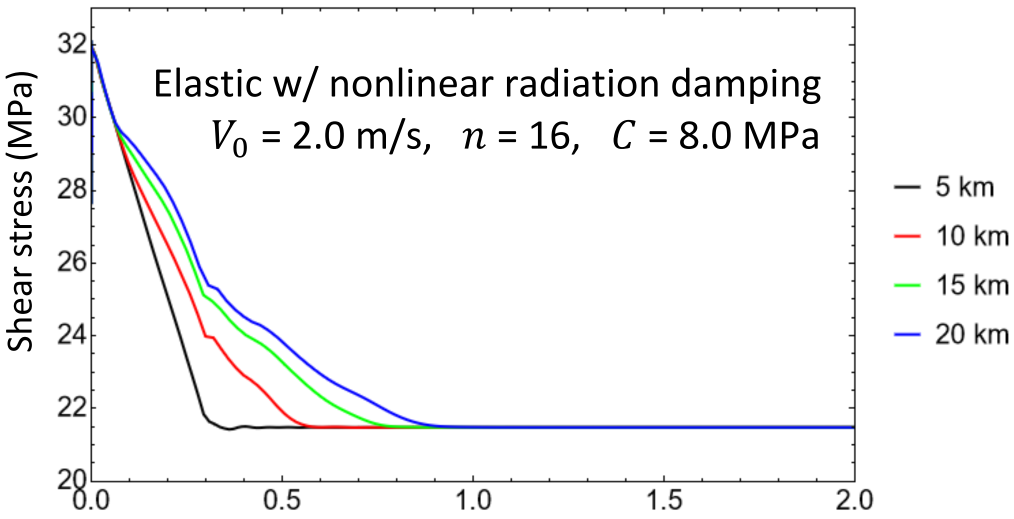


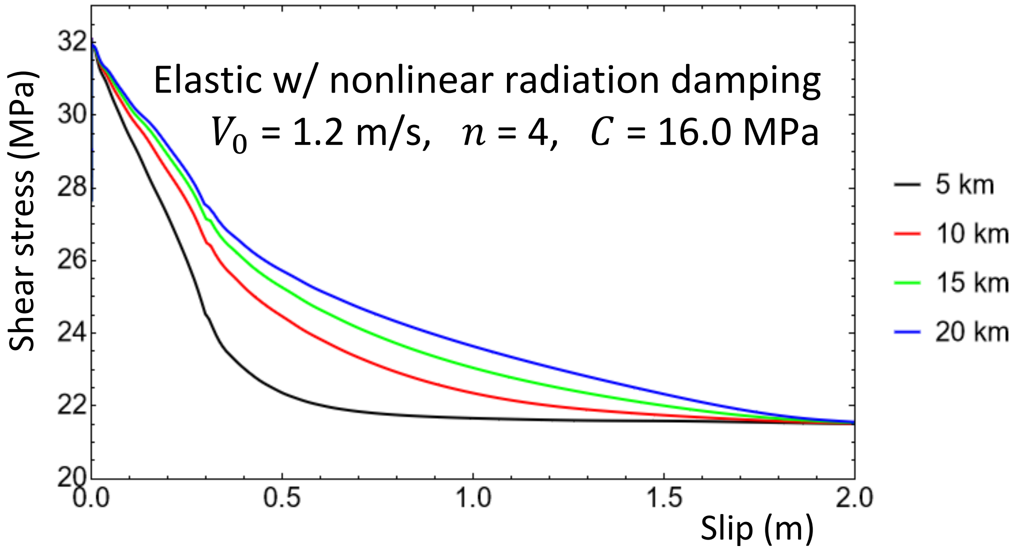
Elastic w/ nonlinear radiation damping
V_0 = 1.2 m/s, n = 4, C = 16.0 MPa



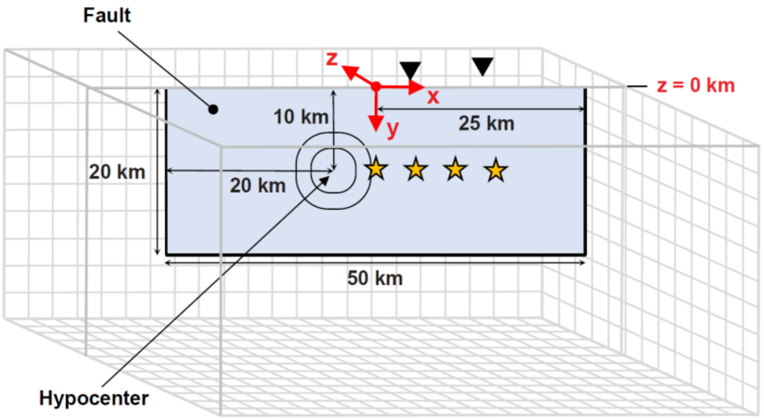
Apparent Increase of Slip-Weakening Critical Distance with Distance from the Hypocenter

Viscoplasticity can make it appear that D_c and fracture energy increase with distance from the hypocenter, because the rate of inelastic energy dissipation increases as the rupture gets larger. Non-linear radiation damping can produce a similar effect.

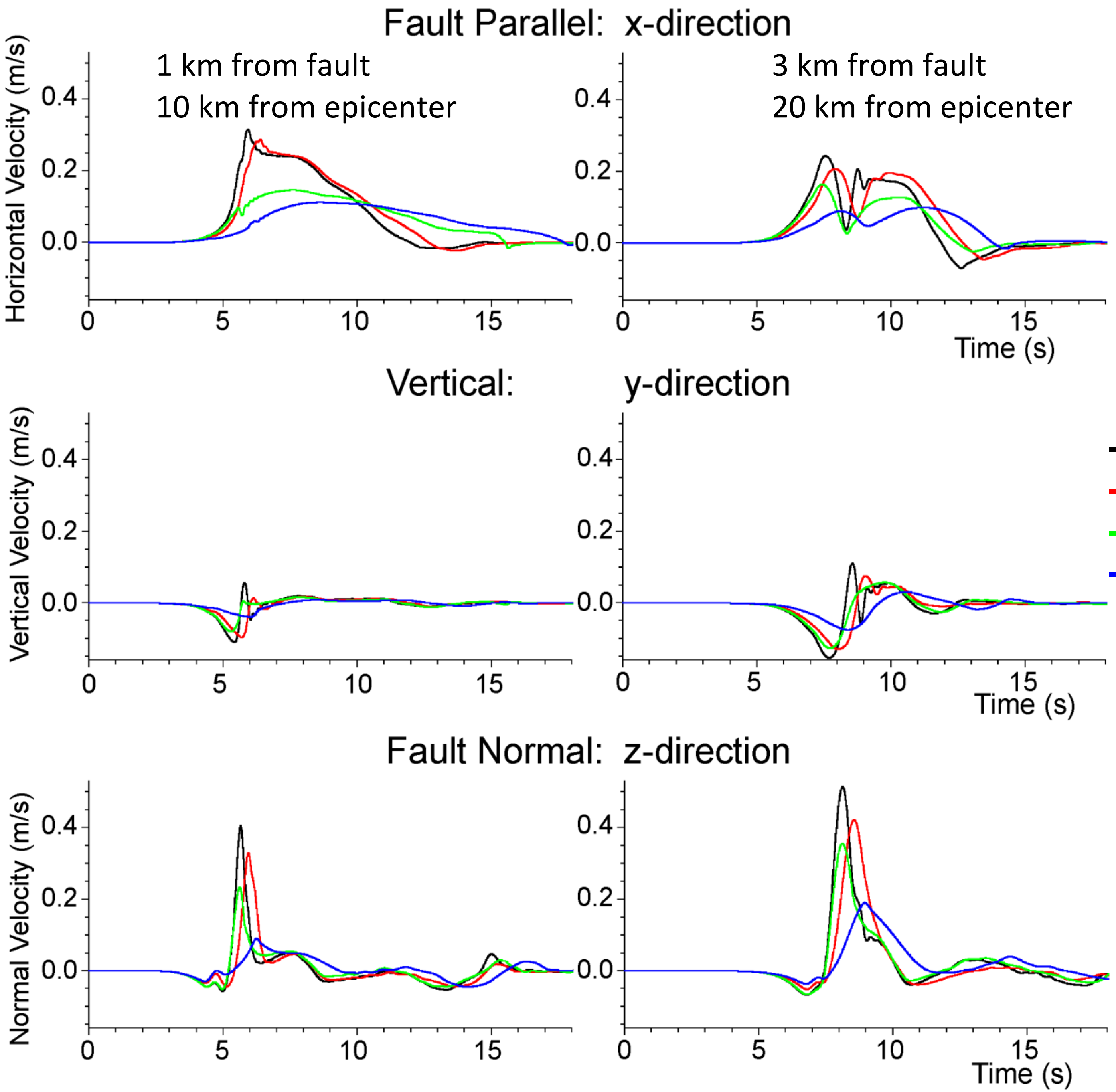


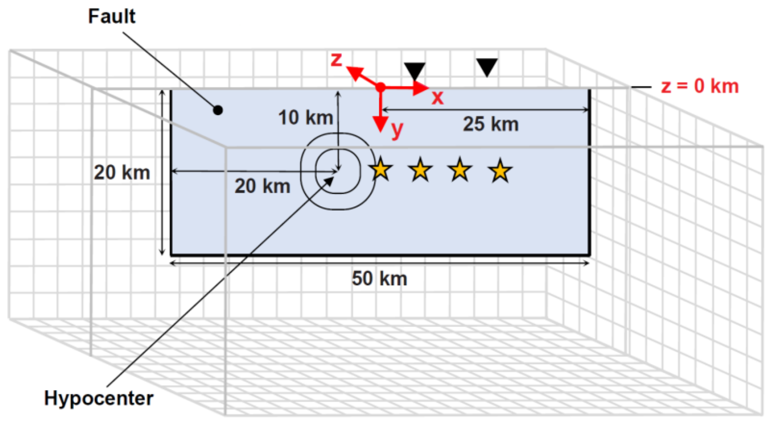


- Plots show shear stress versus slip at various distances from the hypocenter.
- The x-axis is slip in m.
- The y-axis is shear stress in MPa.
- The *apparent critical slip distance* D_c is where the shear stress reaches its final value.
- The *apparent fracture energy* is the area under the curve and above the final value.
- The slip-weakening critical distance in the friction law is fixed at $D_c = 0.3$ m.



Reduction in Ground Motions Near the Fault





- Plots show three components of particle velocity at two locations on the Earth's surface near the fault (triangles in figure above).
- The elastic case has highest PGV (peak ground velocity).
- The viscoplastic case has lower PGV.
- The two radiation damping cases have even lower PGV.

Conclusions

1. By adding non-linear radiation damping to a linear elastic dynamic rupture simulation, we can emulate some of the effects of viscoplasticity.

2. Showed examples of what non-linear radiation damping can do:

- Reduce peak slip rates, and make them tend to level off.
- Reduce radiated seismic energy.
- Produce an apparent D_c that increases with distance from the hypocenter.
- Reduce ground motions (PGV) near the fault.

3. Advantages of our approach:

- Easy to implement.
- Has a small number of parameters.
- Retains the simplicity and efficiency of a linear elastic simulation.

4. Limitations:

- Cannot reproduce effects of viscoplasticity at “hard” fault endpoints or other geometrical complexities.
- Only emulates the effects of inelastic yielding close to the fault.