Blending data and dynamics into equilibrium for the Community Stress Model

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This talk will focus on the time-independent mean stress field in S CA, not on the time-dependence.

With 3-D tensor models of mean stress throughout the lithosphere of southern California, SCEC could:

- **determine the shear stresses on active faults to constrain the physics of their slip**;
- **predict Coulomb stress changes for operational earthquake forecasting**; *and*
- **test the realism of long-term earthquake sequence simulators**.

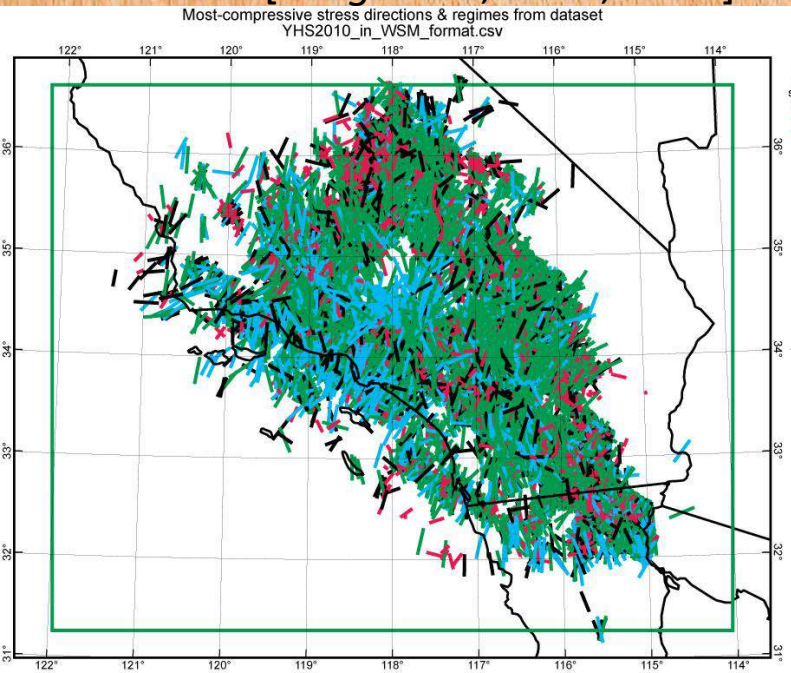
One basis for models is data:

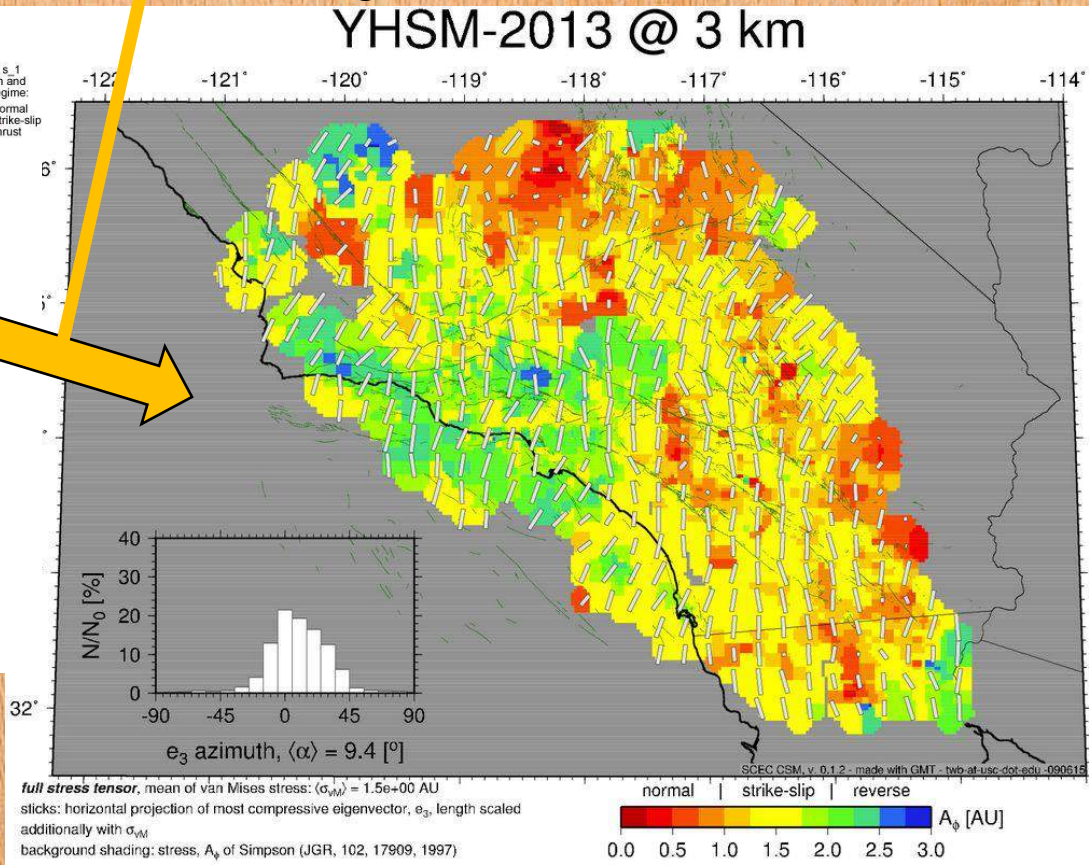
- stress directions (from focal mechanisms and boreholes), and
- stress intensity (only from boreholes).

179,000 focal mechanisms
by HASH method of
[*Hardebeck & Shearer*, 2003]:
YHS2010 [*Yang et al.*, 2012, *BSSA*]

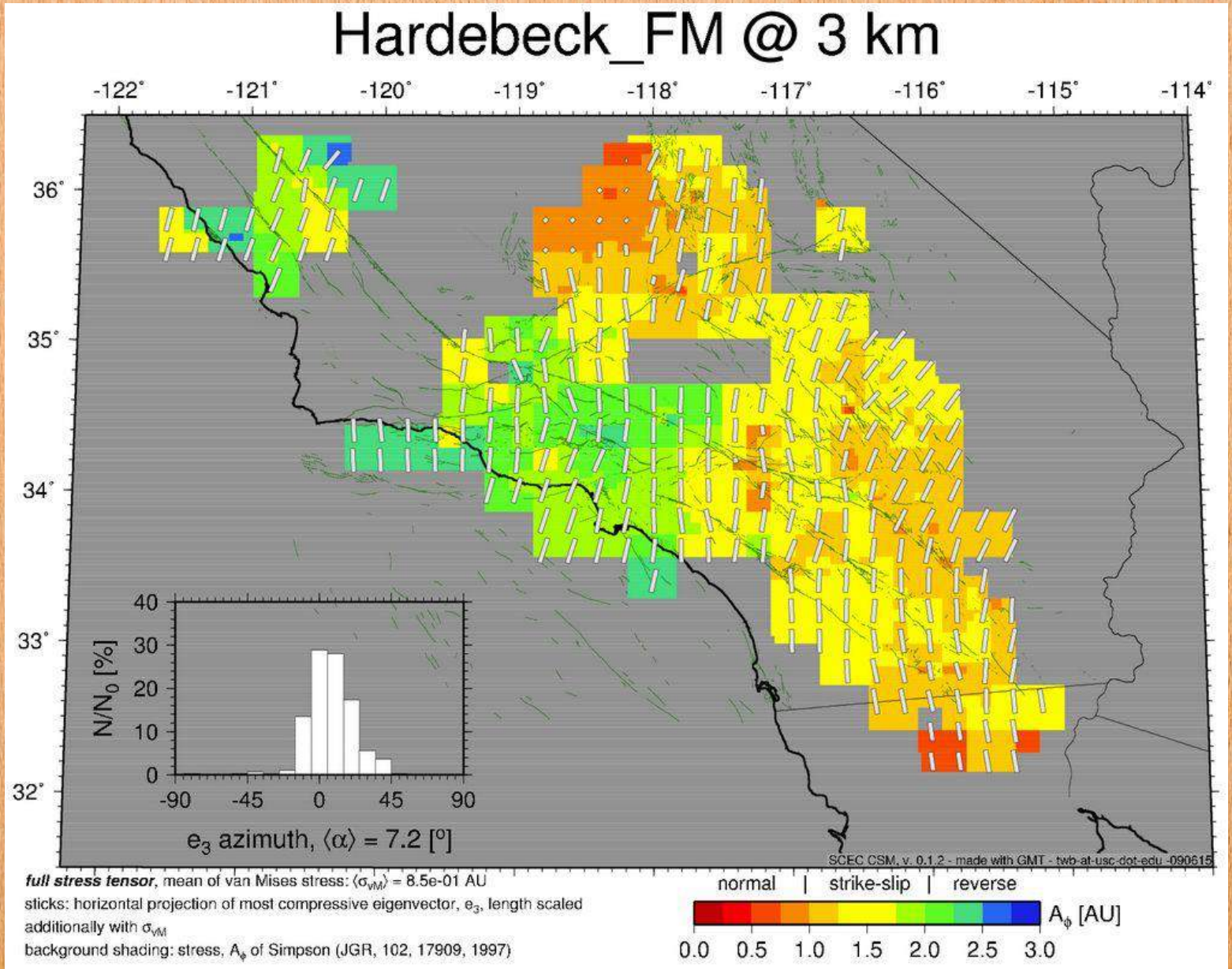
SATSI program
[*Hardebeck & Michael*, 2006]

[*Yang & Hauksson*, 2013, *GJI*]





An alternative inversion of the same data
by Jeanne Hardebeck [2012, CSM web site],
using variable 3-D binning, and damping:



But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km. Therefore, stress data must be supplemented by dynamic models which require <u>multiple input datasets</u>:

- laboratory flow laws (with real-world calibration):

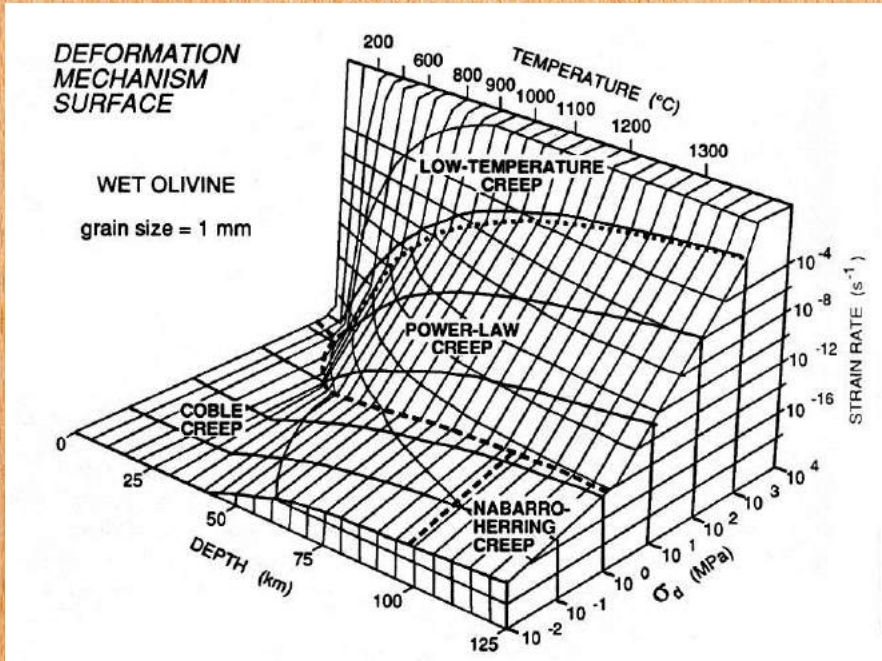
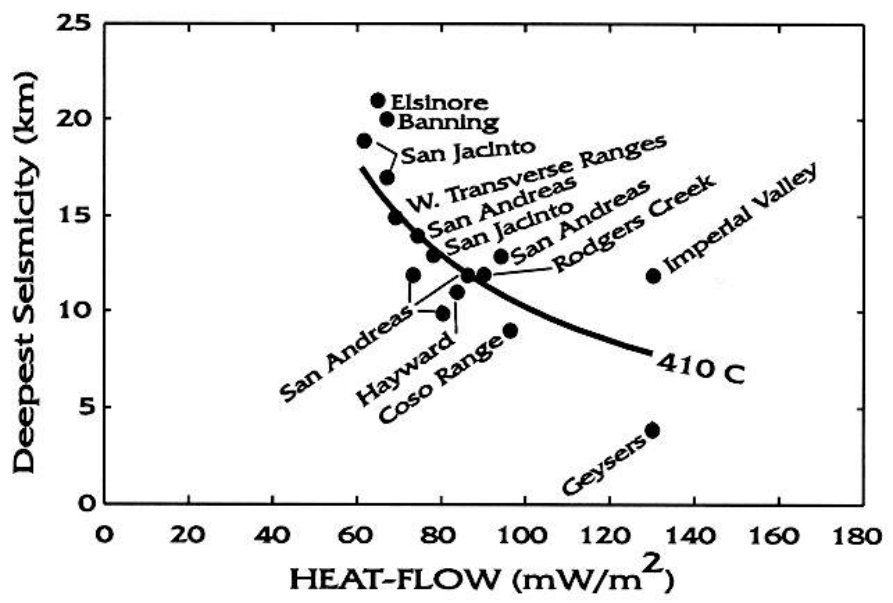


Fig. 4–8. Deformation mechanism surface showing flow regimes and processes for wet olivine (adapted from Tsenn and Carter, 1987).

Engelder [1993]



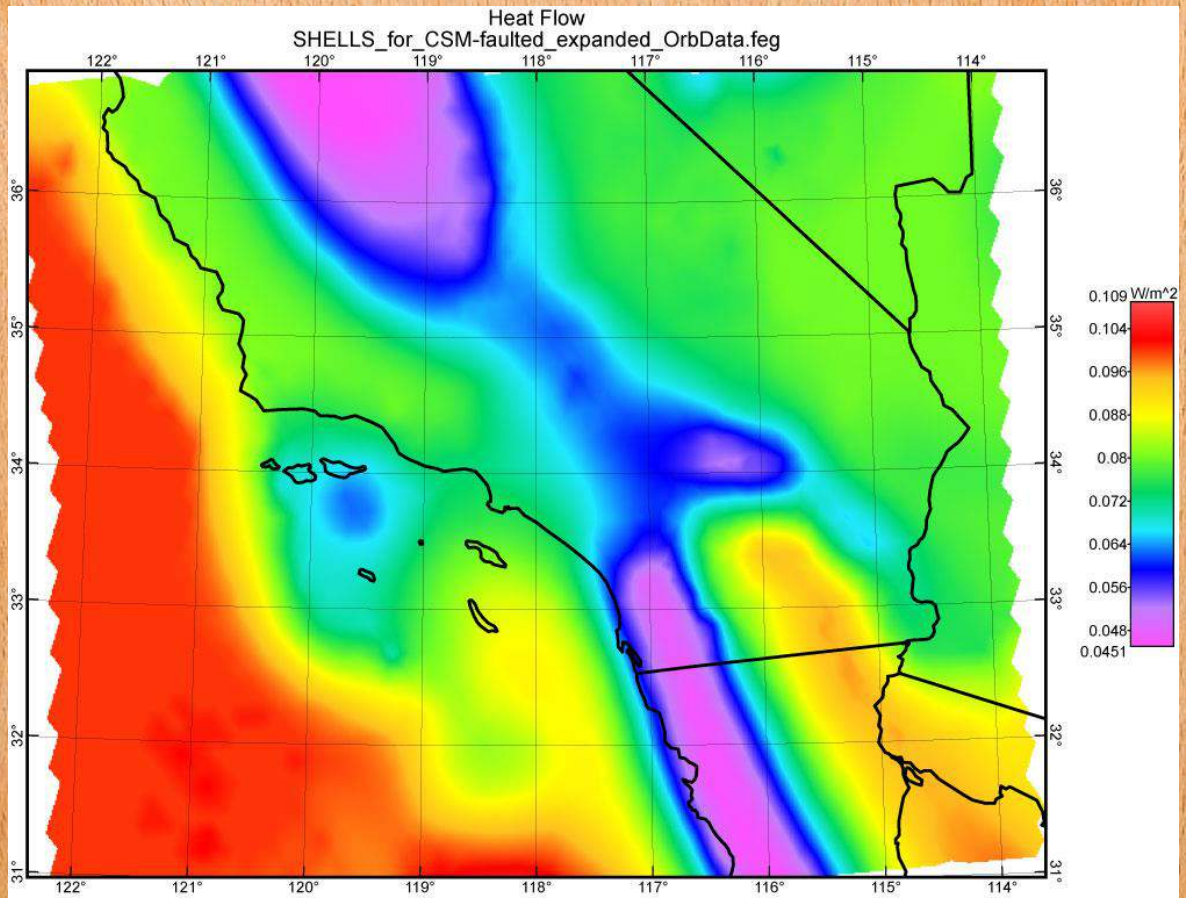
[*Bird & Kong*, 1994, *GSAB*]

But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km.

Therefore, data must be supplemented by dynamic models which require <u>multiple input datasets</u>:

- laboratory flow laws,
- a geotherm model:

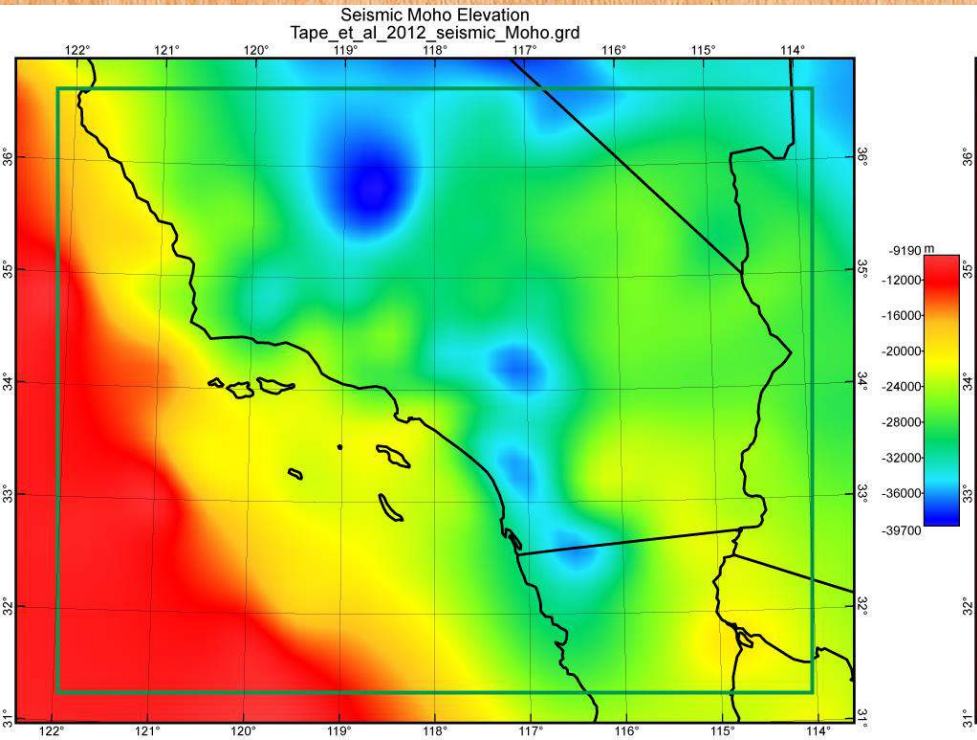
Geothermal Map of North America [*Blackwell & Steele*, 1992]:



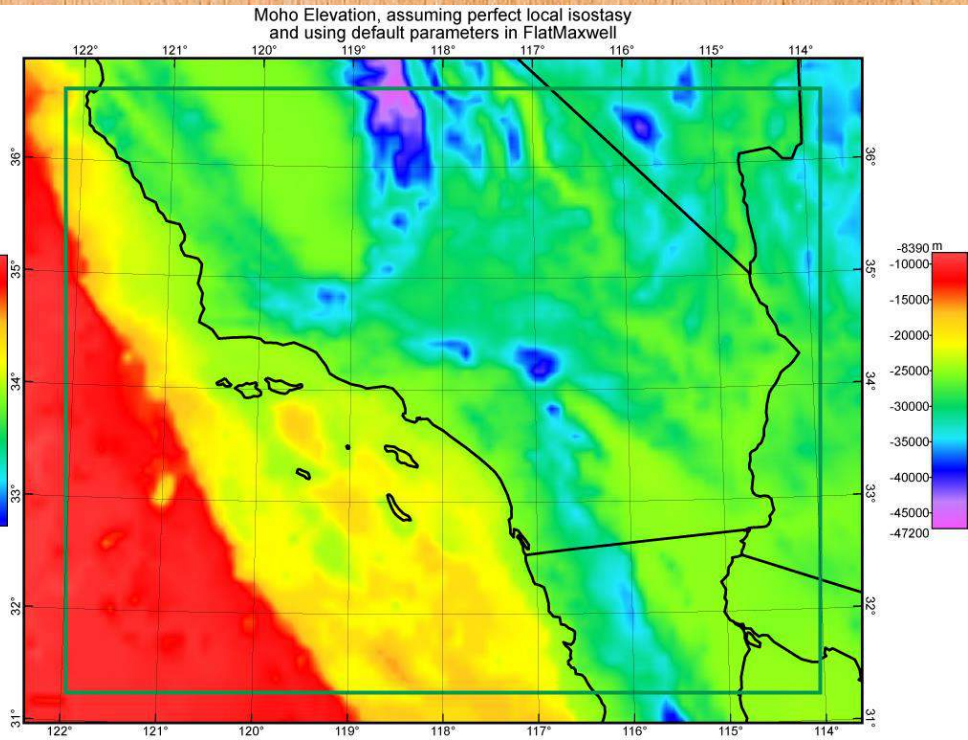
But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km.

Therefore, data must be supplemented by dynamic models which require <u>multiple input datasets</u>:

- laboratory flow laws,
- a geotherm model,
- a Moho model



[*Tape et al.*, 2012] seismic Moho



FlatMaxwell isostatic Moho

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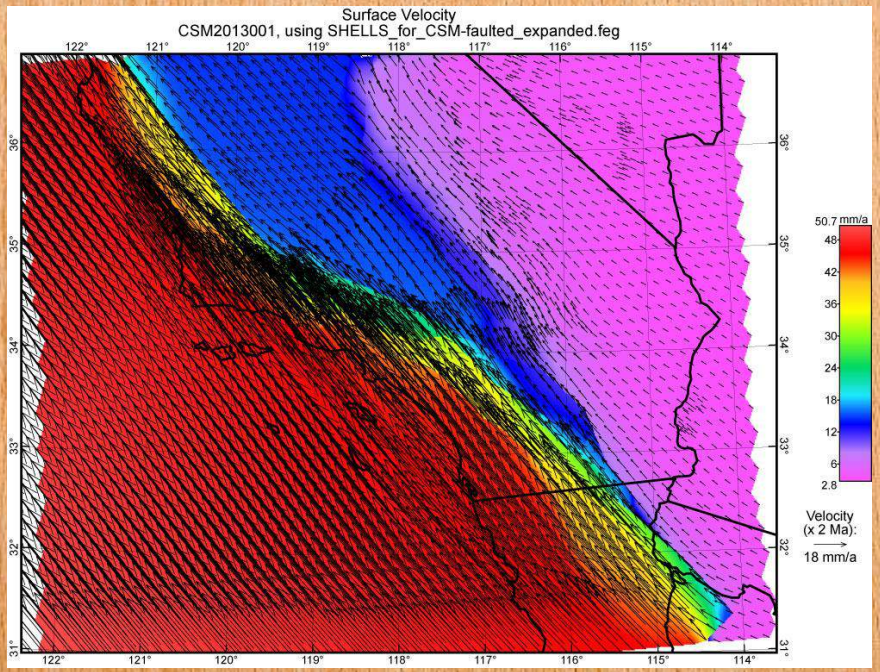
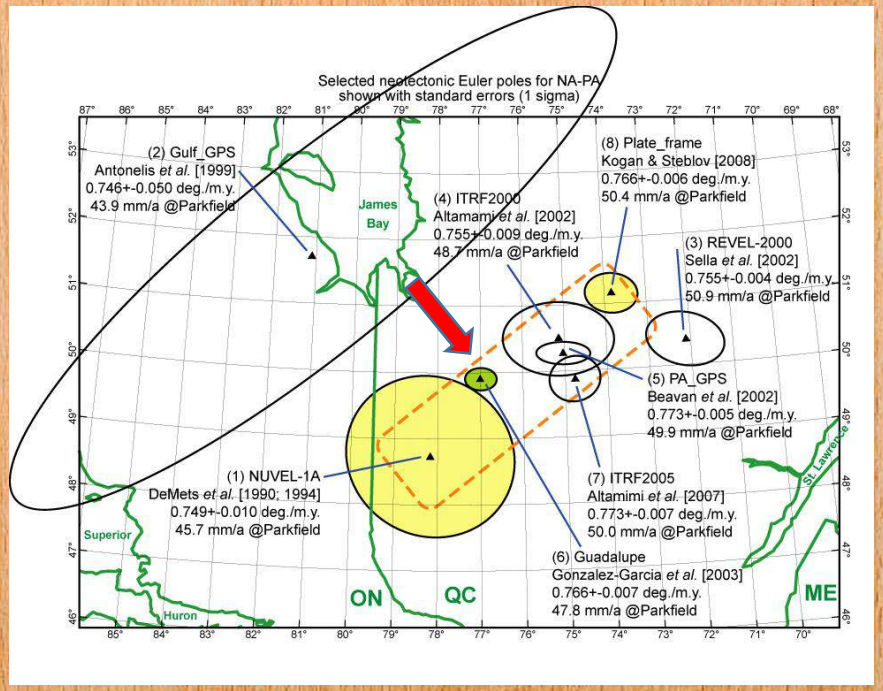
Therefore, data must be supplemented by dynamic models which require <u>multiple input datasets</u>:

- laboratory flow laws,
- a geotherm model,
- a Moho model,
- relative plate motions (PA-NA):



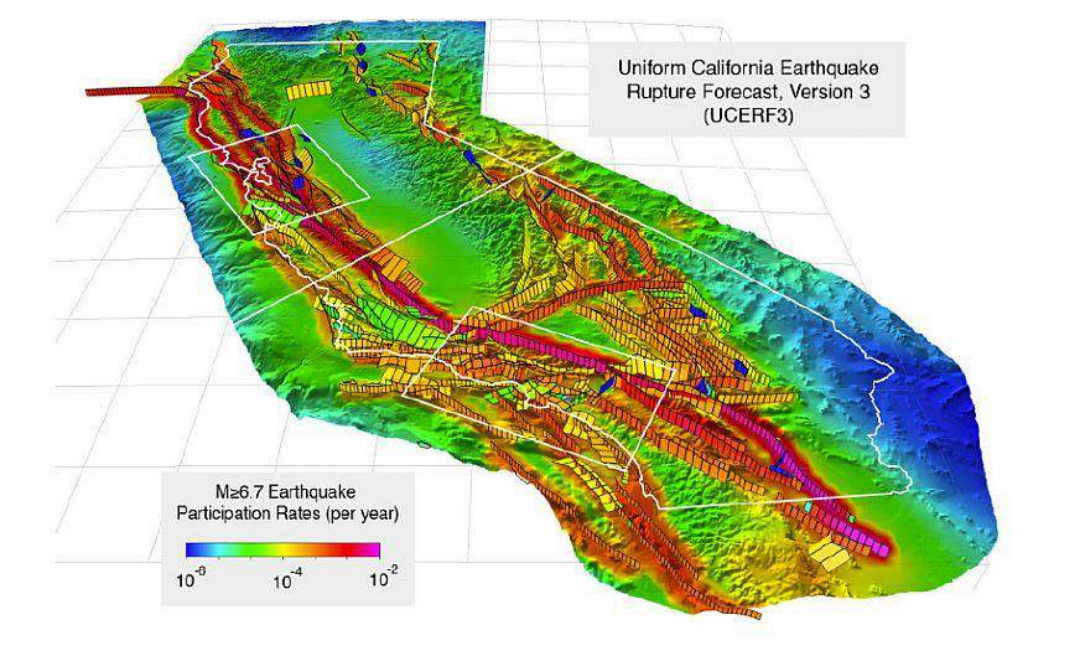


But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km.

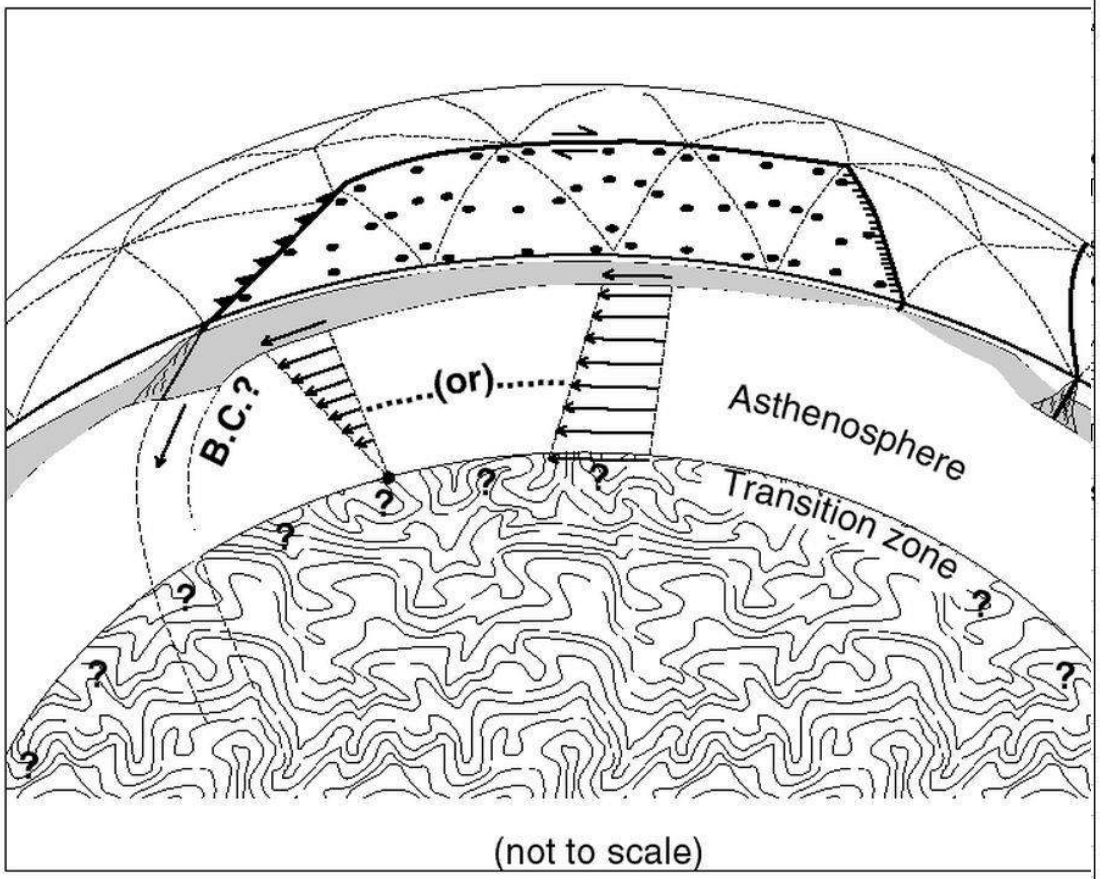
Therefore, data must be supplemented by dynamic models which require <u>multiple input datasets</u>:

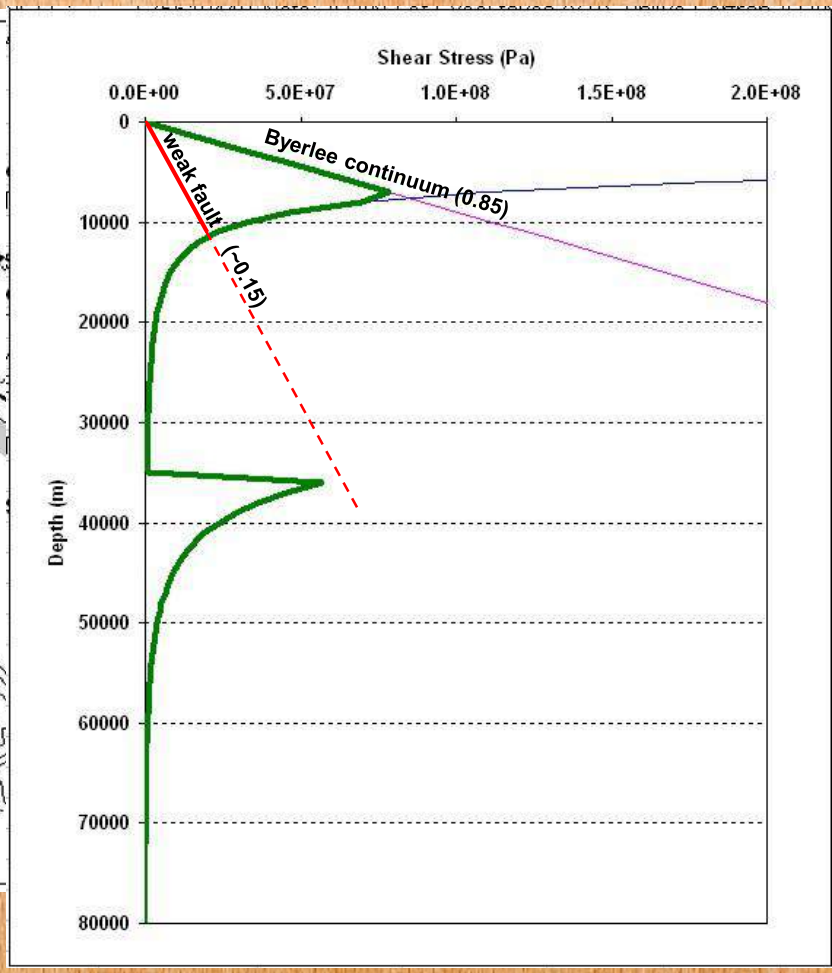
- laboratory flow laws,
- a geotherm model,
- a Moho model,
- relative plate motions, and
- locations of active faults:

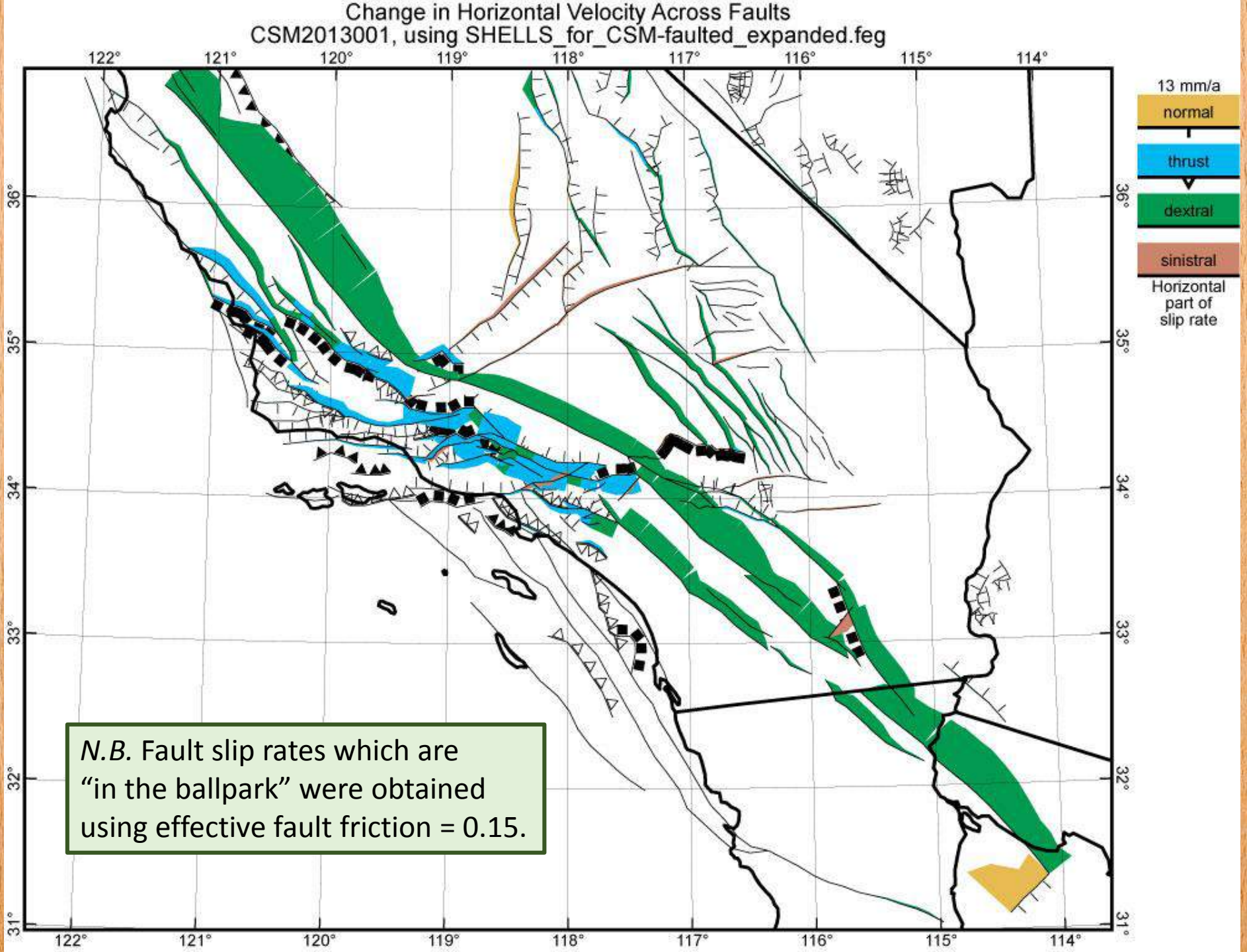
[*Field et al.*, 2013]:



One dynamic model uses code ***Shells***, which solves for 2-D equilibrium of vertically-integrated stresses using 2-D velocity models and 3-D thermal/density/rheologic structure.

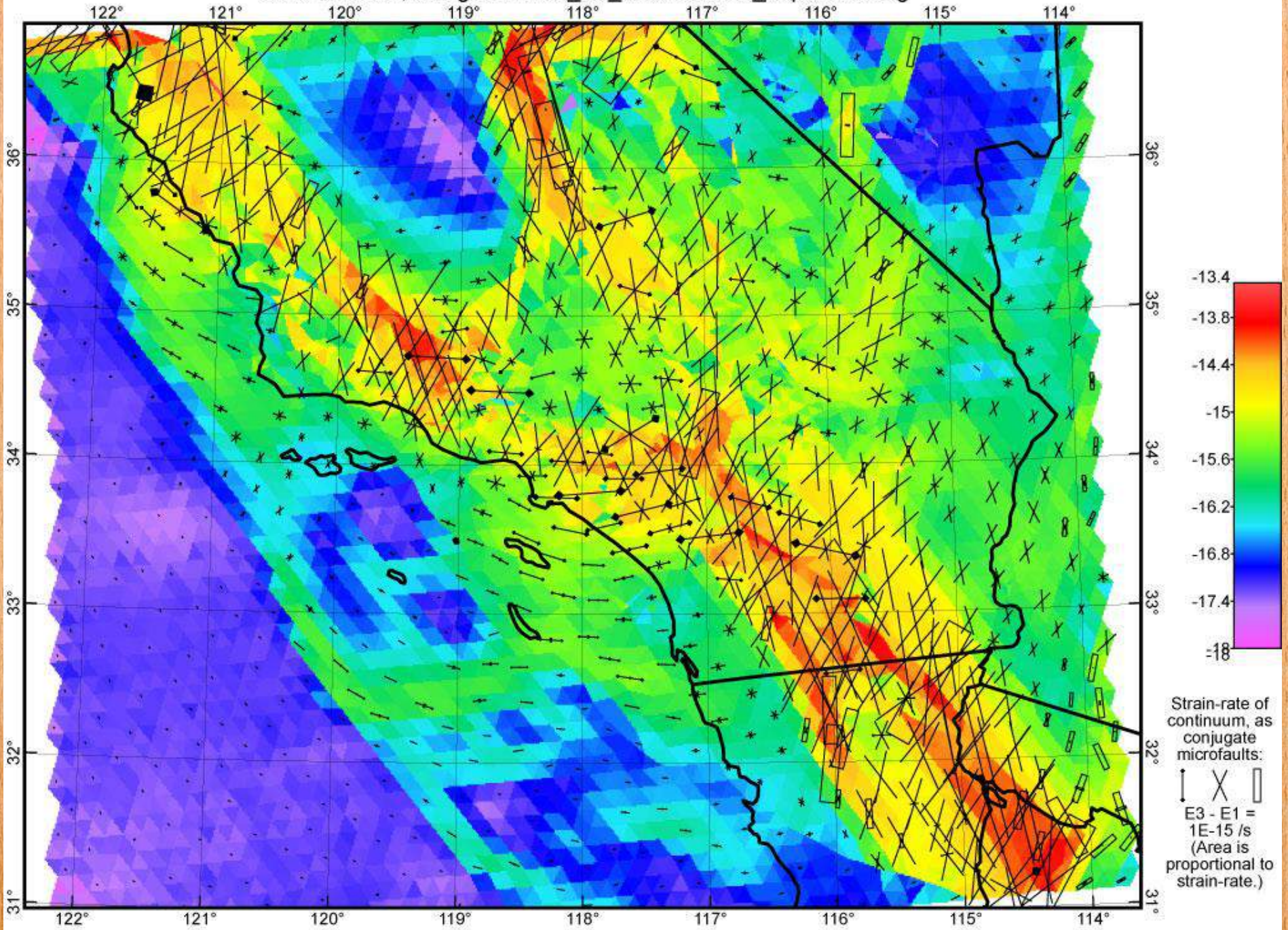




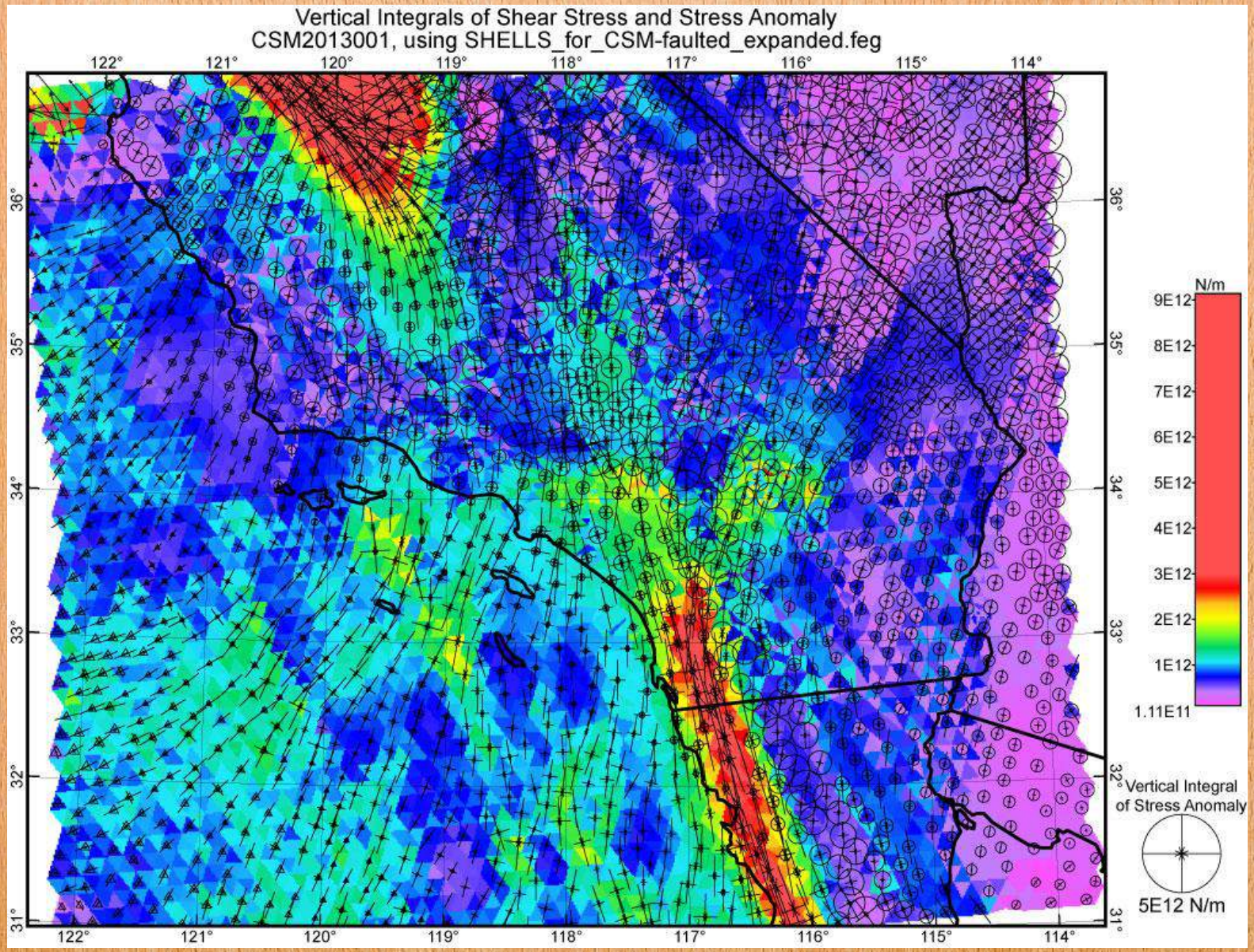


N.B. Fault slip rates which are "in the ballpark" were obtained using effective fault friction = 0.15.

Common Log of [Largest (Absolute Value) Principal Strain-Rate * 1 s]
CSM2013001, using SHELLS_for_CSM-faulted_expanded.feg



This *Shells* model is on the CSM web site. It predicts full stress tensors, ***but*** they are discontinuous and noisy, and only ***approximate 2-D*** stress equilibrium:



A newer approach is to model the stress anomaly field as the sum of topographic and tectonic stress anomaly fields.

In program ***FlatMaxwell***,

- the topographic stress is defined as the convolution of topography (and deep density anomalies) with analytic solutions for point loads on/in an elastic half-space; and
- the tectonic stress is modeled by sums of particular second-derivatives of a Maxwell vector potential field.

Assumptions & Approximations

- Flat-Earth approximation (Cartesian coordinates), with preferred map-projection used to translate (*lon*, *lat*) to (*x*, *y*).
- Model volume is a rectangular solid with sides of 750 × 600 × 75~100 km (= SCEC area x lithosphere thickness, + top of asthenosphere).
- Gravity is the only body force, and is exactly parallel to *z*.
- Quasi-static equilibrium (in times between earthquakes, eruptions, impacts, landslides, *etc.)*

In this Cartesian model space, the quasi-static momentum equation (or stress-equilibrium equation) is

$$\begin{cases} \dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{xy}}{\partial y} + \dfrac{\partial \sigma_{xz}}{\partial z} = 0 \\ \dfrac{\partial \sigma_{yx}}{\partial x} + \dfrac{\partial \sigma_{yy}}{\partial y} + \dfrac{\partial \sigma_{yz}}{\partial z} = 0 \\ \dfrac{\partial \sigma_{zx}}{\partial x} + \dfrac{\partial \sigma_{zy}}{\partial y} + \dfrac{\partial \sigma_{zz}}{\partial z} = \rho g \end{cases}$$

In terms of the stress tensor $\tilde{\sigma}$, gravity $\vec{g}$, and density ρ.

Next, stress $\tilde{\sigma}$ is expressed as a sum of 3 components:

$$\tilde{\sigma} \equiv -P_0(z)\tilde{I} + \tilde{\mu} + \tilde{\tau}$$

where $P_0 \equiv g\int_z^{\infty} \rho_0(s)\,\mathrm{d}s$ is a ***reference lithostatic pressure curve***,

based on a 1-D reference density model $\rho_0(\mathrm{z})$,

$\tilde{\mu}$ is the ***topographic stress anomaly***,

and $\tilde{\tau}$ is the ***tectonic stress anomaly***.

Specifically, I define $\tilde{\mu}$ as any *convenient* solution to the

Inhomogeneous quasi-static momentum equation driven by density anomaly

$\Delta\rho(x,y,z) \equiv \rho(x,y,z) - \rho_0(z)$:

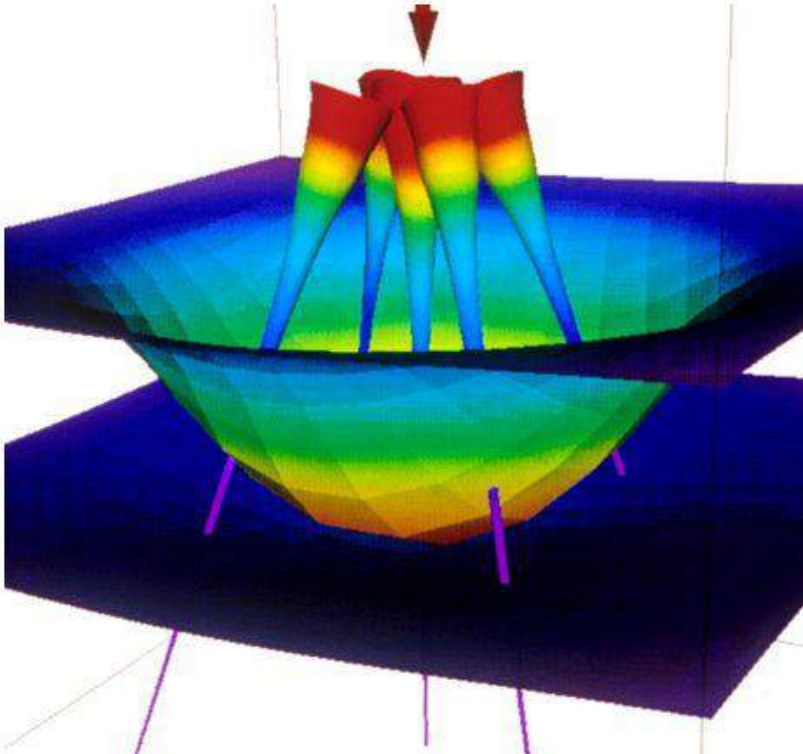
$$\begin{cases} \dfrac{\partial \mu_{xx}}{\partial x} + \dfrac{\partial \mu_{xy}}{\partial y} + \dfrac{\partial \mu_{xz}}{\partial z} = 0 \\ \dfrac{\partial \mu_{yx}}{\partial x} + \dfrac{\partial \mu_{yy}}{\partial y} + \dfrac{\partial \mu_{yz}}{\partial z} = 0 \\ \dfrac{\partial \mu_{zx}}{\partial x} + \dfrac{\partial \mu_{zy}}{\partial y} + \dfrac{\partial \mu_{zz}}{\partial z} = \Delta\rho g \end{cases}$$

and $\tilde{\tau}$ as any solution to the complementary homogeneous quasi-static momentum equation:

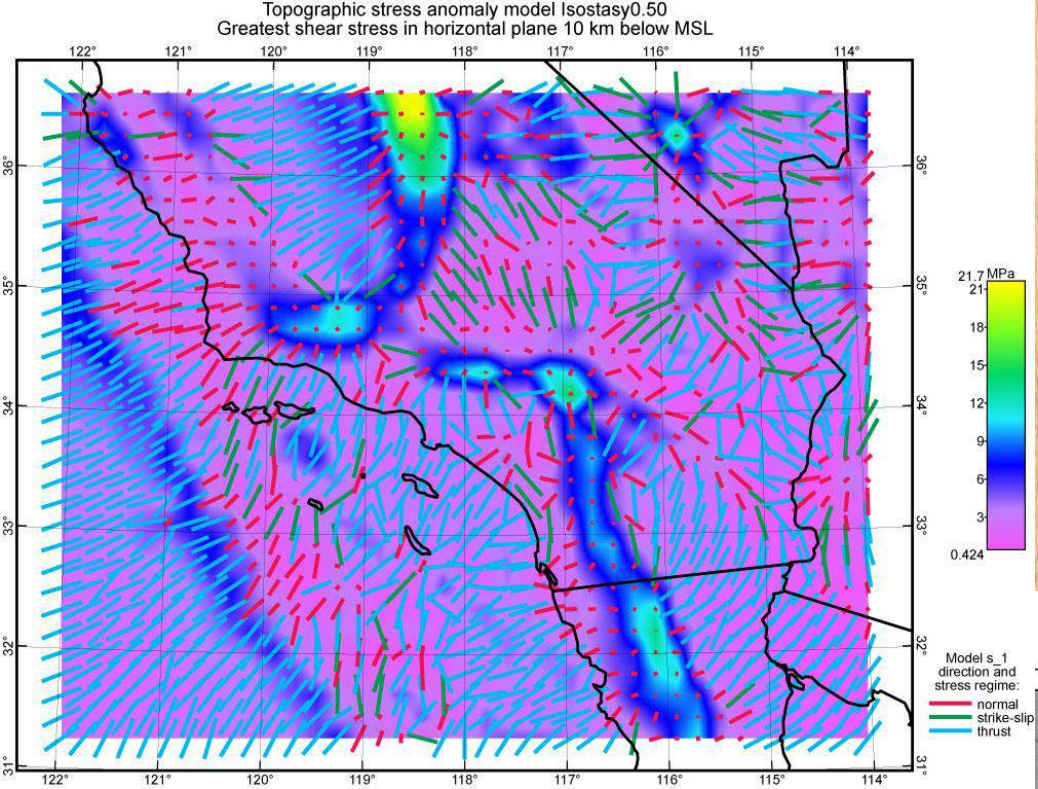
$$\begin{cases} \dfrac{\partial \tau_{xx}}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} + \dfrac{\partial \tau_{xz}}{\partial z} = 0 \\ \dfrac{\partial \tau_{yx}}{\partial x} + \dfrac{\partial \tau_{yy}}{\partial y} + \dfrac{\partial \tau_{yz}}{\partial z} = 0 \\ \dfrac{\partial \tau_{zx}}{\partial x} + \dfrac{\partial \tau_{zy}}{\partial y} + \dfrac{\partial \tau_{zz}}{\partial z} = 0 \end{cases}$$

The most convenient solutions for the topographic stress anomaly $\tilde{\mu}$ come from classic published solutions for an isotropic and homogenous elastic half-space, with no density or pre-stress, but subject to:

- Vertical surface point loads (Boussinesq);
- Horizontal surface point loads (Cerruti); and
- Vertical internal point loads (Mindlin).

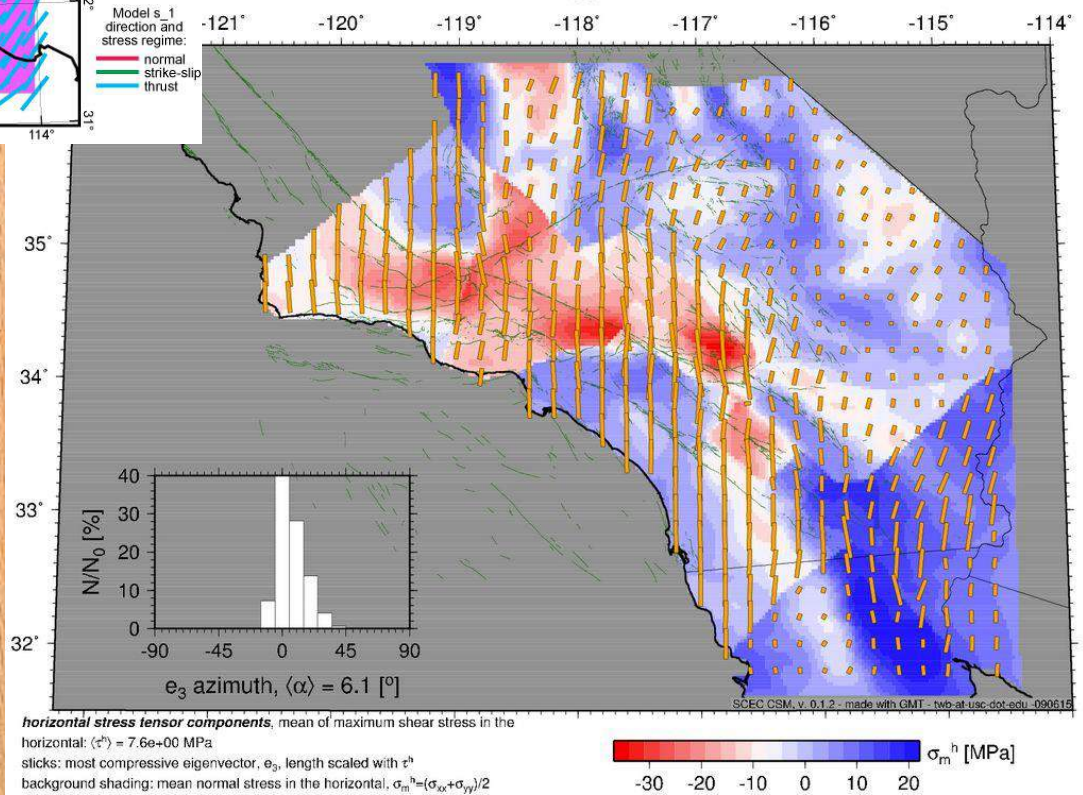


tractions and deflections caused by point load orthogonal to flat surface of elastic half-space



Luttrell, Smith-Konter, & Sandwell [SCEC, 2012; CSM web site] uses a similar method for topographic stress, then adds minimum tectonic stress necessary to match focal mechanisms along each active fault:

Luttrell @ 3 km



FlatMaxwell topographic stress anomaly (ONLY) when:
*isostatic Moho is used; and
*Poisson ratio is 0.50.

The tectonic stress anomaly $\tilde{\tau}$ satisfies the homogeneous quasi-static momentum equation, and therefore it can be obtained from particular second-derivatives of a continuous vector field $\vec{\Phi}$ by:

$$\begin{cases} \tau_{xx} = \dfrac{\partial^2 \Phi_z}{\partial y^2} + \dfrac{\partial^2 \Phi_y}{\partial z^2} \quad \tau_{yy} = \dfrac{\partial^2 \Phi_x}{\partial z^2} + \dfrac{\partial^2 \Phi_z}{\partial x^2} \quad \tau_{zz} = \dfrac{\partial^2 \Phi_y}{\partial x^2} + \dfrac{\partial^2 \Phi_x}{\partial y^2} \\ \tau_{xy} = \tau_{yx} = -\dfrac{\partial^2 \Phi_z}{\partial x \partial y} \quad \tau_{yz} = \tau_{zy} = -\dfrac{\partial^2 \Phi_x}{\partial y \partial z} \quad \tau_{xz} = \tau_{zx} = -\dfrac{\partial^2 \Phi_y}{\partial x \partial z} \end{cases}$$

Fig. 1. James Clerk Maxwell, 24 years old in 1855, about the time he embarked on unifying electrostatics, electrodynamics, and electrical induction. He was inspired by Faraday's intuition and by Fourier's mathematics of diffusion. (Photograph by W.H. Hales; courtesy of Cavendish Laboratories.)

Narasimhan [2003]

Maxwell [1870] came 15 years later…

I explore possible vector fields $\vec{\Phi}$ which are formed as weighted sums of $i = 1, \ldots, N$ basis functions,

$$\vec{\Phi}(x, y, z) = \sum_{i=1}^{N} c_i \vec{\Phi}^i(x, y, z)$$

I have designed a complete and complementary set of basis functions which provide for:

(1) spatially-constant values of each tectonic stress component;

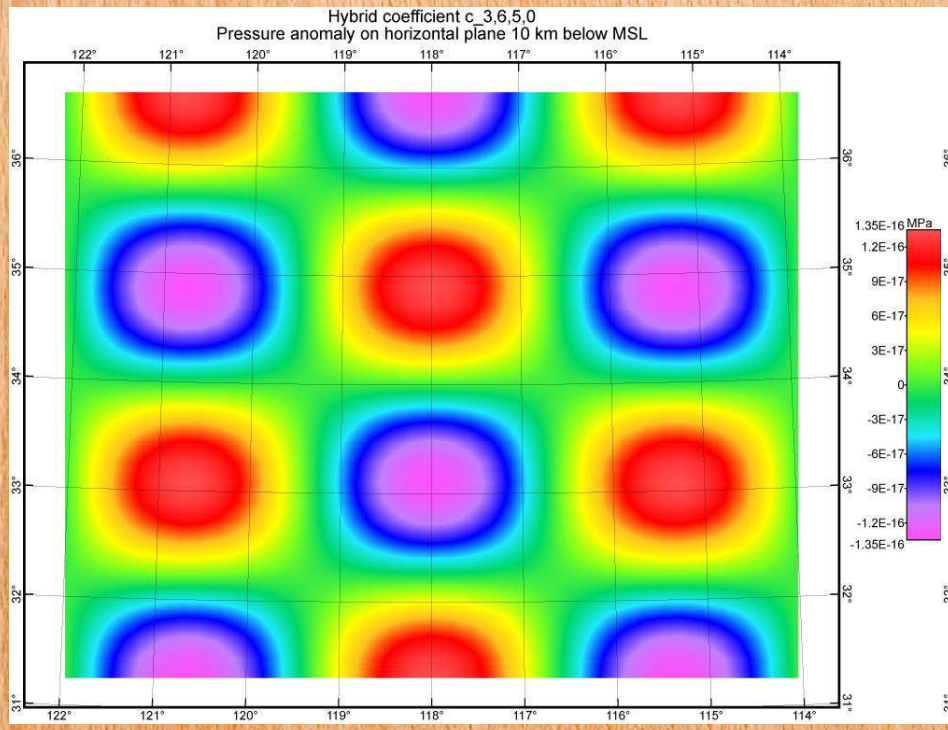
(2) values of any tectonic stress component that may vary linearly along any spatial axis;

(3) stress-potential vectors of arbitrary direction that vary harmonically as a function of any one space direction ("stress waves");

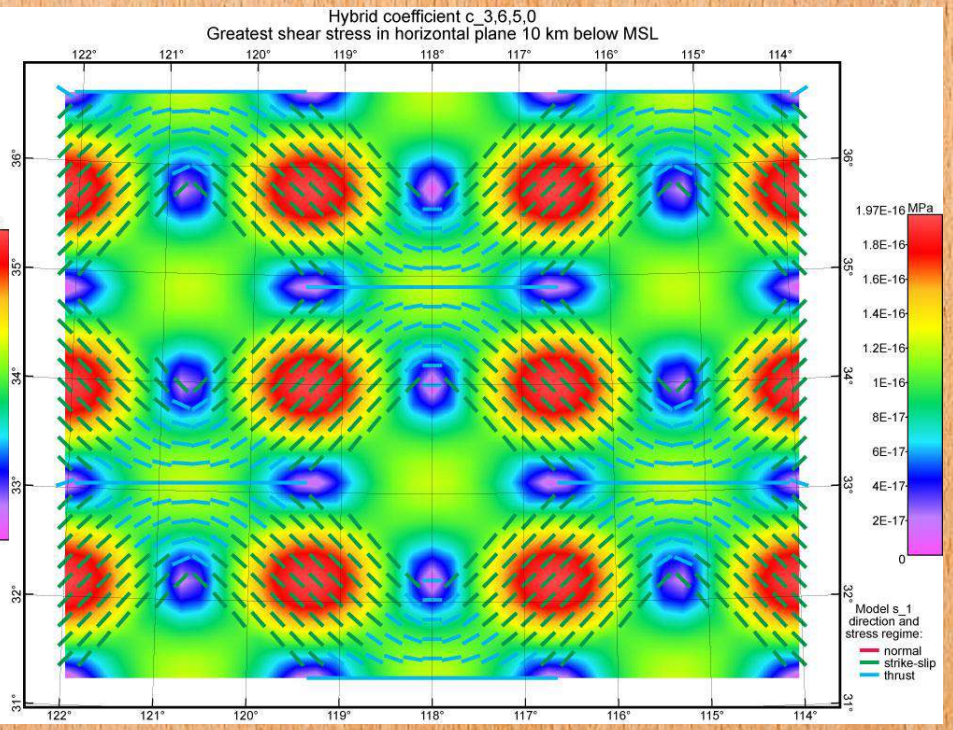
(4) stress-potential vectors that vary harmonically as a function of any two space directions ("stress quilts"); and

(5) stress-potential vectors that vary harmonically as function of all three space directions ("stress crystals").

A generic “stress quilt” (in 2-D) or
“stress crystal” (in 3-D) looks like this:



pressure anomaly

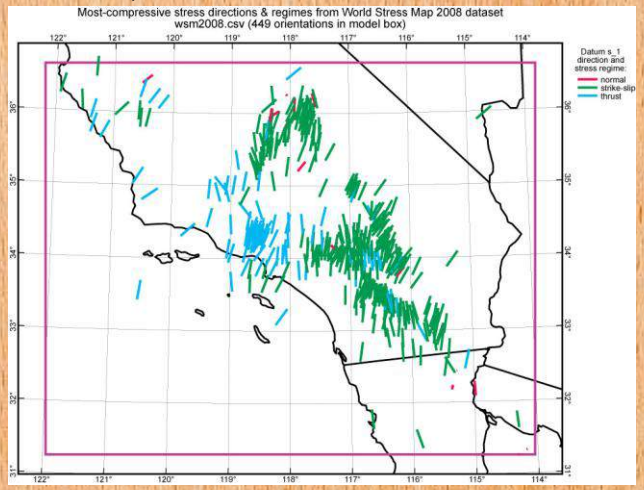


greatest shear stress (color) &
most-compressive axis (bars)

The whole stress field is then best-fit (by weighted least squares) to both:

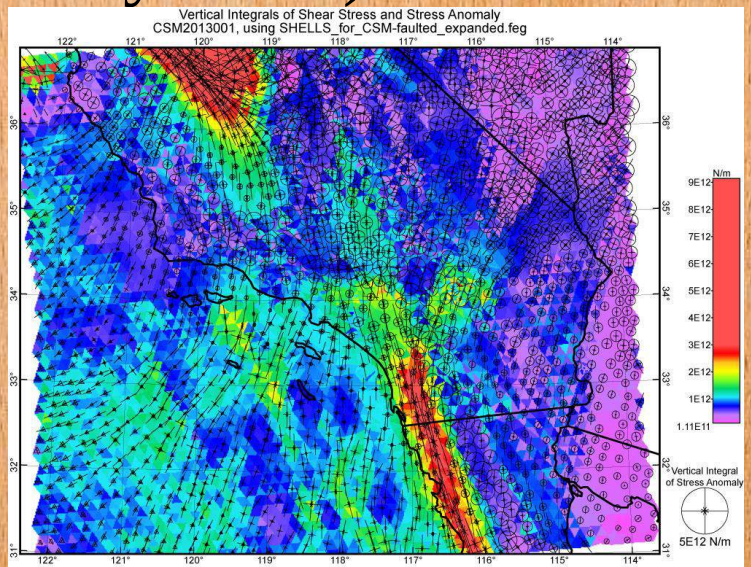
- data (at shallow depths),

***World Stress Map (480 FMs from larger earthquakes)*:**

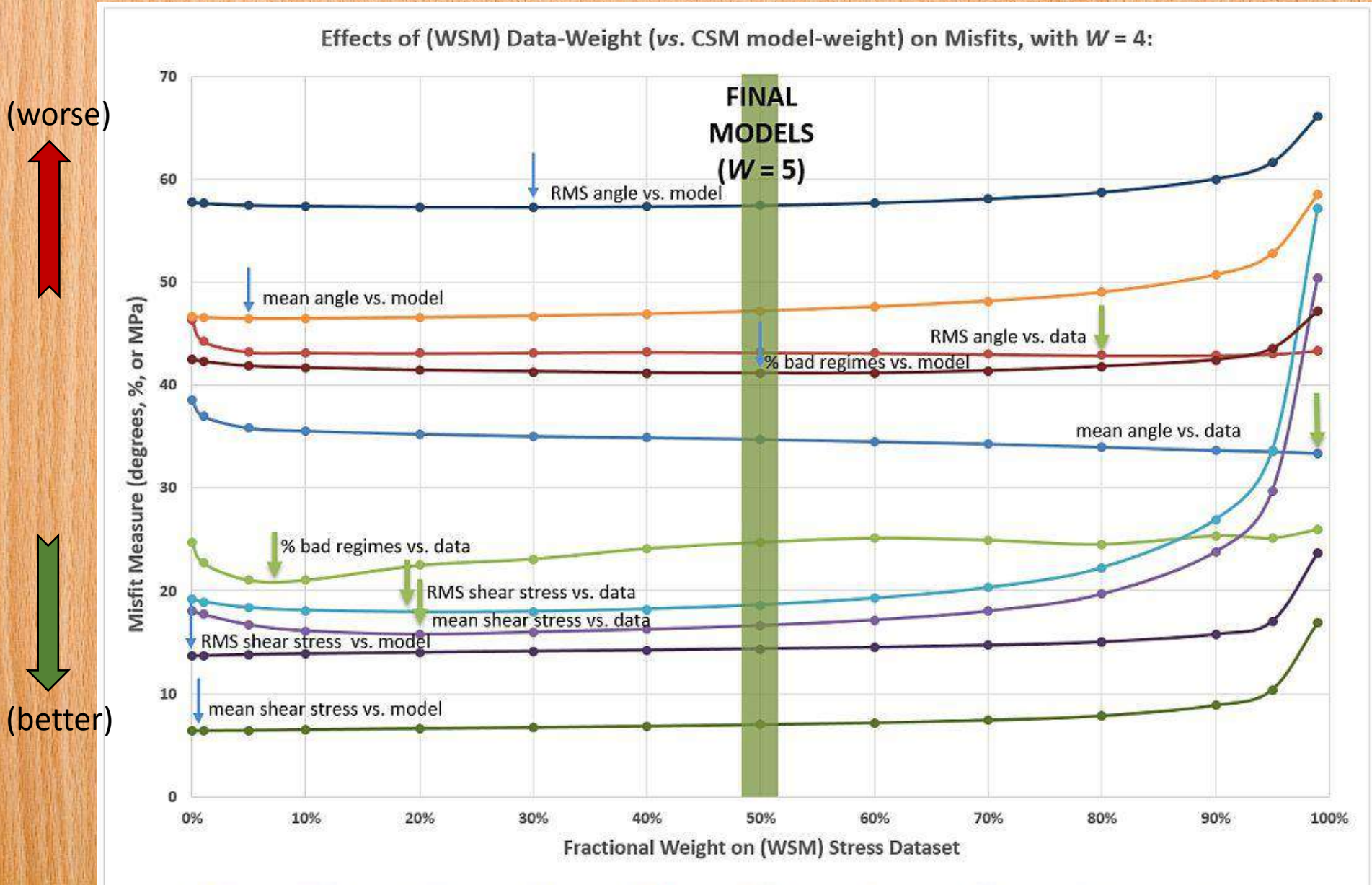


- and the dynamic model (everywhere).

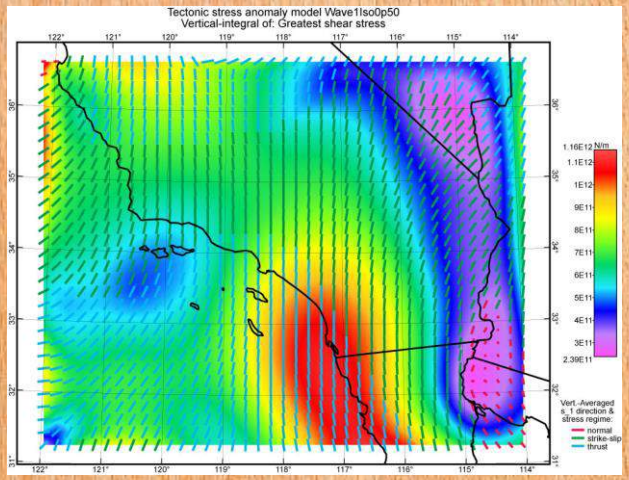
Shells:



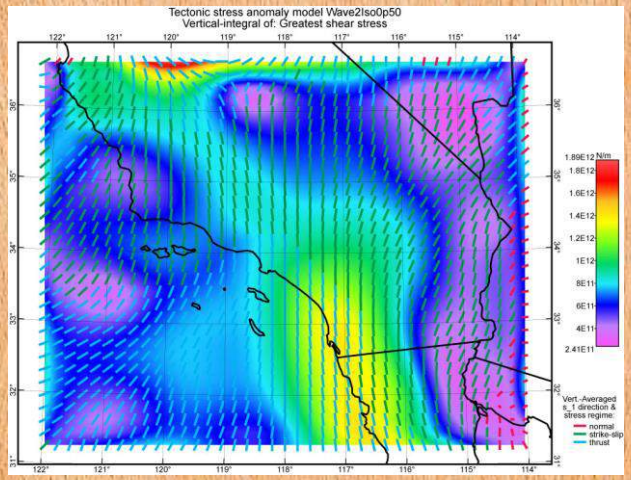
Fortunately, the relative weighting of data *vs.* dynamics has little effect on residual misfits:



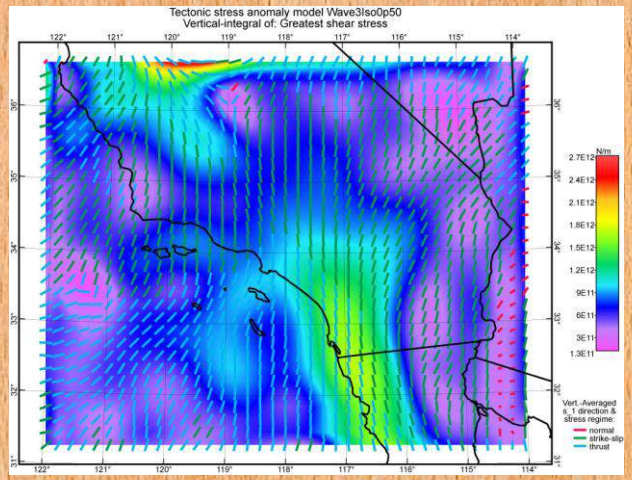
In practice, ***FlatMaxwell*** models are limited in spatial resolution (of tectonic stress) to no more than 6 wavelengths along each side of the model domain (including the depth axis).



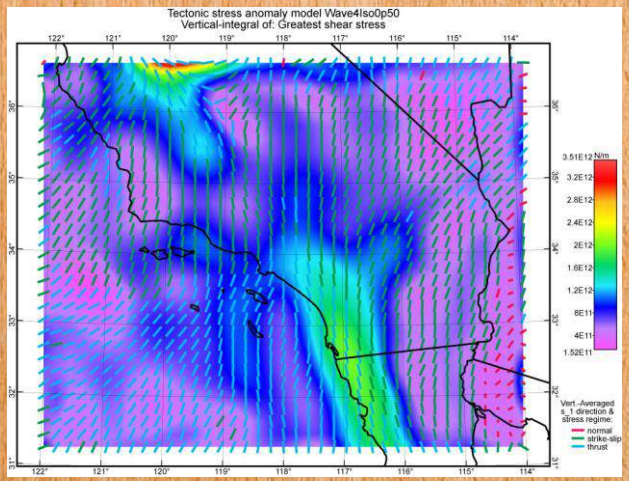
***W* = 1**



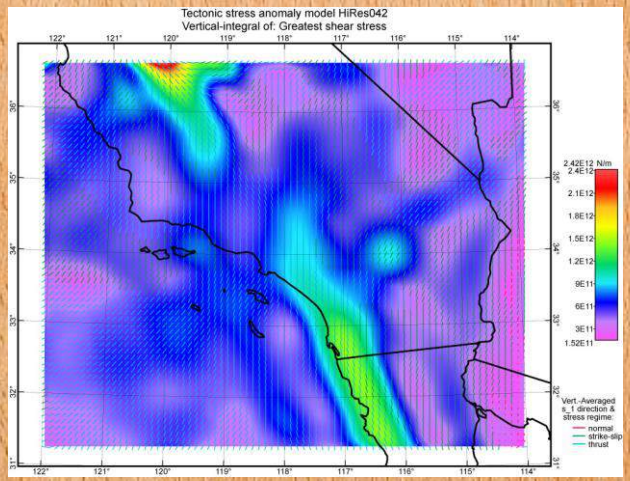
***W* = 2**



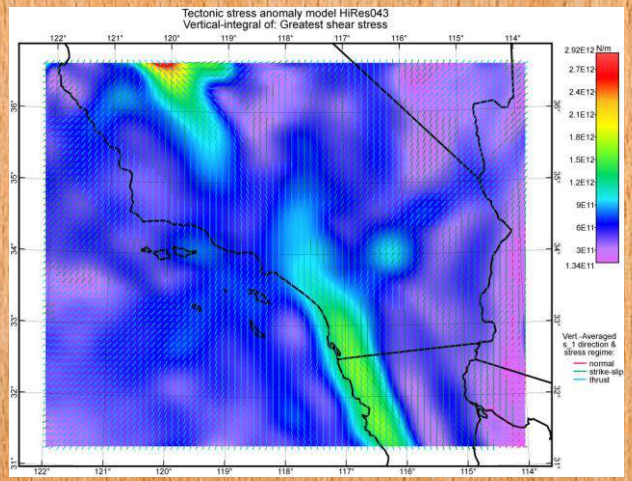
***W* = 3**



***W* = 4**



***W* = 5**



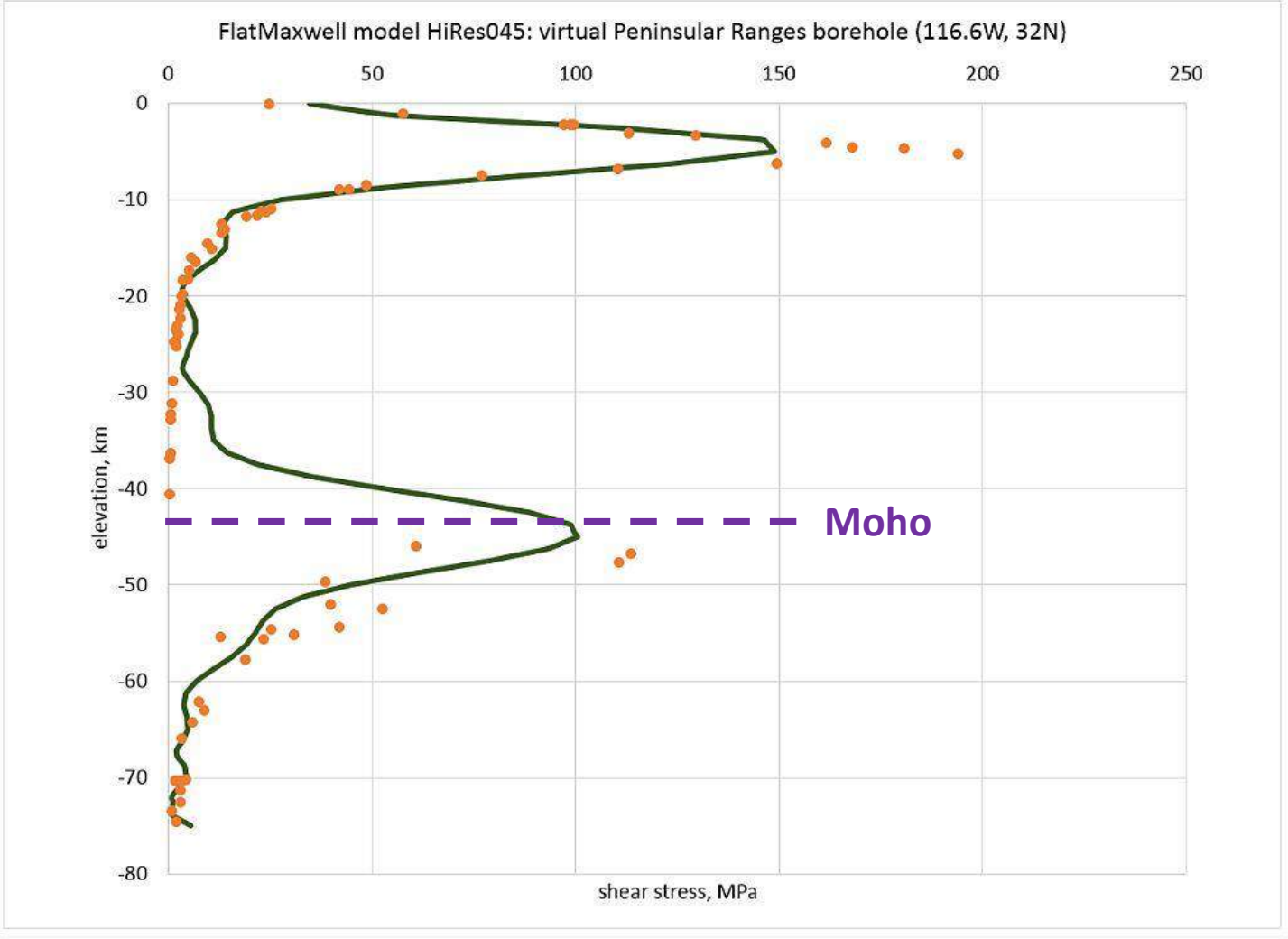
***W* = 6**

Thus, ***FlatMaxwell*** models are quite smooth, and (at present) cannot accurately represent:

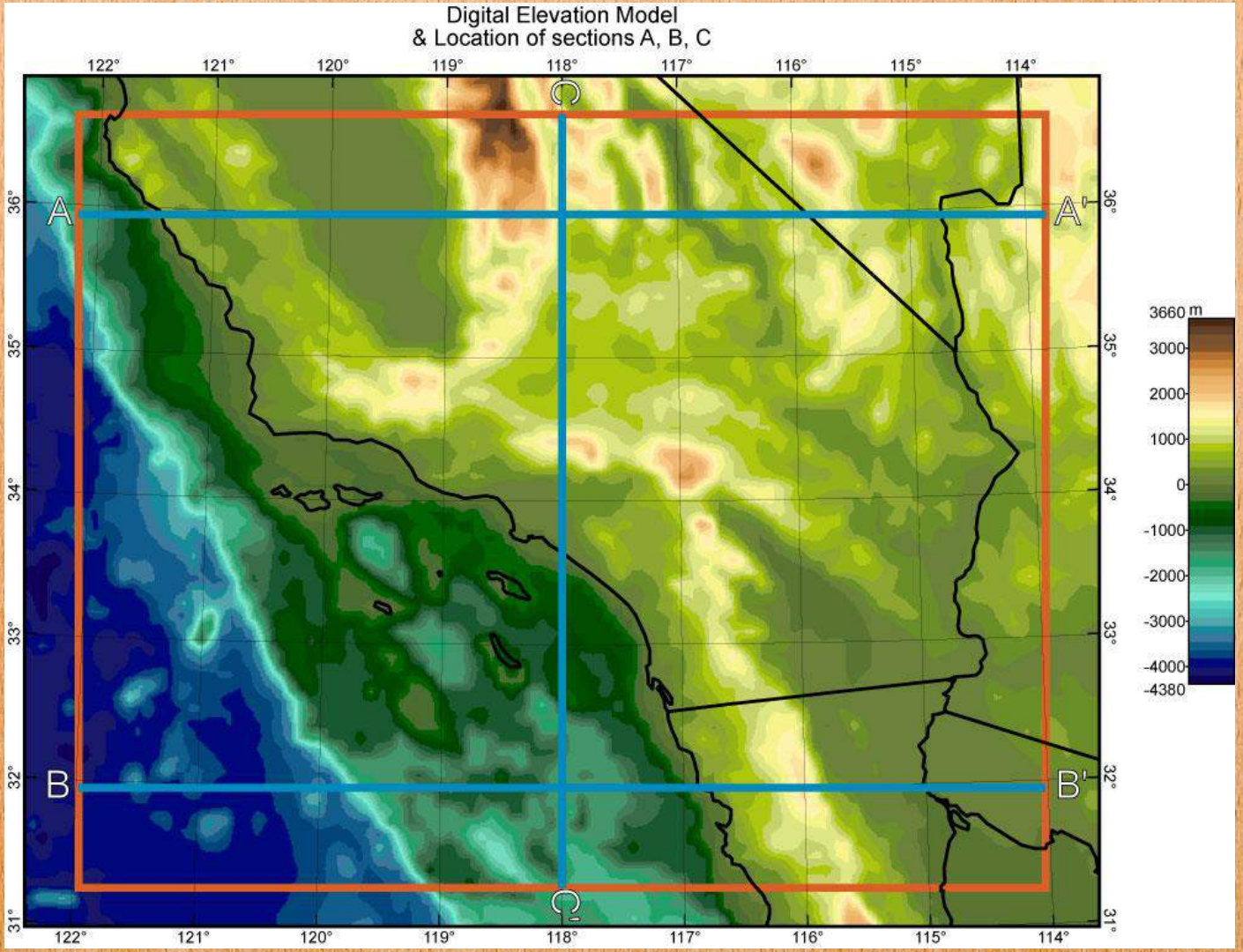
-peak shear stresses in the shallow upper crust; or

-discontinuous shear stresses at the Moho

which are predicted by the ***Shells*** model.



Results to date show a low-amplitude stress anomaly, with peak shear stress of 120~150 MPa and peak vertically-integrated shear stress of 2.9~4.1×10^{12} N/m.



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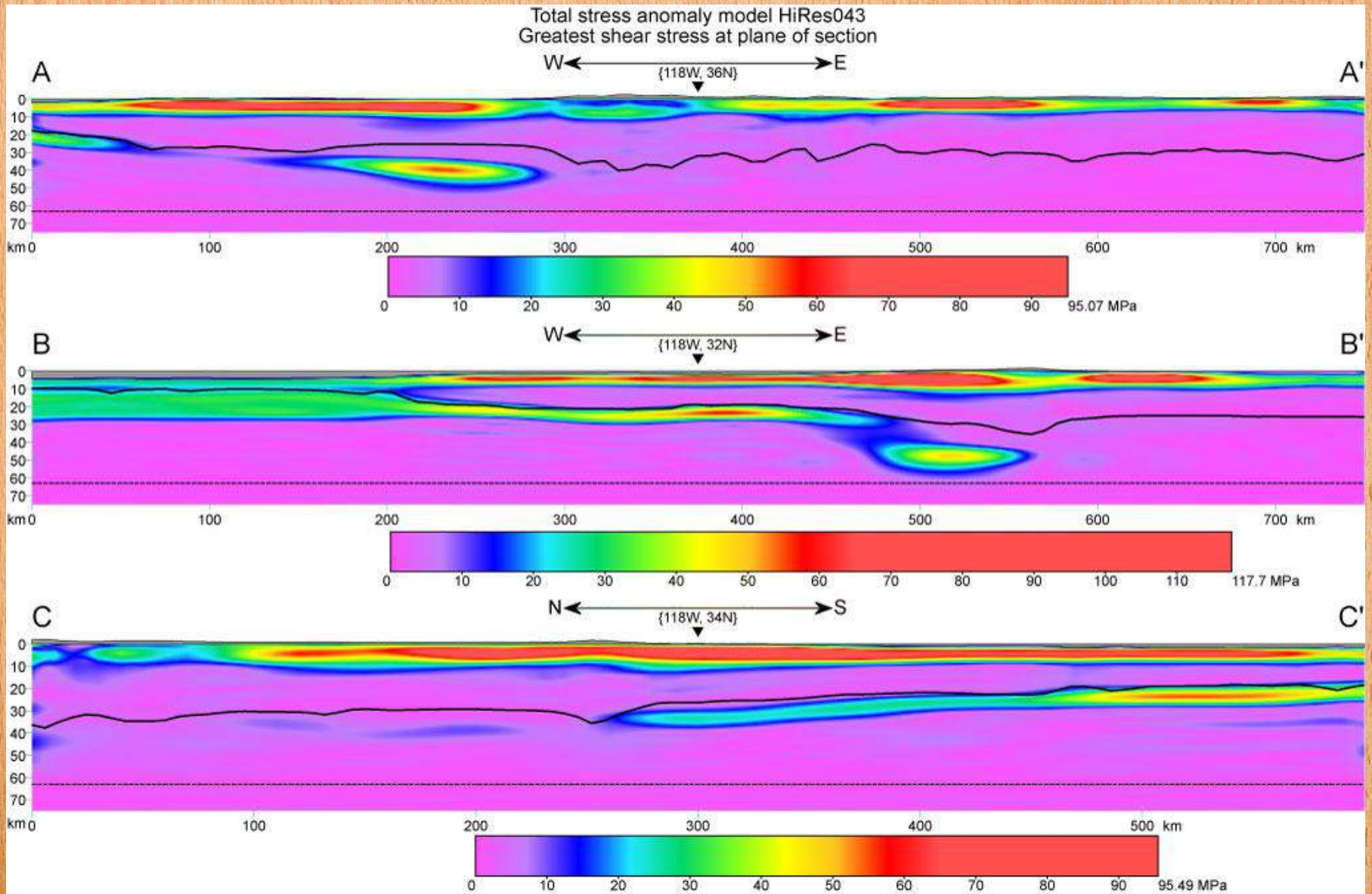
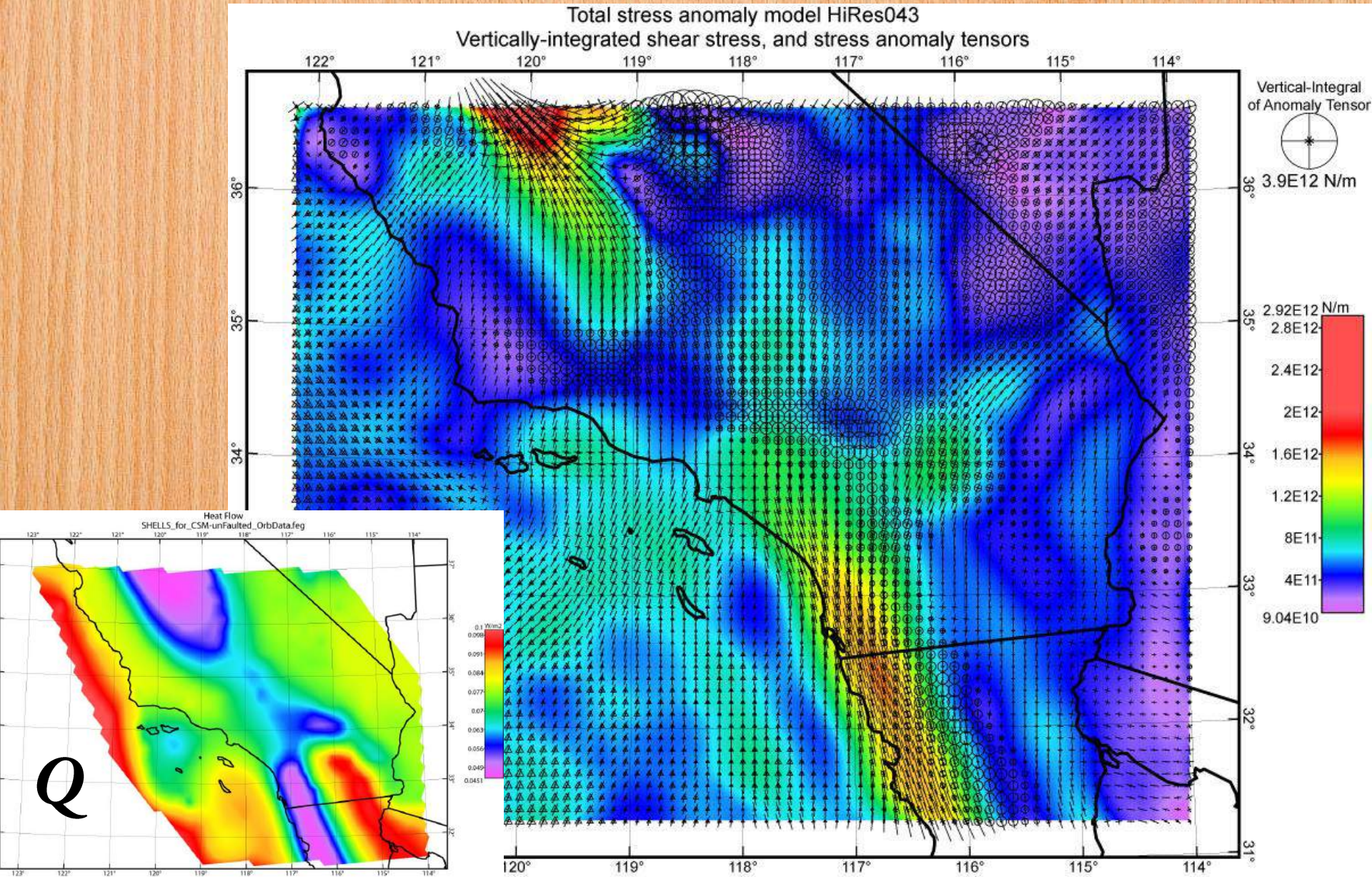


Channeling of deviatoric stress along the strong Peninsular Ranges and Great Valley is seen.



In southern California, deviatoric stress and long-term strain-rate are <u>negatively</u> <u>correlated</u> because regions of low heat-flow act as stress guides while deforming very little.

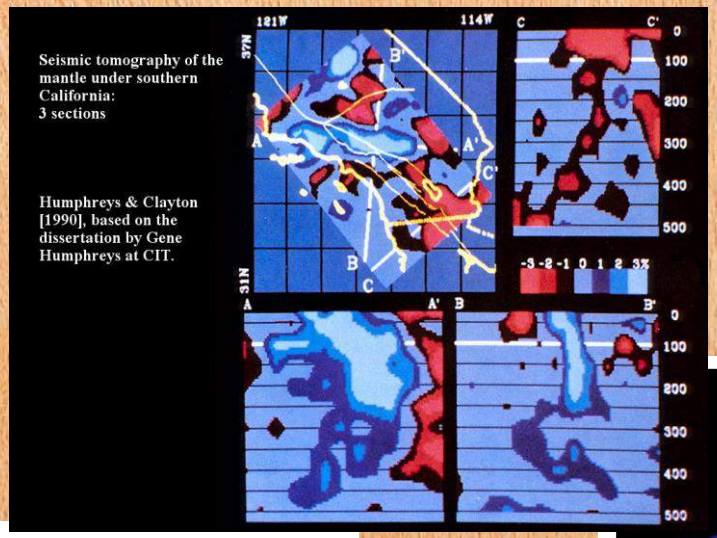
In contrast, active faults lie in areas with higher heat-flow, and their low strength keeps deviatoric stresses locally modest.

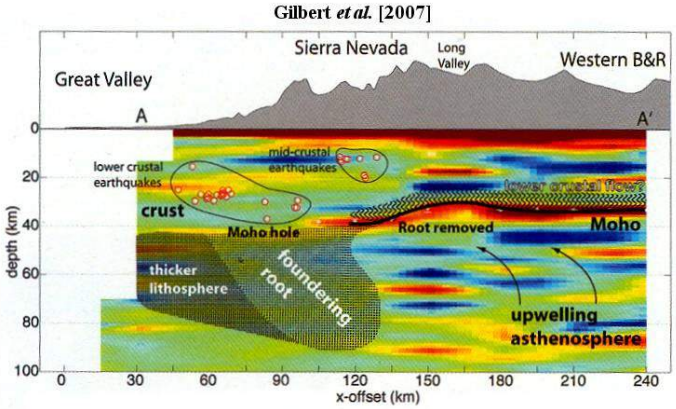
Opportunities for future CSM advances include:

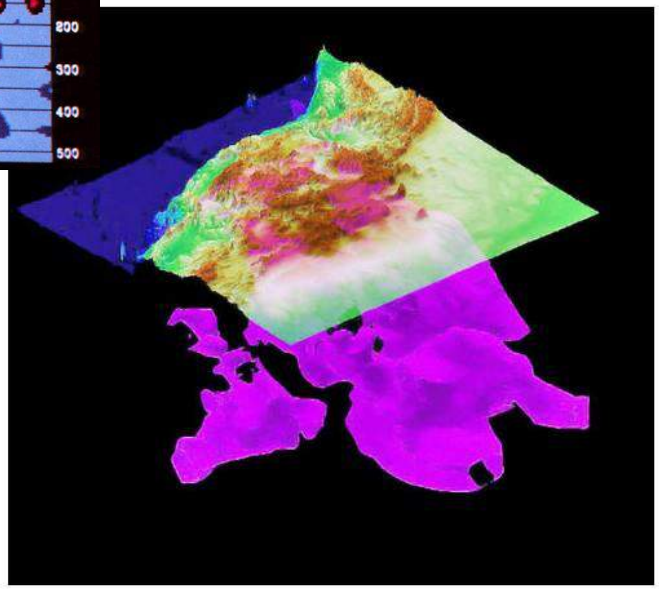
[1] Collecting more data (especially stress magnitudes from boreholes);

[2] Tuning the ***Shells*** dynamic model;

[3] Using a **DIFFERENT** dynamic model, with 3-D upper mantle:







Farallon plate
Karin Sigloch *08
University of Munich

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[1] Collecting more data (especially stress magnitudes from boreholes);

[2] Tuning the ***Shells*** dynamic model;

[3] Using a **DIFFERENT** dynamic model, with 3-D upper mantle; and

[4] Applying a similar Maxwell equilibrium filter to the stressing-**<u>rate</u>** models!