Microseismicity simulated on asperity-like fault patches: on scaling of seismic moment with duration and seismological estimates of stress drops

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Microseismicity is a window for investigating fault properties of large events

Microseismicity provides us with a unique window into the stress environment, structure, and physical properties of the crust and their changes over time or due to large earthquakes. We can combine the observations of microearthquakes and the dynamic modeling to understand fault properties and their changes.

Outline
- Average stress drop of microearthquakes determined from observations
- Seismic clusters with constant source duration but variable seismic moment observed by the borehole seismometer array in Taiwan
- Physical interpretation of those clusters based on the dynamic modeling
- Conclusions
Stress drop of microearthquakes

Stress drop based on Hooke’s law:

\[
\Delta \sigma = C \mu \left( \frac{\bar{D}}{L} \right)
\]

- \(\Delta \sigma\): strain
- \(L\): a characteristic rupture dimension
- \(\bar{D}\): the average displacement
- \(\mu\): the shear modulus
- \(C\): a constant

- Stress drop reveals average slip per rupture dimension
- Scaling of stress drop between small and large earthquakes

Since we are able to calculate stress drop precisely from dynamic models, it is a great opportunity to compare the stress drop estimates from observations and models.
Procedure for estimating average stress drop

\[ \Delta \sigma = C \mu \frac{D}{L} \]

Stress drop for a circular fault

\[ \Delta \sigma = C \frac{M_0}{A^{3/2}} = \frac{7M_0}{16r^3} \]

\[ M_0 = \mu \overline{DA} \]

(Eshelby, 1957)

Source-time function

\[ t_w = \frac{r}{\nu_R} \]

\[ f_c = \frac{k \nu_R}{r} \]

Assumptions:
- Circular rupture shape
- Axisymmetrical expansion
- A constant rupture speed
- Spatially uniform stress drop

Sato and Hirasawa, 1973
Madariaga, 1976

Assumption:
\[ \nu_R = 0.9 \beta \]
Stress drop estimation seismologically: Spectral fitting

\[ \text{Obs}(f) = S(f) \cdot P(f) \]

Theoretical source spectrum:

\[ S(f) = \frac{\Omega_0}{1 + (f/f_c)^n} \]

where \( \Omega_0 = \frac{F}{4\pi \rho v^3 d} M_0 \)

- \( n=2 \) \( \omega \)-square source model (commonly use)
- \( n=3 \) \( \omega \)-cube source model

\( t_w = \frac{r}{\nu_R} \)

\( f_c = \frac{k \nu_R}{r} \)

\( \Delta \sigma = \frac{7M_0}{16r^3} \)

(Abercrombie, JGR, 1995)
Earthquake scaling and moment-invariant stress drops

\[ M_0 f_c^3 = \text{constant} \]
\[ \rightarrow M_0 \propto f_c^{-3} \]
\[ t_w \propto M_0^{1/3} \]

(Allmann & Shearer, JGR, 2009)

-Stress drop is moment-invariance
-General ranges: 0.1~100 MPa

\[ r_1 < r_2 \]

Constant rupture speed

(Modified from Prieto et al., JGR, 2004)

The rupture process between the small and large earthquake is fundamentally similar. (Shearer, 2009)
Taiwan Chelungpu-fault Drilling Project (TCDP)
Borehole seismometers

1999 Mw 7.6 Chi-Chi

Locked

(Lin et al., GJI, 2012)
Constant source durations for earthquakes of different magnitude within a seismic cluster

\[
t_w = 0.070 \text{ s} \\
\text{Cluster A Displacement} \\
\text{Average } P_{\text{d}_{\text{rs}}} = 0.070
\]

\[
t_w = 0.065 \text{ s} \\
\text{Cluster B Displacement} \\
\text{Average } P_{\text{d}_{\text{rs}}} = 0.065
\]

\[
t_w = 0.043 \text{ s} \\
\text{Cluster C Displacement} \\
\text{Average } P_{\text{d}_{\text{rs}}} = 0.043
\]

Width of a far-field P-wave displacement should represent the duration of the event moment-rate function.

\[
t_w \propto M_0^{1/3}
\]

\[M_w \leq 0.3: t_w \sim 0.01 \text{ s} \]

\[M_w \leq 2.0: t_w \sim 0.07 \text{ s} \]

for \(\Delta \sigma = 0.2 \text{ MPa}\)

(Duputal et al., EPSL, 2013)

Similar observations were obtained in Parkfield (Harrington & Brodsky, BSSA, 2009) and in Turkey (Bouchon et al., Science, 2011).
Path effect correction and stress drop scaling

We applied 3 methods for path effect correction:
1. No path effect (P-wave duration = source duration)
2. Futterman Q correction
3. Empirical Green’s function

\[ t_w = \frac{r}{\nu_R} \]

\[ \Delta \sigma = \frac{7M_0}{16r^3} \]

Cluster A

\[ \Delta \sigma = 0.0007 \text{~} 0.32 \text{ MPa} \]

(Lin et al., GJI, 2016)
Slip per unit characteristic length of these events increases with seismic moment, which reveals different kinematic behaviors between small and large earthquakes ranging from Mw 0.0 to 2.0.

\[ t_w = \frac{r}{v_R} \]
\[ f_c = \frac{k v_R}{r} \]
\[ \Delta \sigma = \frac{7 M_0}{16 r^3} \]
A question raises: What if the earthquake source is more complex than a uniform circular fault?

Build our own models

Dynamic modeling can help!

Slip distribution of an M 2.1 repeating earthquake in Parkfield

Complexity of repeating earthquakes

(Dreger et al., GRL, 2007)

(Rubinstein et al., JGR, 2012)
Dynamic modeling

Events occur spontaneously!

(Lapusta & Liu, JGR, 2009)
Rate-and-state friction

“Rate”: Slip rate $V$
“State”: State variable $\theta$

- Well-known empirical law derived from laboratory experiments on rocks
  (Derived by Dieterich, Ruina, Tullis, Marone, and others)

Constitutive law: $\tau = \sigma f = \sigma \left( f_o + a \ln \frac{V}{V_o} + b \ln \frac{V_o \theta}{L} \right)$; $\frac{d\theta}{dt} = 1 - \frac{V \theta}{L}$

- In steady-state, reduces to $\tau \rightarrow \tau_{SS} = \sigma \left[ f_o + (a - b) \ln(V / V_o) \right] \rightarrow (a - b)$ is important

Velocity-strengthening: $a - b > 0$

- Friction increases with increasing $V \rightarrow$ Stable sliding
- Creeping!

Velocity-weakening: $a - b < 0$

- Friction decreases with increasing $V \rightarrow$ Potential instability
- Seismic!
Model with rate-and-state friction and heterogeneous normal stress

Heterogeneous fault model
Effective normal stress
- Background: $\sigma = 40$ MPa
- Blue area: $2 \times \sigma$
- Orange area: $3 \times \sigma$
- Red area: $4 \times \sigma$

Such variations could result from slightly non-planar interfaces being compressed into full contact.

(Lin & Lapusta, GRL, 2018)
Simulated seismic clusters

500 CPUs, 7 days

(Lin & Lapusta, GRL, 2018)
Circular source with directivity (LE)

Event 38

$\bar{\delta} = 0.35 \text{ cm}$

$\nu_R = 0.8\beta$

$M_W = 2.0$

$\Delta\sigma_E = 2.9 \text{ MPa}$

(Lin & Lapusta, GRL, 2018)
Ring-like source model (SE)

\[ \bar{\delta} = 0.06 \text{ cm} \]

\[ \nu_R = 0.8 \beta \]

\[ M_W = 1.0 \]

\[ \Delta \sigma_E = 2.9 \text{ MPa} \]

(Lin & Lapusta, GRL, 2018)
Exploring far-field seismic signal

Far-field displacement in a homogeneous elastic medium for each subfault for a station $u$ at point $x$:

$$u(x, t) = \frac{1}{4\pi\rho\alpha^3} A^p \frac{1}{R} M_0 \left( t - \frac{R}{\alpha} \right) + \frac{1}{4\pi\rho\beta^3} A^s \frac{1}{R} M_0 \left( t - \frac{R}{\beta} \right)$$

\[\]

(Aki and Richards, 2002; Kaneko and Shearer, GJI, 2014; Kaneko and Shearer, JGR, 2015)

(Lin & Lapusta, GRL, 2018)
Comparison of the on-fault durations of both events

Numerical models of heterogeneous faults reproduce observations of earthquake clusters with constant duration but variable seismic moment.

Average stress drop estimated from the on-fault durations

\[ t_w = \frac{r}{\nu_R} \]
\[ \Delta\sigma = \frac{7M_0}{16r^3} \]

On-fault duration:
- Mw 2.0: \( t_w = 0.061 \text{s} \)
- Mw 1.0: \( t_w = 0.057 \text{s} \)

Average stress drop:
- Mw 2.0: 0.1 MPa
- Mw 1.0: 0.006 MPa

\( \Delta\sigma_E = 2.9 \text{ MPa} \)

(Lin & Lapusta, GRL, 2018)
Spectral fitting analysis for $n=2$ and $n$ varies approaches

P-ave. stress drop:
Mw 2.0: 1.2 MPa
Mw 1.0: 0.07 MPa

S-ave. stress drop:
Mw 2.0: 6.5 MPa
Mw 1.0: 0.6 MPa

P-ave. stress drop:
Mw 2.0: 2.5 MPa
Mw 1.0: 0.03 MPa

S-ave. stress drop:
Mw 2.0: 7.6 MPa
Mw 1.0: 0.1 MPa

\[
S(f) = \frac{\Omega_0}{1 + (f/f_c)^n}
\]

(Lin & Lapusta, GRL, 2018)
Comparison of theoretical and seismologically inferred stress drops

\[ \Delta \sigma = \frac{7M_0}{16r^3} \]

Stress drop of small earthquake was underestimated, producing significant trend.

(Allmann & Shearer, JGR, 2009)
Seismologically estimated stress drops are much smaller than the actual ones mainly due to overestimated rupture area.

\[ A_{\text{seis}} \gg A_{\text{actual}} \]

\[ \Delta \sigma_{\text{seis}} \ll \Delta \sigma_{\text{actual}} \]

\[ \Delta \sigma = C_1 \frac{M_0}{A^{3/2}} \]

The simple circular source models commonly used are inadequate to capture increasingly detailed observations.

Symmetrical circular source model (e.g., Madariaga, BSSA, 1976)

Actual rupture area

Area assumed in seismological analysis (Lin & Lapusta, GRL, 2018)
Conclusions

- Numerical models of heterogeneous faults reproduce observations of earthquake clusters with constant duration but variable seismic moment.

- The constant duration is controlled by the fault size while the variable seismic moment depends on the fraction of the fault rupturing.

- On-fault stress drops are constant at ~3 MPa but their seismological estimates vary from 0.005 to 10 MPa due to the misinterpretation of source complexity.

Thank you very much!
Variation of high-frequency fall-off rate $n$

Small and large events within a cluster have different falling rates in high frequency band, suggesting they have different rupture behavior.

Why?
Dynamic modeling can help!

$$S(f) = \frac{\Omega_0}{1+(f/f_c)^n}$$

(Lin et al., GJI, 2016)

$n$ is not always 2!
Simulations results reproduce the observations in TCDP borehole seismometers

Cluster in TCDPBHS

Peak amplitude
Mw 2.0: $110 \times 10^{-7}$ m
Mw 1.0: $4.2 \times 10^{-7}$ m
Ratio: $\sim 26$

(Lin et al., GJI, 2016)

(Lin & Lapusta, GRL, 2018)
Modeling also reproduces differences in high-frequency falloff rate $n$ between larger and smaller events

Observations (Lin et al., GJI, 2016)
Actual average stress drop from the model

Energy-based stress drop $\Delta \sigma_E$ from dynamic model:

$$\Delta \sigma_E = \frac{\int \Delta \sigma \cdot \Delta u \, dS}{\int \Delta u \, dS}$$

(Noda et al., GJI, 2013)

Real answer of stress drop from the modeling.

In the following, we will compare the theoretical energy-based stress drop and seismologically inferred stress drop.
Average Path Q Deconvolution

\( t_w = 0.058 \text{ s} \)

- Cluster A Q-correction STF
  - \( t^* = 0.0117, Q = 202 \)

\( t_w = 0.056 \text{ s} \)

- Cluster B Q-correction STF
  - \( t^* = 0.0117, Q = 202 \)

\( t_w = 0.034 \text{ s} \)

- Cluster C Q-correction STF
  - \( t^* = 0.0117, Q = 202 \)

\( M_w 0.3: t_w \approx 0.01\text{s} \)
\( M_w 2.0: t_w \approx 0.07\text{s} \)

for \( \Delta \sigma = 0.2 \text{ MPa} \)

(Lin et al., GJI, 2016)
Empirical Green’s function (lower bound)

$tw = 0.026 \text{ s}$

$M_w 0.3: t_w \sim 0.01 \text{s}$

$M_w 2.0: t_w \sim 0.07 \text{s}$

for $\Delta \sigma = 0.2 \text{ MPa}$

$tw = 0.023 \text{ s}$

(Lin et al., GJI, 2016)
Pdur of nearby event (< 2km)

(Lin et al., GJI, 2016)
Summary of the observations of the clusters with microearthquakes

- The source durations of the events within the seismic clusters observed in TCDP borehole seismometers are essentially constant regardless of seismic moment, which violate the commonly used scaling relation \( t_w \propto M_0^{1/3} \)

- The seismological interpretation suggests that slip per unit characteristic length of these events increases with seismic moment, which reveals different kinematic behaviors between small and large earthquakes ranging from Mw 0.0 to 2.0