Earthquake Source Scaling

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Earthquake Self-Similarity

Matryoshka!
Static measures of earthquakes

- Static stress drop $\Delta \sigma = \text{difference between the stress on the fault before and after the earthquake.}$

\[\Delta \sigma \propto \mu \frac{D}{W}\]

Geometry of the fault dictates strain

At 15 km depth, lithostatic pressure is about 400MPa.

$D = 1\text{m}, W = 10\text{ km}, \mu = 1\text{E}10 \text{ Pa}, \Delta \sigma = 1\text{MPa}.$

Are stress drops similar for small earthquakes and for large earthquakes?
Dynamic measures of earthquakes

• Kinetic energy carried by seismic waves $E_R = \rho c \int V^2(t) dt$ (strong ground motion)
• Fast/high stress drop earthquakes radiated more than slow/low stress drop earthquakes.
• $E_R/M_0$ is scaled energy

Do large earthquakes radiate, proportional to their size, more seismic waves?

Yes ?

No ?

$E_R/M_0$

$G \propto (\Delta \sigma/2 - \mu E_R/M_0)S$

Baltay et al. (2014)

Viesca and Garagash (2015)
Seismic Observations

Far-field displacement seismogram

\[ S(t) \sim M_0 \]

Source spectrum

\[ |\hat{S}(f)| \sim M_0 \]

\[ f_c \propto \frac{1}{T} \]

\[ \log_{10} f \]

\[ M_0 = \mu DA \]

\( \mu \) rigidity (\( \sim 10 \) GPa)

\( D \) displacement

\( A \) fault area (WL?)

\[ L \propto T, W \propto T, D \propto T \]

\[ M_0 \propto T^3 \]

\[ M_0 \propto f_c^{-3} \]
Earthquake scaling

\[ A = \text{area} \]
\[ M_0 = \mu DA \]

\[ A^* = b^2 A \]
\[ M_0^* = b^3 M_0 \]

From Prieto et al, 2004
Earthquake scaling

From Prieto et al, 2004
Earthquake scaling

From Prieto et al., 2004
Measures of stress drop and radiated energy

\[ S_b(f) \sim M_0 \]

\[ L = V_r T = \frac{V_r}{f_c} \]

\[ M_0 = \mu D L^2 \]

\[ \Delta \sigma = \frac{M_0}{L^3} = \frac{M_0}{V_r^3} f_c^3 \]

\[ E_R = \rho c \int |\hat{V}(f)|^2 \, df \]
Earthquake Self-Similarity

Global scale: $M_{5.5-7.1}$
- Constant $\Delta\sigma$
- Constant $E_R/M_0$

Local scale: $M_{0.5-3.4}$

Prieto et al. (2004)

Allmann and Shearer (2009)
Comprehensive view: independent data sets, independent methods

$M_w$

$100$ MPa

$0.1$ MPa

$f_c^{-3}$

Allmann and Shearer (2009)
Main question: are earthquakes self-similar?

1. Source duration $T$ follows a scaling $M_0 \sim T^3$

2. Source spectrum shape is identical (if shifted)

Conclusion for today: NO
Global catalog of M5.5+ since 1990

800+ M<7
126 M>7
15 M>8
Can we do better?
\[ S(f) = \sqrt{\frac{M_0}{1 + \left(\frac{f}{f_1}\right)^2}} \sqrt{1 + \left(\frac{f}{f_2}\right)^2} \]
Best-fit global source models from P waves

Variable spectral shapes breaks self-similarity
Best-fit global source models from P waves

What is $T_1$?
$T_1$ source duration
(see from P waves)

Seismic Moment (Nm)

Time (s)

Magnitude

$M_0 \sim T$

$M_0 \sim T^2$

$M_0 \sim T^3$
source dimension

Fault dimension \( L \propto T \)

Seismic Moment (Nm)

Source Width (km)

Length

Source dimension (km)

Fault dimension

Seismic Moment (Nm)
**source dimension**

Width dimension \( W \sim L^{2/3} \) (Leonard, 2010)

Fault dimension \( L \propto T \)

Geometry breaks self-similarity

\[ L \propto T \]

\[ W \sim L^{2/3} \]
Stress drop with moment

\[
\Delta \sigma (\text{Pa})
\]

(a) Stress drop with moment

(b) Stress drop with depth

Seismic Moment (Nm)

Magnitude

Earthquake depth (km)

Median is 1.9MPa.

\[\text{M9 2011 Tohoku } \Delta \sigma \sim 8\text{MPa (similar to Brown et al., 2015)}\]
\[\text{M9 2004 Sumatra } \Delta \sigma \sim 0.1\text{MPa}\]
\[\text{M8.5 2010 Maule, M8.7 2005 Nias } \Delta \sigma \sim 1\text{MPa}\]
What is $T_2$?
What is the other time scale, $T_2$?

$T_2$ varies between 0.5s and 5s

$T_2 \propto W \Rightarrow$ too short

what we know
1. $T_2$ varies with moment.

How does $T_2$ affect radiated energy?

(1) Adjusted source duration $T'_1$

(2) Adjusted short time scale $T'_2$

(3) Slip-weakening source time function

$T_1 \propto M^{1/5}$

$T_2 \propto M^{1/5}$
Radiated energy

\[ E_R = \int |fS(f)|^2 df \]

\[ E_R > E_R \]
Radiated energy with moment

$$\frac{E_R}{M_0} = 1.5 \times 10^{-5} \text{J/N/m.}$$

$$\tau_a = \mu \frac{E_R}{M_0}$$

$$\mu = 39 \text{GPa}$$

Median is

$$\tau_a = 0.6 \text{MPa}$$
Earthquake Self-Similarity

1. Source duration $T$ follows a scaling $M_0 \sim T^3$ until the fault reaches a natural boundary (free surface – brittle-ductile transition zone)

2. Source spectrum shape is NOT identical (if shifted). It has an intermediate slope that increases radiation

Conclusion for today

$\Delta \sigma$ and $E_R/M_0$ are invariant (or weakly variant) with earthquake size

BUT

This does not imply self-similarity

Outstanding questions

1. What is $T_2$ ? Pulse-like ruptures or dynamic weakening?
2. Do strike slip and normal faults exhibit similar spectral shape?