Blending data and dynamics into equilibrium for the Community Stress Model

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for the SCEC Annual Meeting, September 2015,
Palm Springs, CA
This talk will focus on the time-independent mean stress field in S CA, not on the time-dependence.

With 3-D tensor models of mean stress throughout the lithosphere of southern California, SCEC could:

- determine the shear stresses on active faults to constrain the physics of their slip;
- predict Coulomb stress changes for operational earthquake forecasting; and
- test the realism of long-term earthquake sequence simulators.
One basis for models is data:

- stress directions (from focal mechanisms and boreholes), and
- stress intensity (only from boreholes).

179,000 focal mechanisms by HASH method of [Hardebeck & Shearer, 2003]:

YHS2010 [Yang et al., 2012, BSSA]

SATSI program [Hardebeck & Michael, 2006]

[Yang & Hauksson, 2013, GJI]
An alternative inversion of the same data by Jeanne Hardebeck [2012, CSM web site], using variable 3-D binning, and damping:

**Hardebeck_FM @ 3 km**
But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km. Therefore, stress data must be supplemented by dynamic models which require multiple input datasets:

- laboratory flow laws (with real-world calibration):

[Bird & Kong, 1994, GSAB]
But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km. Therefore, data must be supplemented by dynamic models which require multiple input datasets:

- laboratory flow laws,
- a geotherm model:

![Geothermal Map of North America](Blackwell & Steele, 1992)
But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km. Therefore, data must be supplemented by dynamic models which require **multiple input datasets:**

- laboratory flow laws,
- a geotherm model,
- a Moho model

[Tape et al., 2012] seismic Moho

FlatMaxwell isostatic Moho
But earthquakes rarely occur deeper than 15~20 km, and boreholes rarely go deeper than ~6 km. Therefore, data must be supplemented by dynamic models which require **multiple input datasets**:

- laboratory flow laws,
- a geotherm model,
- a Moho model,
- relative plate motions (PA-NA):
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- laboratory flow laws,
- a geotherm model,
- a Moho model,
- relative plate motions, and
- locations of active faults:

[Field et al., 2013]:

![Map of California with geological features and data]

Uniform California Earthquake Rupture Forecast, Version 3 (UCERF3)
One dynamic model uses code *Shells*, which solves for 2-D equilibrium of vertically-integrated stresses using 2-D velocity models and 3-D thermal/density/rheologic structure.
N.B. Fault slip rates which are “in the ballpark” were obtained using effective fault friction = 0.15.
This *Shells* model is on the CSM web site. It predicts full stress tensors, *but* they are discontinuous and noisy, and only *approximate* 2-D stress equilibrium:
A newer approach is to model the stress anomaly field as the sum of topographic and tectonic stress anomaly fields.

In program *FlatMaxwell*,

- the topographic stress is defined as the convolution of topography (and deep density anomalies) with analytic solutions for point loads on/in an elastic half-space; and

- the tectonic stress is modeled by sums of particular second-derivatives of a Maxwell vector potential field.
Assumptions & Approximations

- Flat-Earth approximation (Cartesian coordinates), with preferred map-projection used to translate (lon, lat) to (x, y).
- Model volume is a rectangular solid with sides of 750 \times 600 \times 75\text{~}100 \text{ km} (= SCEC area x lithosphere thickness, + top of asthenosphere).
- Gravity is the only body force, and is exactly parallel to z.
- Quasi-static equilibrium (in times between earthquakes, eruptions, impacts, landslides, etc.)
In this Cartesian model space, the quasi-static momentum equation (or stress-equilibrium equation) is

\[
\begin{align*}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0 \\
\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= 0 \\
\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= \rho g
\end{align*}
\]

In terms of the stress tensor $\tilde{\sigma}$, gravity $\tilde{g}$, and density $\rho$. 
Next, stress $\tilde{\sigma}$ is expressed as a sum of 3 components:

$$\tilde{\sigma} \equiv -P_0(z)\tilde{I} + \tilde{\mu} + \tilde{\tau}$$

where $P_0 \equiv g \int_0^z \rho_0(s) \, ds$ is a reference lithostatic pressure curve, based on a 1-D reference density model $\rho_0(z)$,

$\tilde{\mu}$ is the topographic stress anomaly,

and $\tilde{\tau}$ is the tectonic stress anomaly.
Specifically, I define $\tilde{\mu}$ as any convenient solution to the

inhomogeneous quasi-static momentum equation driven by density anomaly

$$\Delta \rho(x, y, z) \equiv \rho(x, y, z) - \rho_0(z):$$

$$\begin{align*}
\frac{\partial \mu_{xx}}{\partial x} + \frac{\partial \mu_{xy}}{\partial y} + \frac{\partial \mu_{xz}}{\partial z} &= 0 \\
\frac{\partial \mu_{yx}}{\partial x} + \frac{\partial \mu_{yy}}{\partial y} + \frac{\partial \mu_{yz}}{\partial z} &= 0 \\
\frac{\partial \mu_{zx}}{\partial x} + \frac{\partial \mu_{zy}}{\partial y} + \frac{\partial \mu_{zz}}{\partial z} &= \Delta \rho g
\end{align*}$$
and \( \tilde{\mathbf{T}} \) as any solution to the complementary homogeneous quasi-static momentum equation:

\[
\begin{align*}
\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} &= 0 \\
\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} &= 0 \\
\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} &= 0
\end{align*}
\]
The most convenient solutions for the topographic stress anomaly $\tilde{\mu}$ come from classic published solutions for an isotropic and homogenous elastic half-space, with no density or pre-stress, but subject to:

- Vertical surface point loads (Boussinesq);
- Horizontal surface point loads (Cerruti); and
- Vertical internal point loads (Mindlin).
Luttrell, Smith-Kanter, & Sandwell [SCEC, 2012; CSM web site] uses a similar method for topographic stress, then adds minimum tectonic stress necessary to match focal mechanisms along each active fault:

FlatMaxwell topographic stress anomaly (ONLY) when:
* isostatic Moho is used; and
* Poisson ratio is 0.50.
The tectonic stress anomaly $\tilde{\tau}$ satisfies the homogeneous quasi-static momentum equation, and therefore it can be obtained from particular second-derivatives of a continuous vector field $\vec{\Phi}$ by:

\[
\begin{align*}
\tau_{xx} &= \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} \\
\tau_{yy} &= \frac{\partial^2 \Phi}{\partial z^2} + \frac{\partial^2 \Phi}{\partial x^2} \\
\tau_{zz} &= \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \\
\tau_{xy} &= \tau_{yx} = -\frac{\partial^2 \Phi}{\partial x \partial y} \\
\tau_{yz} &= \tau_{zy} = -\frac{\partial^2 \Phi}{\partial y \partial z} \\
\tau_{xz} &= \tau_{zx} = -\frac{\partial^2 \Phi}{\partial x \partial z}
\end{align*}
\]
Maxwell [1870] came 15 years later...
I explore possible vector fields \( \Phi \) which are formed as weighted sums of \( i = 1, \ldots, N \) basis functions,

\[
\Phi(x, y, z) = \sum_{i=1}^{N} c_i \Phi^i(x, y, z)
\]

I have designed a complete and complementary set of basis functions which provide for:

1. spatially-constant values of each tectonic stress component;
2. values of any tectonic stress component that may vary linearly along any spatial axis;
3. stress-potential vectors of arbitrary direction that vary harmonically as a function of any one space direction ("stress waves");
4. stress-potential vectors that vary harmonically as a function of any two space directions ("stress quilts"); and
5. stress-potential vectors that vary harmonically as function of all three space directions ("stress crystals").
A generic “stress quilt” (in 2-D) or “stress crystal” (in 3-D) looks like this:

Pressure anomaly

Greatest shear stress (color) & most-compressive axis (bars)
The whole stress field is then best-fit (by weighted least squares) to both:

- data (at shallow depths),

World Stress Map
(480 FMs from larger earthquakes):

- and the dynamic model (everywhere).
Fortunately, the relative weighting of data vs. dynamics has little effect on residual misfits:

Effects of (WSM) Data-Weight (vs. CSM model-weight) on Misfits, with $W = 4$: 

- FINAL MODELS ($W = 5$)
- RMS angle vs. model
- mean angle vs. model
- RMS angle vs. data
- % bad regimes vs. model
- mean angle vs. data
- RMS shear stress vs. data
- mean shear stress vs. data
- % bad regimes vs. data

(worse) 
(better)
In practice, FlatMaxwell models are limited in spatial resolution (of tectonic stress) to no more than 6 wavelengths along each side of the model domain (including the depth axis).
Thus, FlatMaxwell models are quite smooth, and (at present) cannot accurately represent:
- peak shear stresses in the shallow upper crust; or
- discontinuous shear stresses at the Moho which are predicted by the Shells model.
Results to date show a low-amplitude stress anomaly, with peak shear stress of 120~150 MPa and peak vertically-integrated shear stress of $2.9\sim4.1 \times 10^{12}$ N/m.
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Channeling of deviatoric stress along the strong Peninsular Ranges and Great Valley is seen.
In southern California, deviatoric stress and long-term strain-rate are negatively correlated because regions of low heat-flow act as stress guides while deforming very little. In contrast, active faults lie in areas with higher heat-flow, and their low strength keeps deviatoric stresses locally modest.
Opportunities for future CSM advances include:

[1] Collecting more data (especially stress magnitudes from boreholes);
[2] Tuning the *Shells* dynamic model;
[3] Using a DIFFERENT dynamic model, with 3-D upper mantle:
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1. Collecting more data (especially stress magnitudes from boreholes);
2. Tuning the Shells dynamic model;
3. Using a DIFFERENT dynamic model, with 3-D upper mantle; and
4. Applying a similar Maxwell equilibrium filter to the stressing-rate models!