Earthquake stress drop estimates: What are they telling us?

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Lots of data for big earthquakes (rupture dimensions, slip history, etc.)

Small earthquakes are only observed from seismograms; no direct measurements of physical properties
Two parameters

displacement = \( D \)

area = \( A \)

Moment \( M_0 = \mu AD \)

Stress drop \( \Delta \sigma = \sigma_{\text{final}} - \sigma_{\text{initial}} \)
Circular crack model

\[ \Delta \sigma = \frac{7 \pi \mu D}{16 r} = \frac{7 M_0}{16 r^3} \]

\[ M_0 = \mu AD = \mu \pi r^2 D \]

Stress drop is proportional to displacement/radius ratio

(Eshelby, 1957; Brune, 1970)
In theory, far-field seismometer will record displacement pulse from small earthquake (can be either $P$ or $S$ wave), ignoring attenuation and other path effects.

Area under displacement pulse $f(h\tau)$ is related to seismic moment $M_0$ (one measure of event strength).

**Pulse width** $\tau$ is related to physical dimension of fault, rise time, and rupture velocity.
Spectral Analysis 101

Time Series

$u(t)$

$\tau$

$h$

FFT

Spectrum

"corner" frequency ($f_c \sim 2/\tau$) is measure of pulse width

$u(\omega)$

$\log(\omega)$

Zero frequency amplitude gives area under $u(t)$
How to get Brune-type stress drop

- Assume rupture velocity and source model
  \( (\text{Brune, Madariaga, Sato & Hirasawa, Kaneko & Shearer, etc.}) \)

- Estimate \( \Omega_0 \) and \( f_c \)

- Correct for geometrical spreading and radiation pattern

- \( \Delta\sigma = \frac{7 M_0}{16 r^3} \)

- Correct for attenuation

- \( \log(u(f)) \)

- \( \log(f) \)

- Original spectrum

- Assume circular crack model

- cubed!
General $\Delta\sigma$ results and issues

- $\Delta\sigma = 0.2$ to 20 MPa from corner frequency studies
- Much less than absolute shear stress levels predicted by Byerlee’s law and rock friction experiments
- Little dependence of average $\Delta\sigma$ on $M_0$, implying self-similar scaling of earthquakes, but possibility of small increase with $M_0$ has been debated
- Some evidence that plate-boundary earthquakes have lower $\Delta\sigma$ than mid-plate earthquakes
- Hard to compare $\Delta\sigma$ results among studies because they often use different modeling assumptions and are based on small numbers of earthquakes
UCSD/Caltech spectral analysis

- Online database of seismograms, 1984–2003
- > 300,000 earthquakes
- $P$ and $S$ multi-taper spectra computed for all records
- 60 GB in special binary format

Egill Hauksson
Isolating Spectral Contributions

\[ d_{ij} \approx e_i + s_j + x_{k(i,j)} + r_{ij} \text{ (residual)} \]
\[ d_{ij} \approx e_i + s_j + x_{k(i,j)} \]

- > 60,000 earthquakes, >350 stations
- 1.38 million \( P \)-wave spectra (STN > 5, 5-20 Hz)
- Iterative least squares approach with outlier suppression
Assumed source model

- Madariaga (1976), Abercrombie (1995)

We fit data (solid lines) between 2 and 20 Hz, using:

\[ u(f) = \frac{\Omega_0}{1 + (f/f_c)^n} \]

\[ f_c = \frac{0.42 \beta}{(M_0/\Delta\sigma)^{1/3}} \]

(assumes rupture velocity = 0.9 \( \beta \))

Model prediction (dashed lines) is for \( \Delta\sigma = 1.60 \text{ MPA (constant)} \)
Calculated Earthquake Stress Drops

- 65,070 events
- > 300,000 spectra
- 1989–2001
- > 4 spectra/event
- 5 - 20 Hz band

Red = fewer high frequencies, lower stress drop or high near-source attenuation

Blue = more high frequencies, higher stress drop or low near-source attenuation
Empirical Green’s Function (EGF)

Subtract small event from big event to get estimate of true source spectrum for big event.
Source-specific EGF method

For each event, find 500 neighboring events:

Fit moment binned spectra to $\Delta \sigma$ and EGF

Then subtract EGF from target event spectrum and compute $\Delta \sigma$ for this event
Observed source $\Delta \sigma$ using spatially varying EGF method

Previous result using constant EGF method

New results
How variable are earthquake stress drops?

- Harder to resolve high $\Delta\sigma$ events due to high corner frequencies
- Results are more reliable when more stations are stacked
- $\Delta\sigma = 0.2$ to 20 Mpa
- $\sim$10x local scatter
- $\sim$10x regional variations
Earthquake scaling

uniform scaling of all parameters (self similarity)

Constant $\Delta \sigma$

or

different scaling of parameters

Variable $\Delta \sigma$

Small Earthquake

Big Earthquake
Median stress drop does not vary with $M_w$
Stress drop versus depth

- Average $\Delta \sigma$ increases from 0.6 to 2 MPa from 0 to 8 km
- But slower rupture velocities at shallow depths could also explain trend
- Nearly constant from 8 to 18 km
- Large scatter at all depths
Stress drop versus type of faulting

3895 high-quality focal mechanisms from J. Hardebeck (2005)
Landers Aftershocks

- Along-strike changes in $\Delta \sigma$
- Related to mainshock slip?

Profiles for slip model of Wald & Heaton (1994)
Comparison to Landers Slip Model

Red = low $\Delta\sigma$
Blue = high $\Delta\sigma$

Slip model from Wald & Heaton (1994)
Landers Slip Models

Cohee & Beroza (1991)

Hernandez (1999)

Cotton & Campillo (1991)

Zeng & Anderson (1999)

Wald & Heaton (1994)

Aftershock stress drops

from www.seismo.ethz.ch/srcmod/
Average $\Delta\sigma$ (smoothed over 500 events)

- 0.5 to 5 MPa
- Coherent patterns
- What does it mean?
- Does this say anything about absolute stress?
Conclusions for Southern California

- Stress drops range from 0.2 to 20 MPa for $M_L = 1$ to 3.4 earthquakes, with no dependence on moment.
- Spatially coherent patterns in average stress drop (0.5 to 5 MPa), no consistent decrease near active faults.
- Shallow earthquakes radiate less high frequencies than deeper events, implying slower rupture velocities or lower stress drops.
- Landers aftershocks have strong along-strike variations in stress drop with possible correlation to slip models.
- Hard to resolve any temporal changes.
1989-2001 $b$-values

- Computed for each event and 500 nearest neighbors
- $M = 2$ to 4
- median $b = 1.12$
not much correlation!