Seismic Source Inversion and Back Projection: (1) Introduction of uncertainty of Green's function into waveform inversion for seismic source processes

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Fukahata, Y. (DPRI, Kyoto University)

To Estimate the seismic source process:

Yagi & Fukahata (2011, GJI)

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We calculate a Green's function under the assumption of a simplified earth structure model.

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Our Solution: Introducing "Uncertainty of a Green's function" into waveform inversion.

Yagi & Fukahata (2011, GJI)

Observation Equation

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$$\mathbf{u}(\mathbf{x},t) = \sum_{q=1}^{2} \int_{\Sigma} \mathbf{G}_q(\mathbf{x},\boldsymbol{\xi},t) \; * \; \dot{D}_q(\boldsymbol{\xi},t) \; dS + F(t) * \mathbf{e}_0(t)$$

Observation Equation

Seismic wave

Slip-rate

$$\mathbf{u}(\mathbf{x},t) = \sum_{q=1}^{2} \int_{\Sigma} \mathbf{G}_q(\mathbf{x},\boldsymbol{\xi},t) \; * \; \dot{D}_q(\boldsymbol{\xi},t) \; dS + F(t) * \mathbf{e}_0(t)$$

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Green's function

Uncertainty of Green's function

$$\mathbf{G}_q(\mathbf{x},\boldsymbol{\xi},t) = F(t) * Q(t) * \left[\hat{\mathbf{g}}_q(\mathbf{x},\boldsymbol{\xi},t) + \delta\mathbf{g}_q(\mathbf{x},\boldsymbol{\xi},t) \right]$$

Observation Equation

Seismic wave

Slip-rate

$$\mathbf{u}(\mathbf{x},t)=\sum_{q=1}^{2}\int_{\Sigma}\mathbf{G}_q(\mathbf{x},\boldsymbol{\xi},t)\ *\ \dot{D}_q(\boldsymbol{\xi},t)\ dS+F(t)*\mathbf{e}_0(t)$$

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Error term of Green's function

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Error term of Green's function

$$\sum_{q=1}^{2}\sum_{k=1}^{K}\dot{D}_q(\boldsymbol{\xi}_{\mathrm{k}},t)\ *\ \delta\mathbf{g}'_{qk}(\mathbf{x},t)\ *F(t)*Q(t)$$

Point of our formulation 1

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If we introduce uncertainty of Green's function into Kernel matrix.

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$$
\begin{aligned}
\mathbf{d} &= (\mathbf{H} + \delta\mathbf{H})(\mathbf{a} + \delta\mathbf{a}) + \mathbf{F}\mathbf{e}_0 \\
&= \mathbf{H}\mathbf{a} + \delta\mathbf{H}\mathbf{a} + \mathbf{H}\delta\mathbf{a} + \delta\mathbf{H}\delta\mathbf{a} + \mathbf{F}\mathbf{e}_0 \\
&\simeq \mathbf{H}\mathbf{a} + \delta\mathbf{H}\mathbf{a} + \mathbf{F}\mathbf{e}_0
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Impossible?

We divide term of uncertainty of Green's function into slip-rate function (matrix) and Gaussian error term (vector)

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Possible!

Covariance matrix $\mathbf{C}_d\left(\mathbf{a}, \sigma_g^2, \chi^2\right) = \sigma_g^2\left[\mathbf{P}(\mathbf{a})\mathbf{P}^t(\mathbf{a}) + \chi^2\mathbf{F}\mathbf{F}^t\right]$

Point of our formulation II

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Two hyper parameters: smoothing and modeling error

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We can estimate hyper parameter using Akaike Bayesian Information Criteria (ABIC).

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\begin{aligned}
ABIC\left(\alpha^2,\gamma^2\right) = N\log s\left(\mathbf{a}^*\right) - \log\alpha^2\left|\mathbf{G}_1+\chi^2\mathbf{G}_2\right| \\
+\log\left|\mathbf{H}^T\mathbf{C}_d^{-1}\left(\alpha^2,\mathbf{a}^i\right)\mathbf{H}+\alpha^2\left(\mathbf{G}_1+\chi^2\mathbf{G}_2\right)\right| \\
+\log\left|\mathbf{C}_d^{-1}\left(\alpha^2,\mathbf{a}^i\right)\right| + C
\end{aligned}
$$

Importance of Error Structure

Yagi & Fukahata (2011, GRL)

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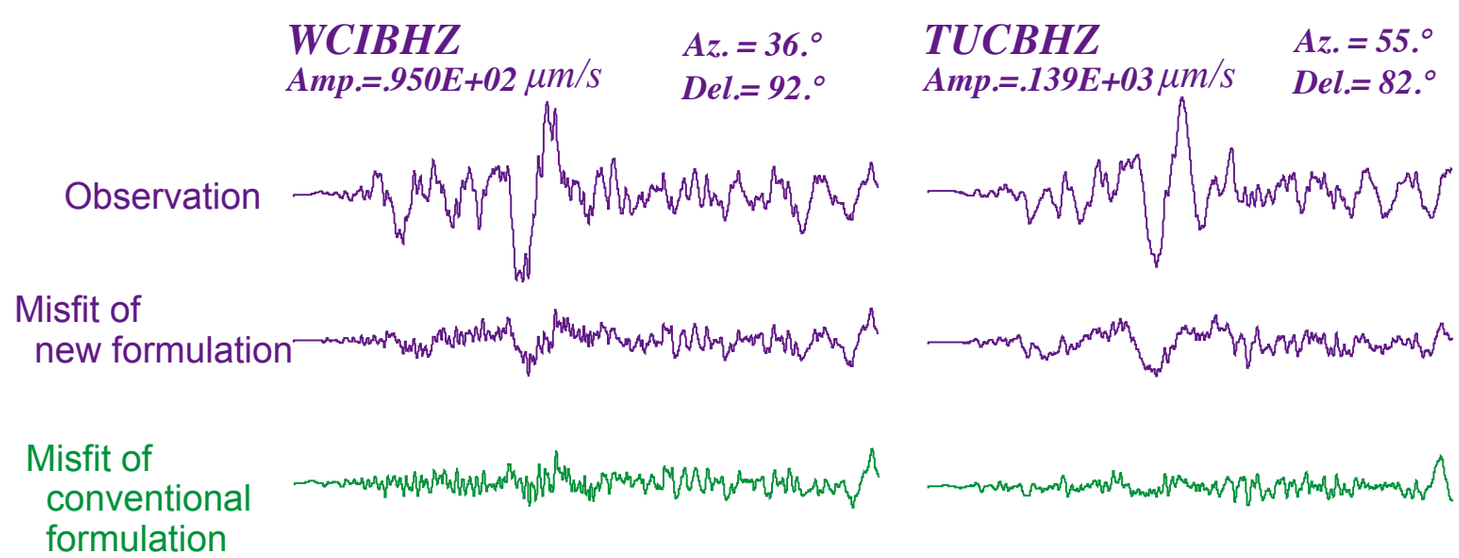

Large L2 norm

Small L2 norm

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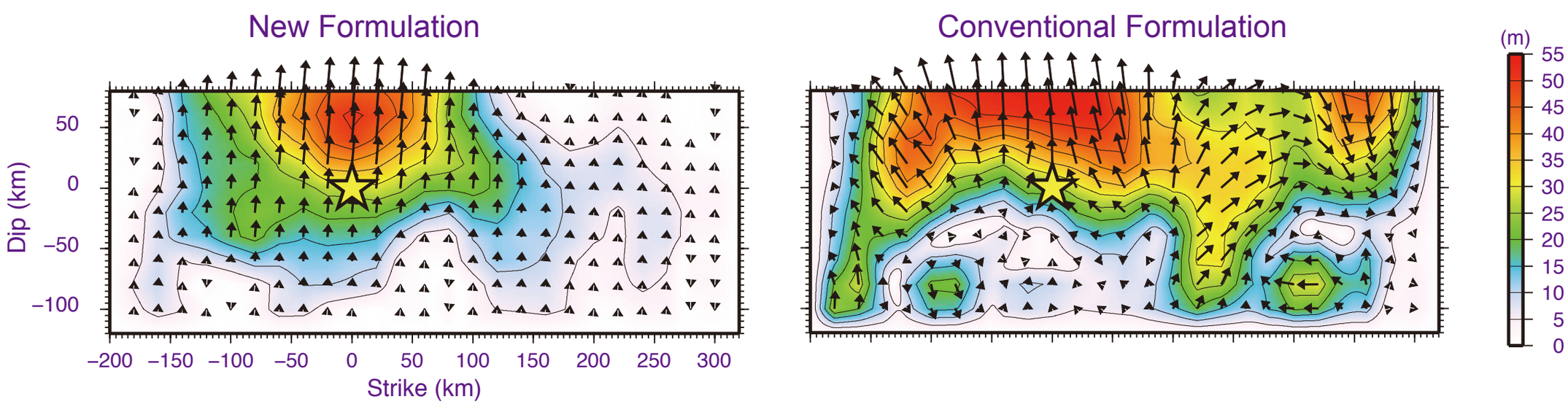
2011 Tohoku-oki Earthquake



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Spin-off

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We can get a plausible solution without the Non-Negative constraint.

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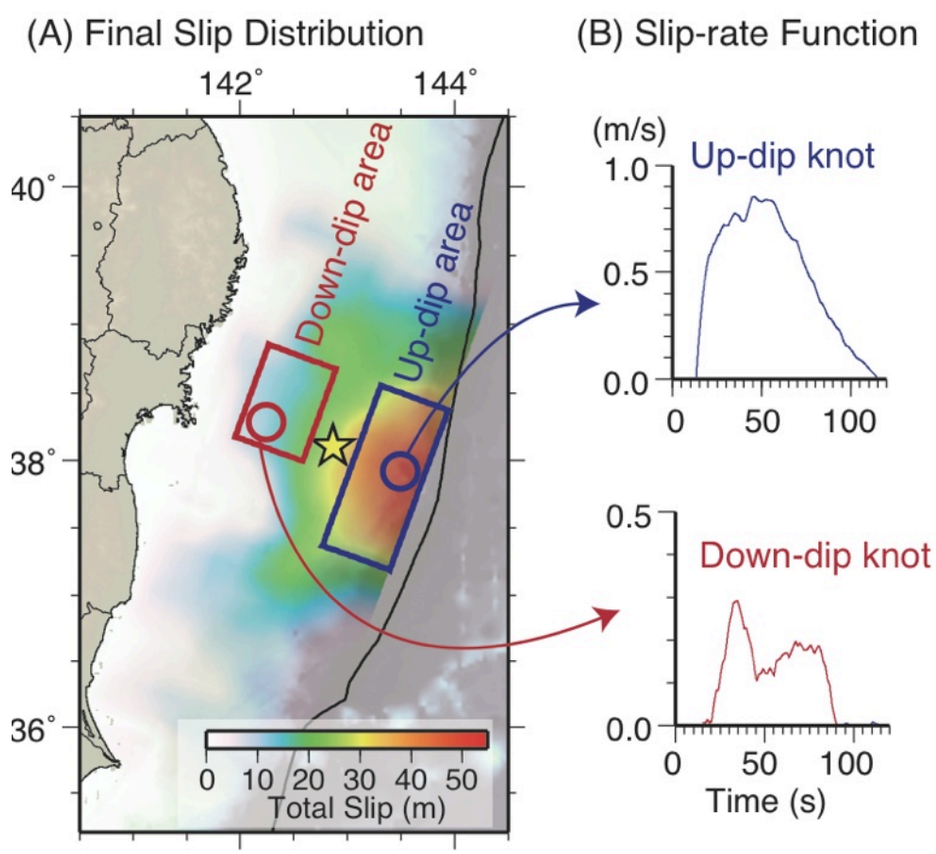


Number of model parameters:
over 40,000

Yagi et al., (2012, EPSL)

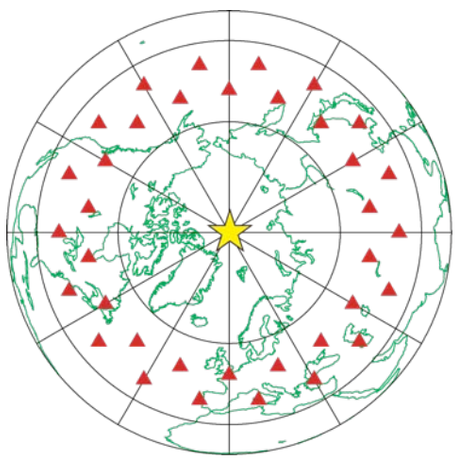
Numerical Test

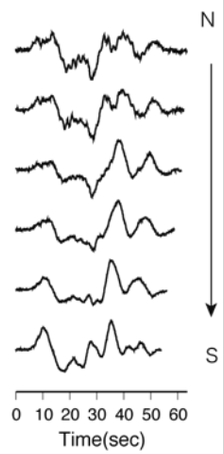
Numerical Test

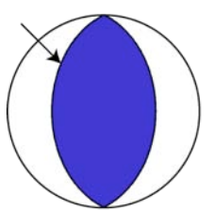
INPUT

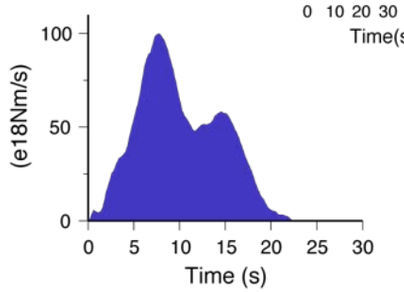
$$\sigma_g = 2\%$$

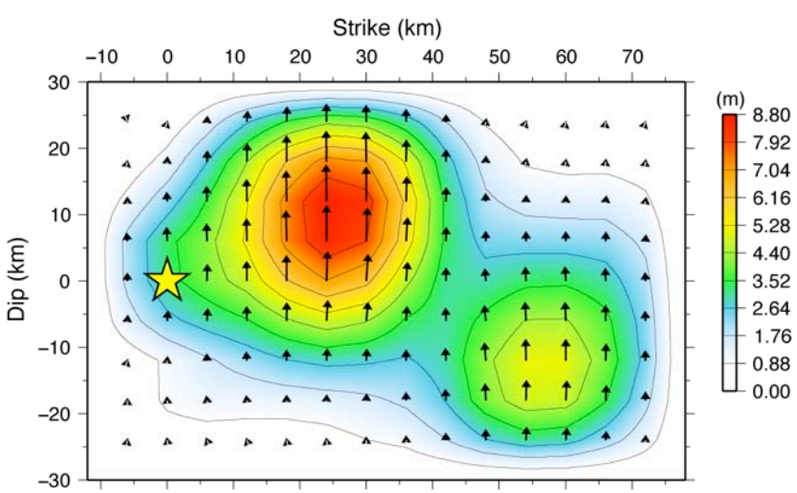
$$\sigma_0 = 3.8\ \mu m$$











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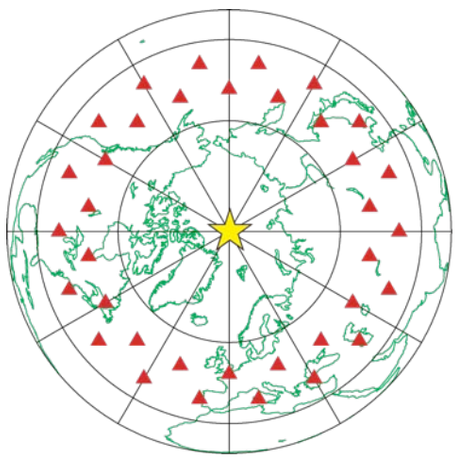
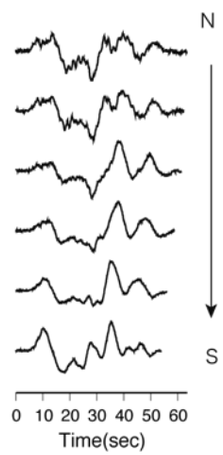
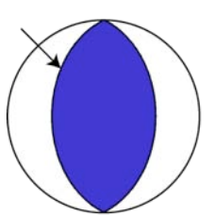
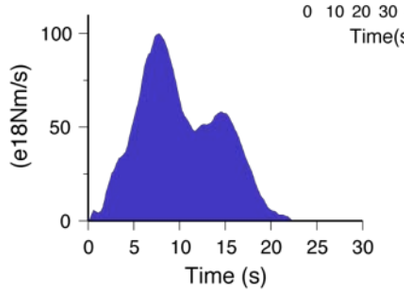
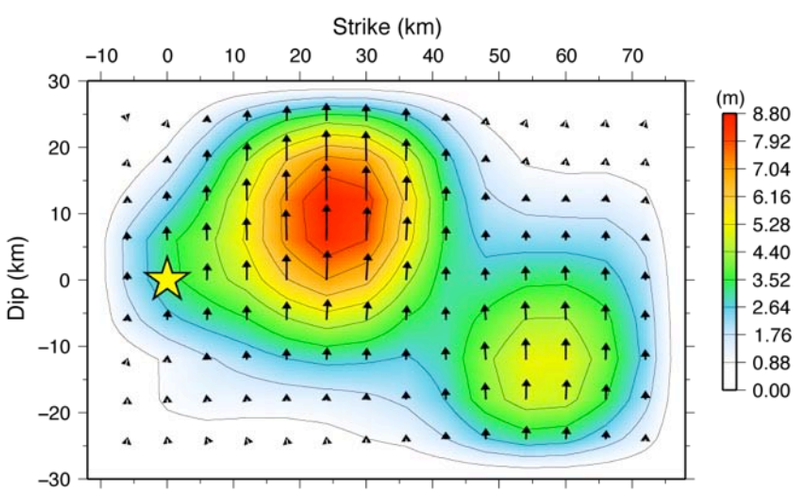






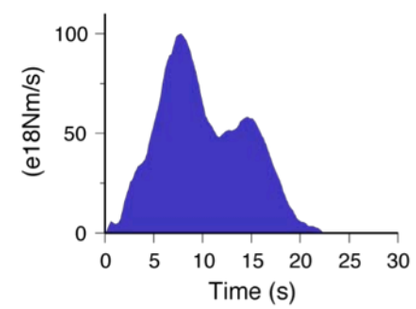


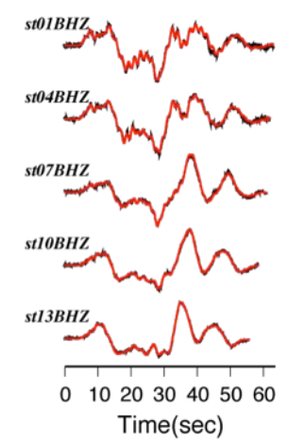


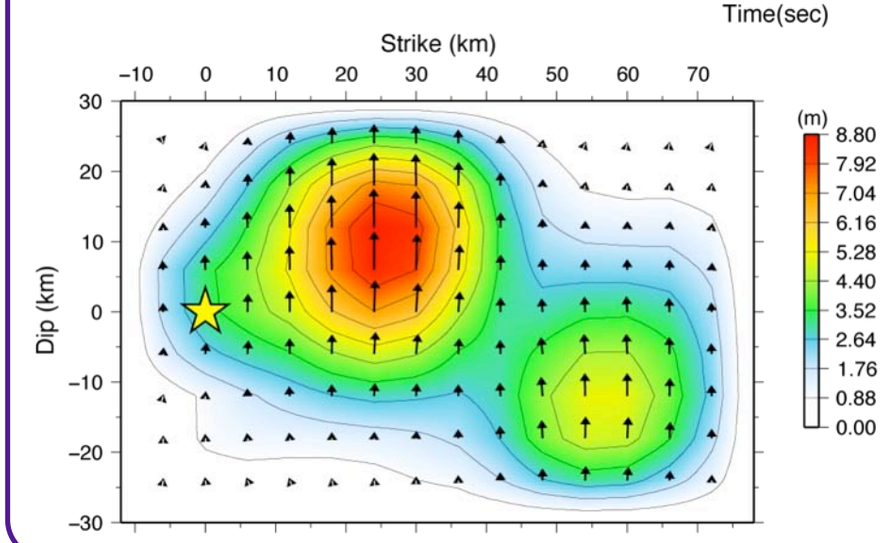
OUTPUT

$\sigma_g = 1.9\%$

$\sigma_0 = 3.7\ \mu m$







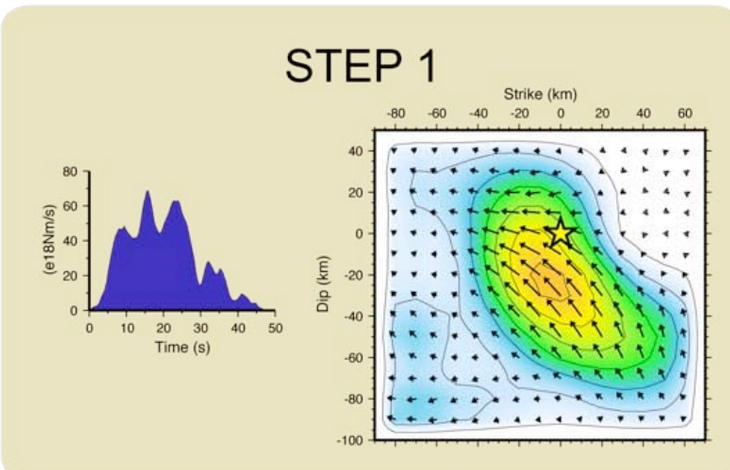
Application to real data set

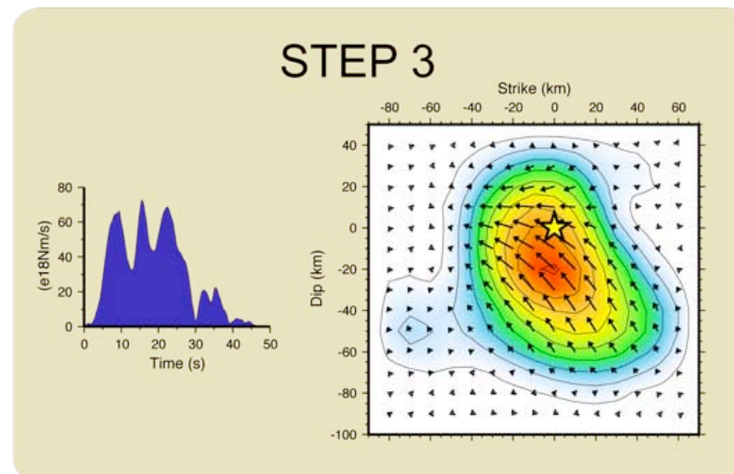
Check Point:

1. Convergence
2. Dependence on initial value
3. Sampling and resolution
4. Critical Case

1. Convergence

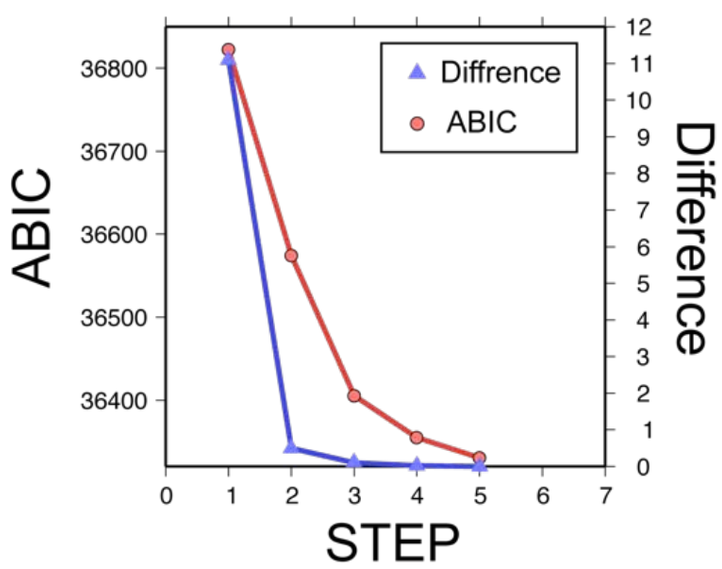
2006 Tonga Earthquake

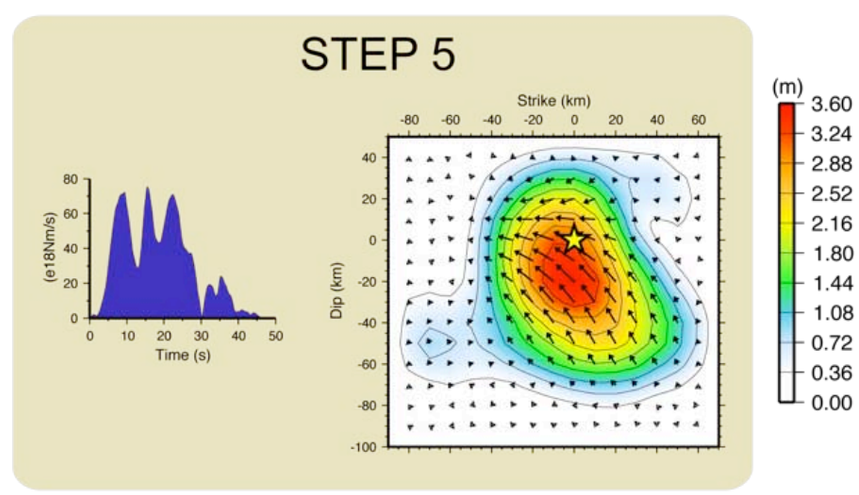




$$Diff = \frac{\sum_m \left(a_m^k - a_m^{k-1}\right)^2}{\sum_m \left(a_m^{k-1}\right)^2}$$

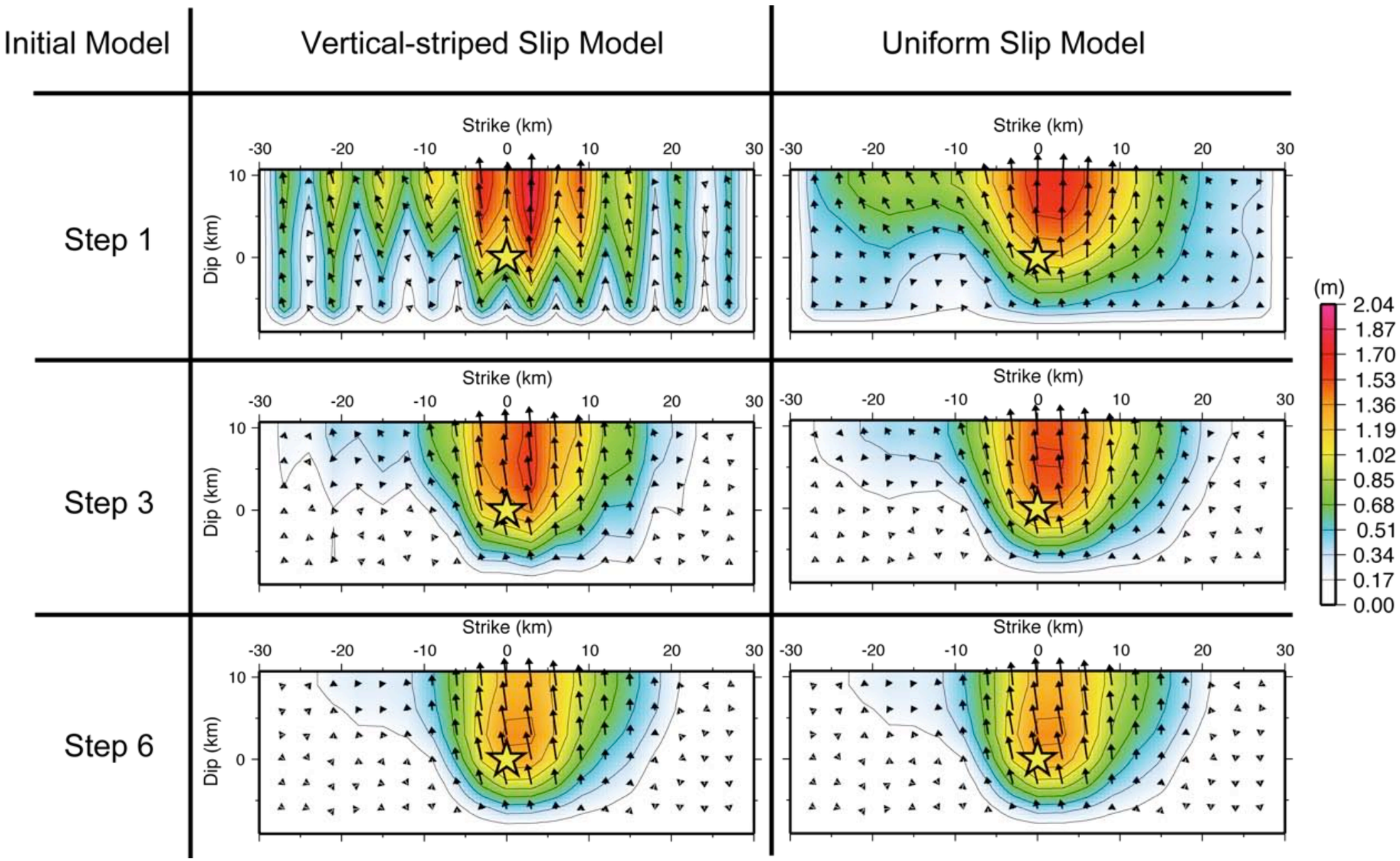
k : step number





2. Dependence on Initial Value

2008 Iwage-Miyagi, Japan earthquake (Mw 6.9)



3. Sampling interval and resolution

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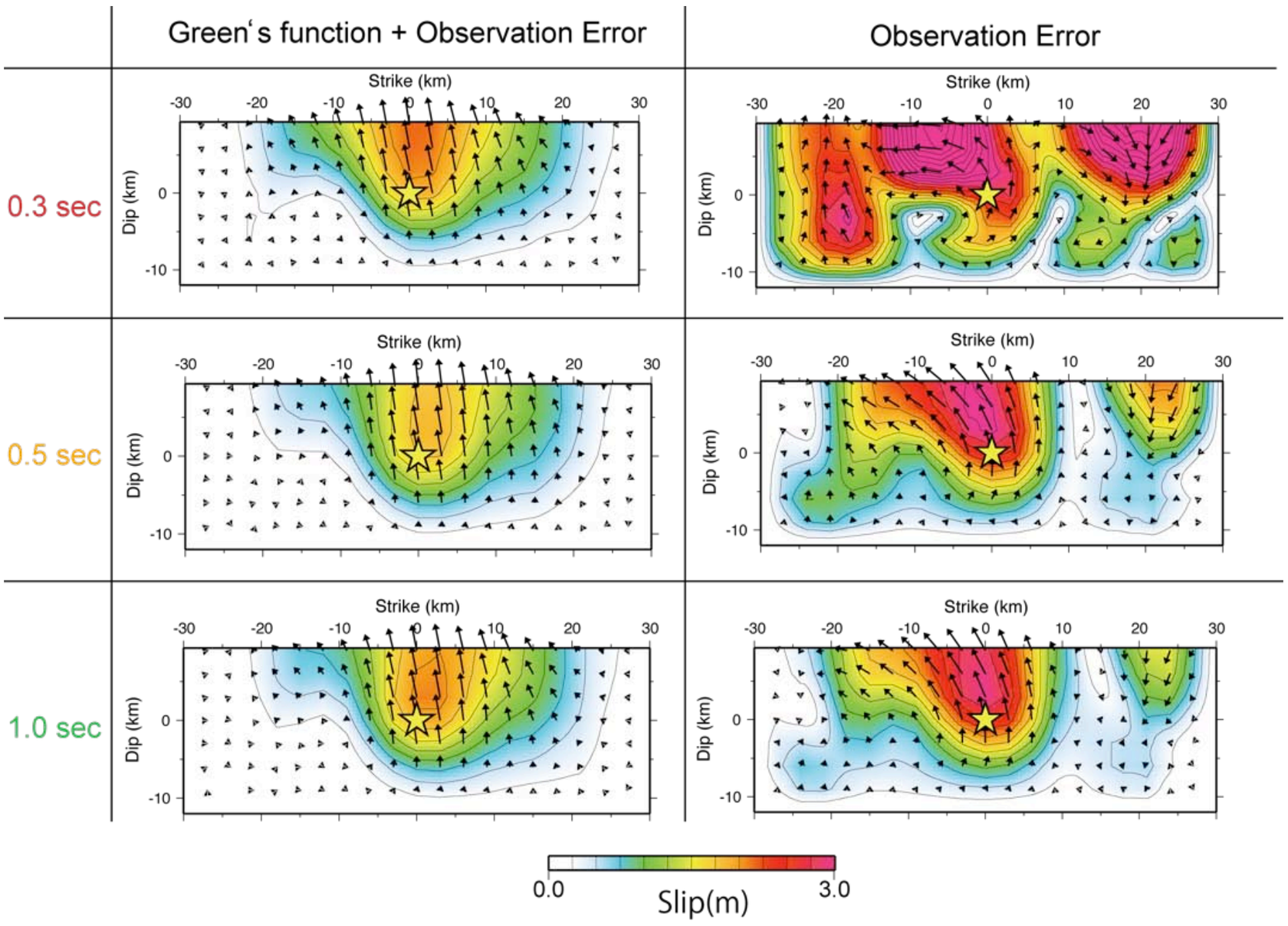

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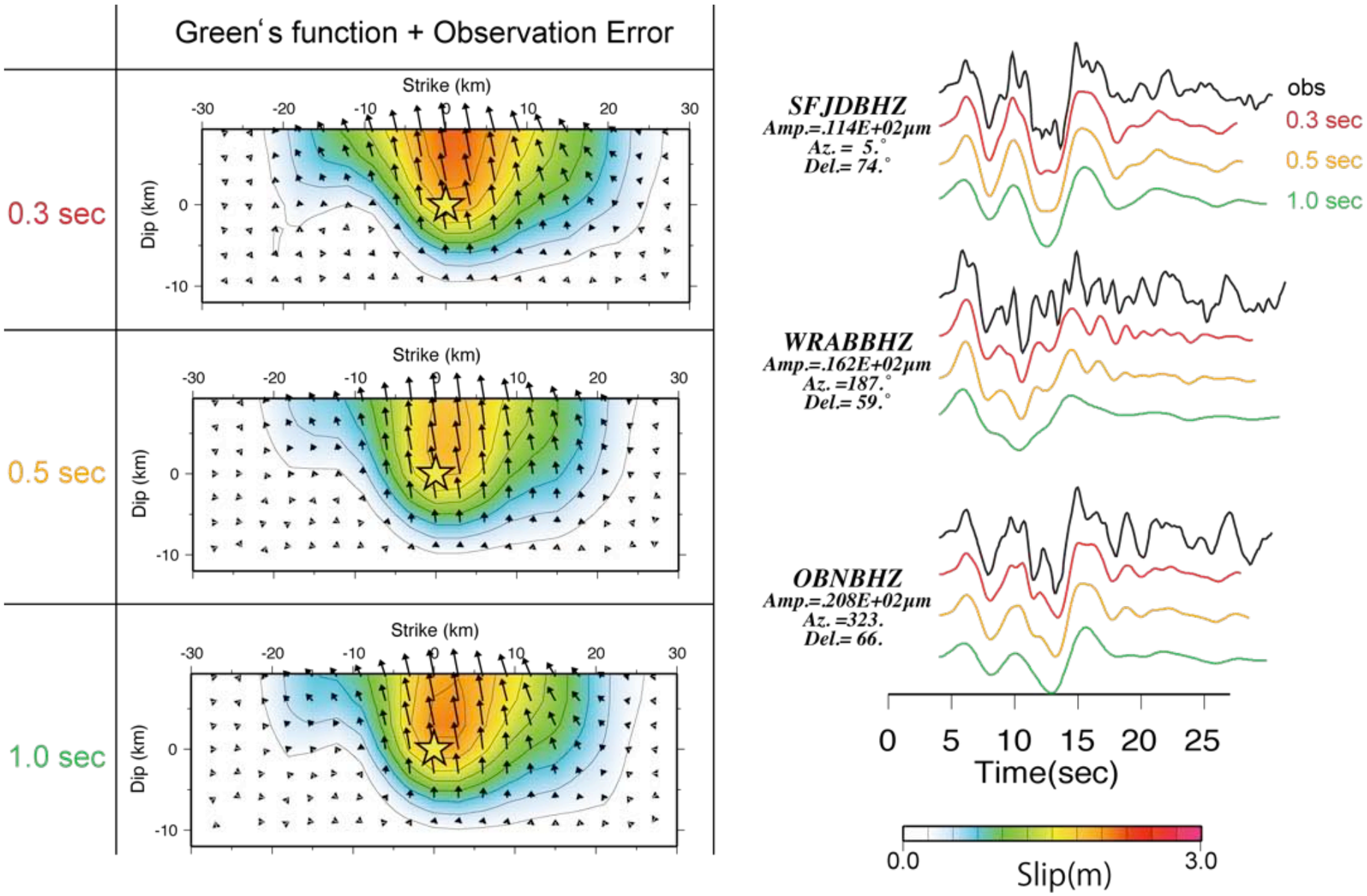
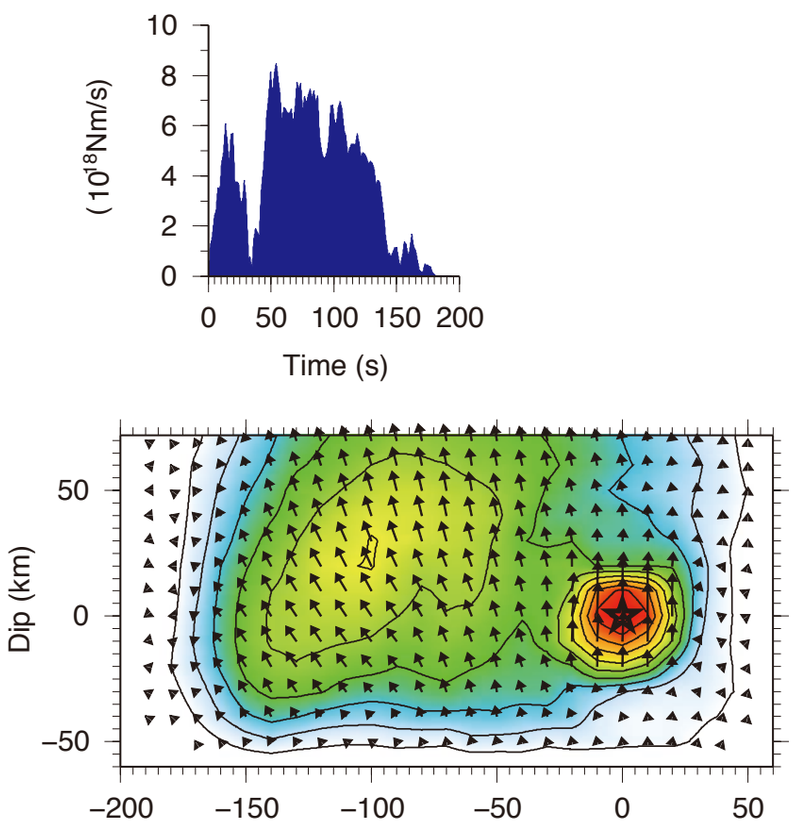


4. Critical Case

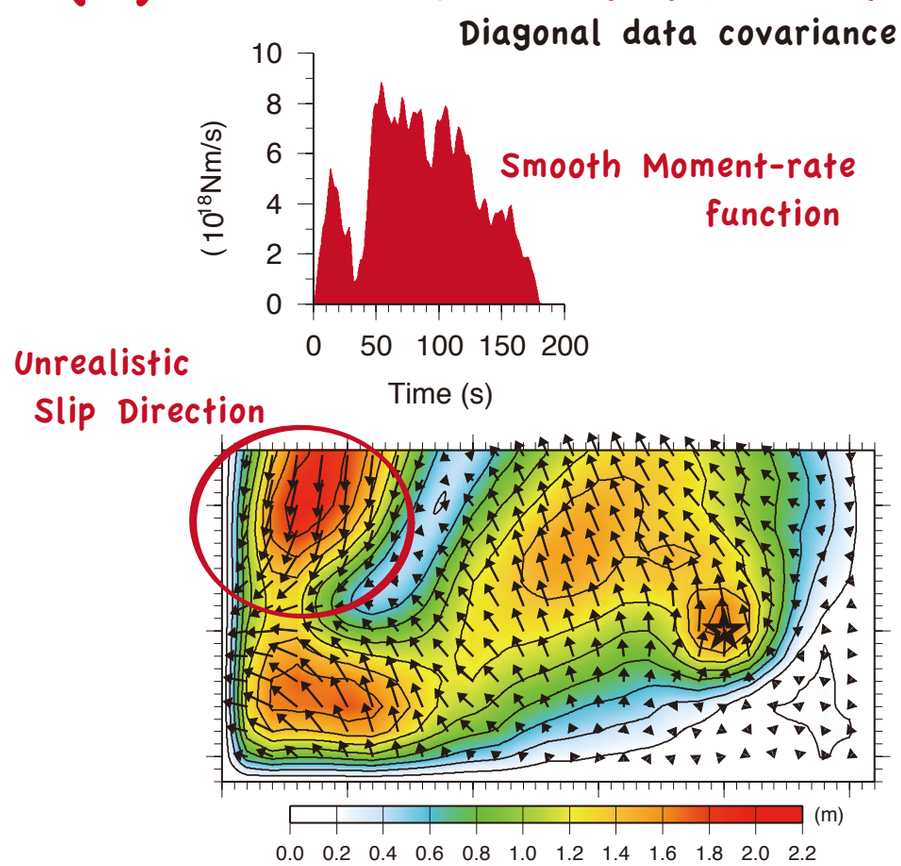
Long source time function

E.g.) 2006 Java Tsunami earthquake

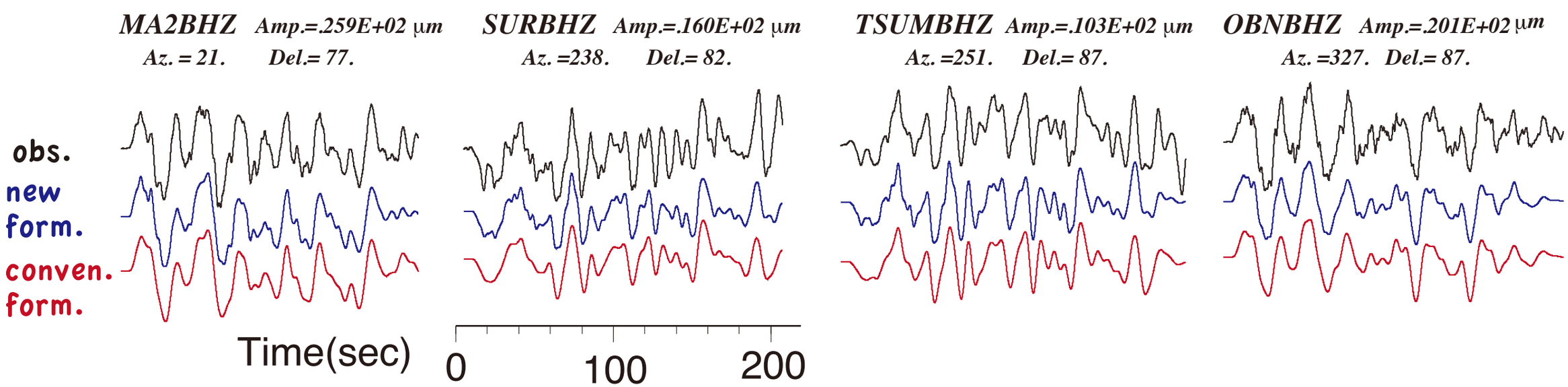
(a) New formulation



(b) Conventional formulation



Observation and Calculation



L2 norm	New formulation	conventional formulation
high freq.	large	small
	good fitting	no good fitting

Conclusion I

We introduced the uncertainty of Green's function into waveform inversion analyses.

It well work for real tele-seismic data set.

We can get a plausible solution without the Non-Negative constraint. (=> high resolution)

L2-norm is not good criterion of solution.

It's critical in large earthquake analysis such 2011 Tohoku-oki earthquake.

Seismic Source Inversion and Back Projection:
(2) Theoretical relationship between back-projection imaging and inverse solutions

Fukahata, Yagi & Rivera (submitted to GJI)
Yagi et al. (2012, EPSL)

Theoretical Background of BP

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

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Basic Equation of the BP

$$s_l^{BP}(t) = \sum_j c_j \dot{d}_j \left(t + t^P{}_{lj}\right)$$

$$= \sum_j c_j \sum_i \left(\ddot{a}_i * G_{ij}\right)\left(t + t^P{}_{lj}\right)$$

d_j : displacement for station "j"

$t^P{}_{lj}$: Travel time from source grid "l" to station "j"

a_i : Slip function on grid "l"

G_{ij} : Green's function from grid "l" to station "j"

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Important assumption in BP:
Waveform due to slips on the grid except for the grid "l" are cancelled out earth other (Ishii et al., 2005).

$$s_l^{BP}(t) \approx \sum_j c_j \left(\ddot{a}_l * G_{lj}\right)\left(t + t^P{}_{lj}\right) \quad \text{or} \quad \approx \sum_j c_j \left(\dot{a}_l * \dot{G}_{lj}\right)\left(t + t^P{}_{lj}\right)$$

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Green's function is assumed to be like the delta function, since we only use predicted travel time in BP.

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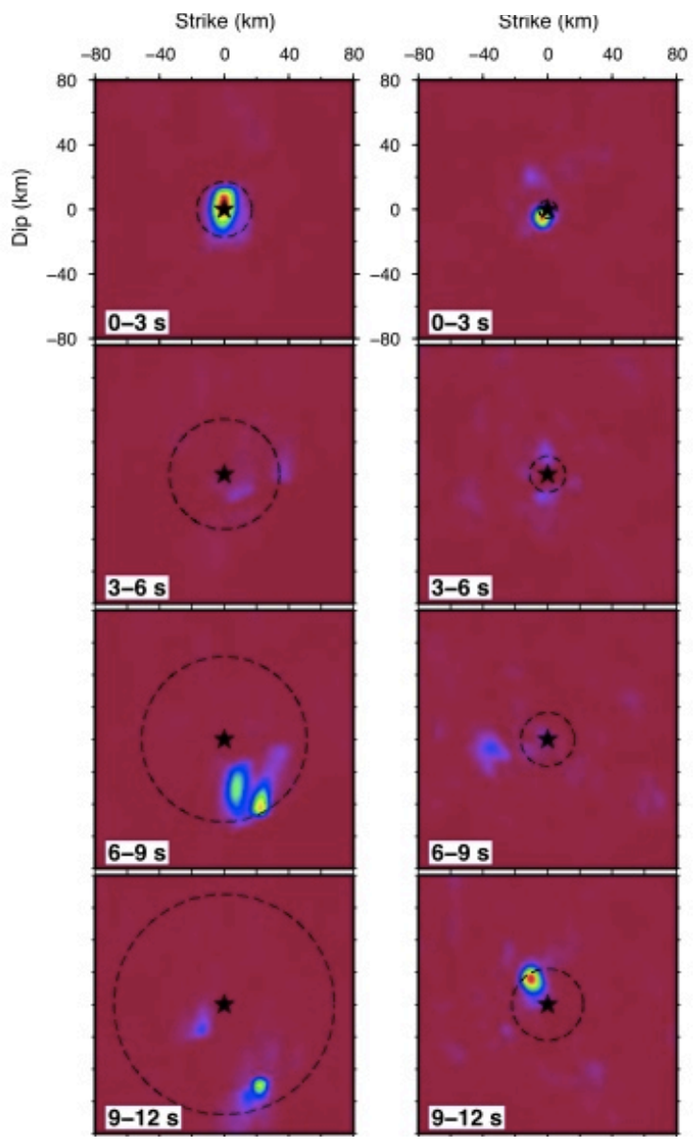
BP is directly related not to seismic energy release but to slip-acceleration (or slip-rate) on fault plane!

~ Yao et al. (2012 GJI)

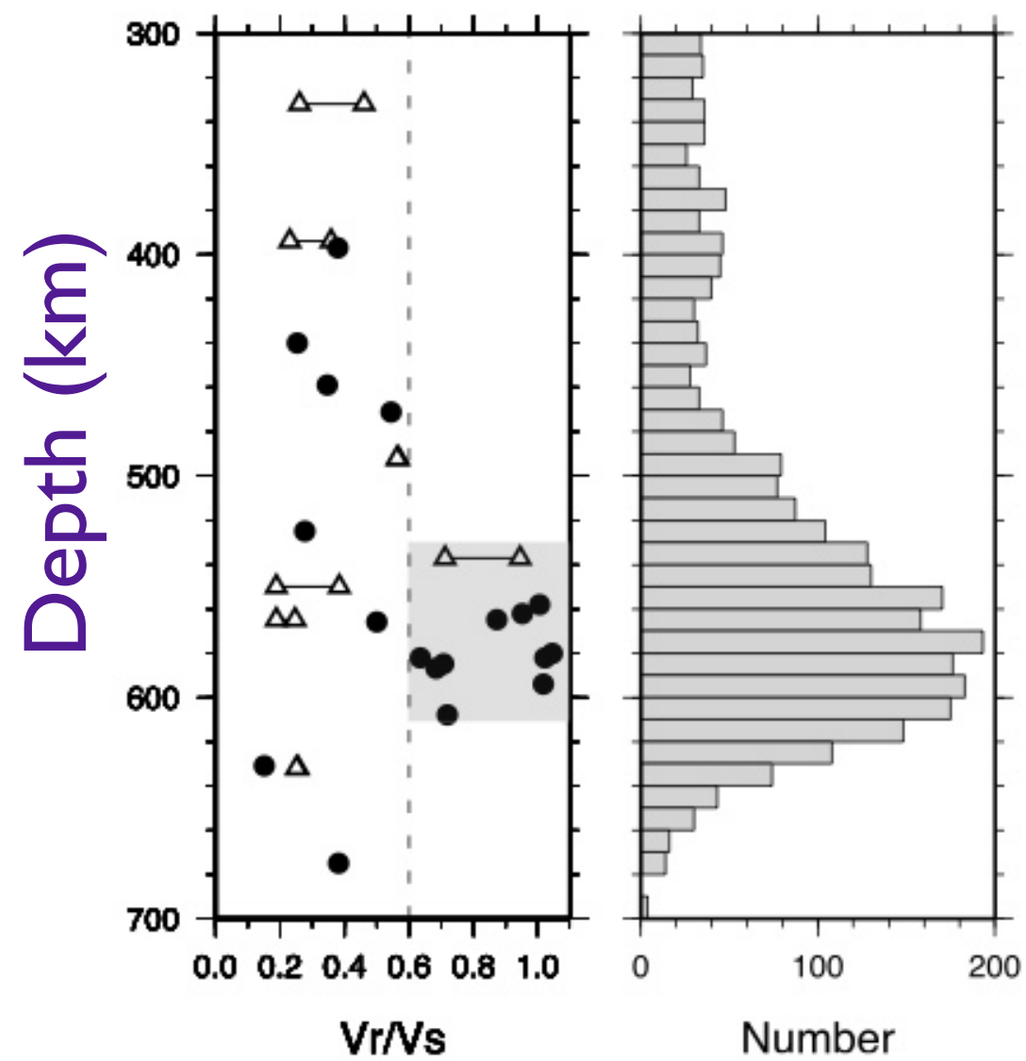
BP well work for deep earthquake

Green's function can be assumed to be like the delta function (Suzuki & Yagi, 2011, GRL)

Fast Slow



Rupture velocity



Hybrid Back-Projection (HBP)

By Yagi; Nakao; Kasahara (2012, EPSL)

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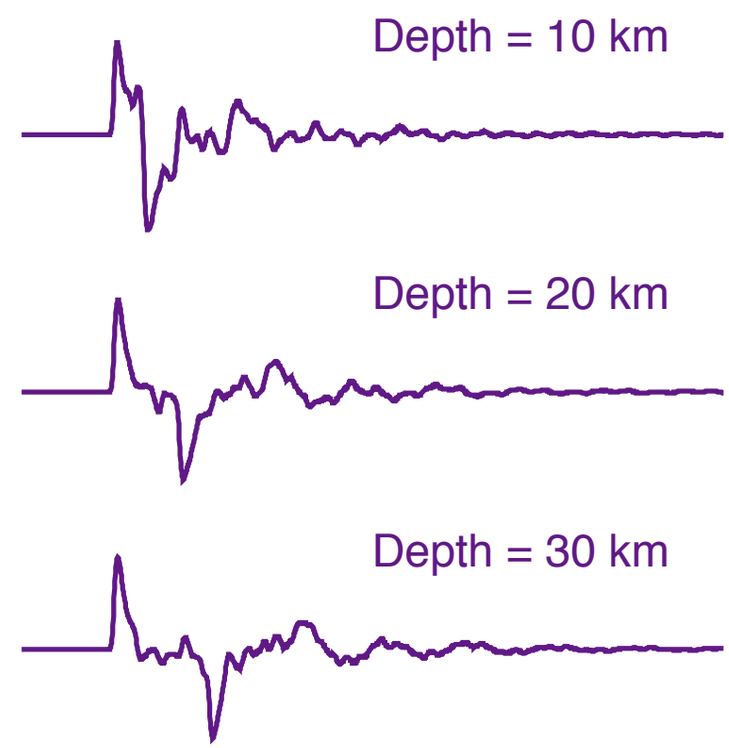

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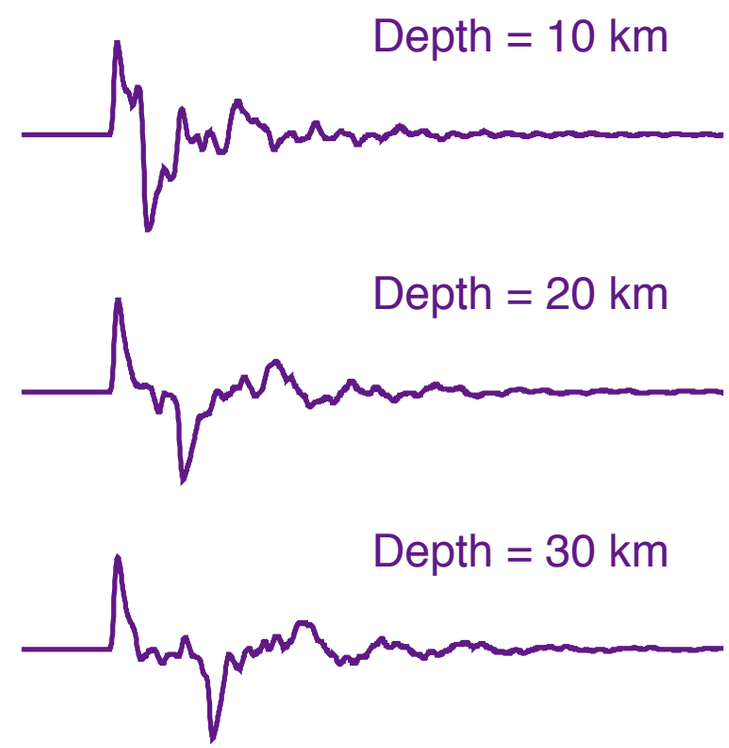

Solution:
Cross-correlation of an observed waveform with the theoretical Green's function

Theoretical Background of HBP

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

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Basic Equation of the HBP

$$s_l^{HBP}(t) = \sum_j c_j \left(\dot{u}_j \,\hat{*}\, \dot{G}_{lj} \right)(t) = \sum_j c_j \sum_i \left[\left(\dot{a}_i * \dot{G}_{ij} \right) \hat{*}\, \dot{G}_{lj} \right](t)$$

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Important assumption in HBP:

Waveform due to slips on the grid except for the grid "*l*" are cancelled out earth other.

$$s_l^{HBP}(t) \approx \dot{a}_l(t) * \sum_j c_j \left(\dot{G}_{lj} \hat{*} \dot{G}_{lj} \right)(t)$$

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Auto correlation of greens function is assumed to be the delta function.

$$s_l^{HBP}(t) \sim \sum_j c_j \, \dot{a}_l(t)$$

In Vector Form

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

In Vector Form

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Relation of data to slip

$$\dot{d}_j = \sum_i \left(\dot{G}_{ij} * \dot{a}_i \right)(t) \qquad \mathbf{d}_j = \mathbf{G}_j \mathbf{a} \qquad \boxed{\mathbf{d} = \mathbf{G}\mathbf{a}}$$

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Inverse Solutions

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

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Model Resolution Matrix

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

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In terms of model resolution matrix,
BP and HBP are inferior to LSS.

HBP and Damped LSS

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In BP, the Green's function in "$\mathbf{G}$" is further approximated by the delta function.

Simple Numerical Examples

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Condition: $\mathbf{d} = \mathbf{G}$

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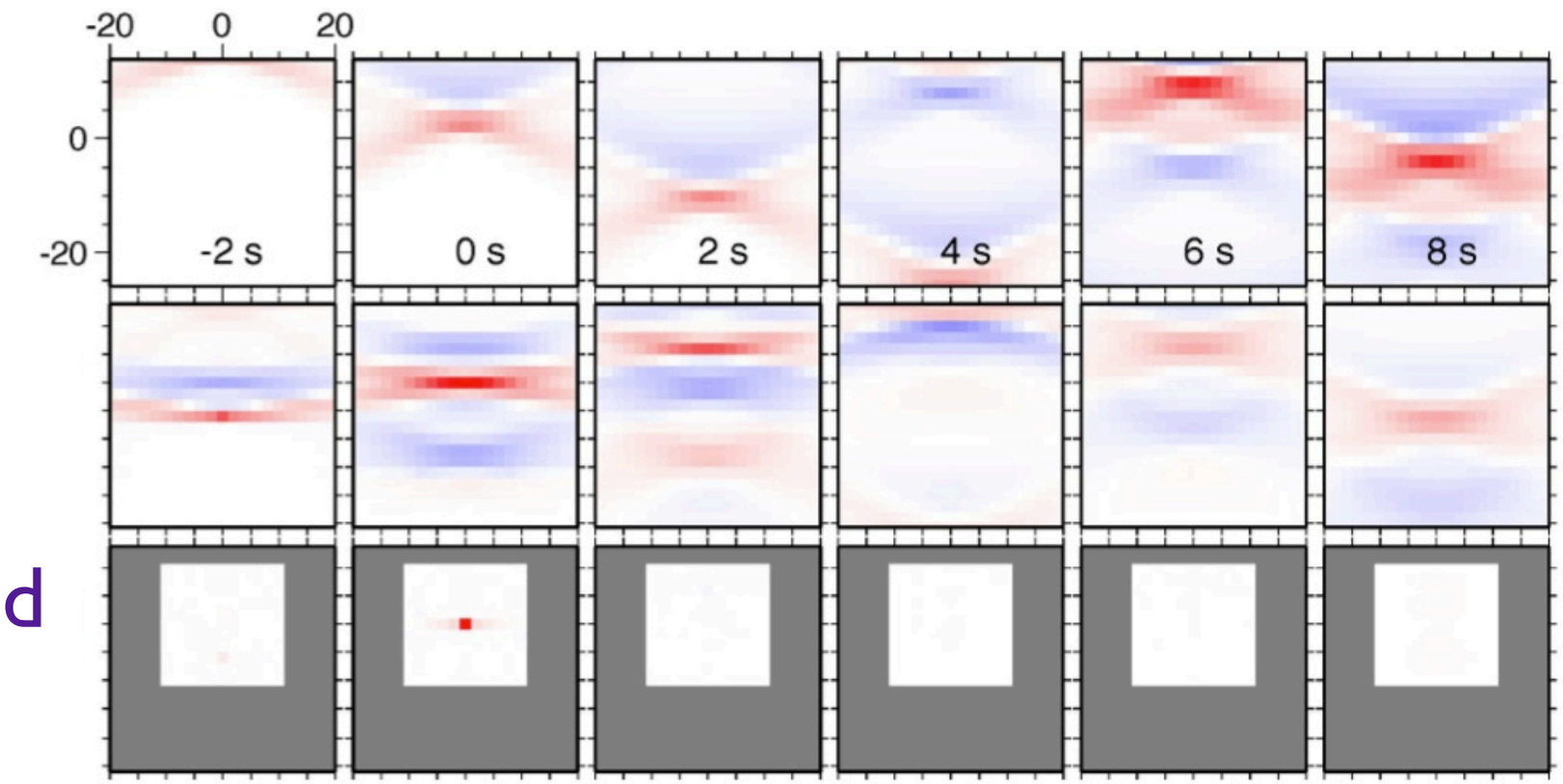

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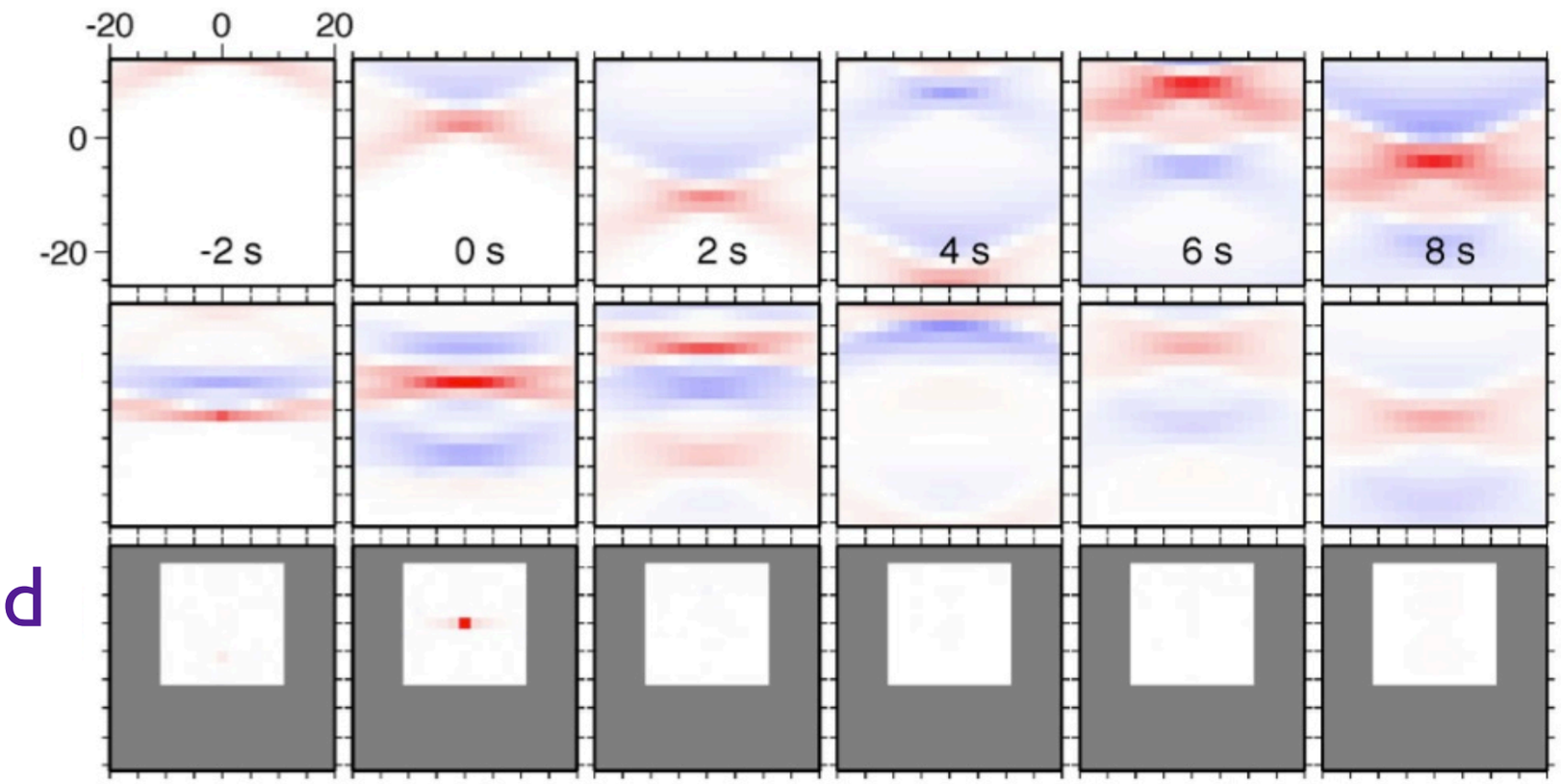

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HBP and Inversion

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Source model from
Yagi and Fukahata (2011, GRL)

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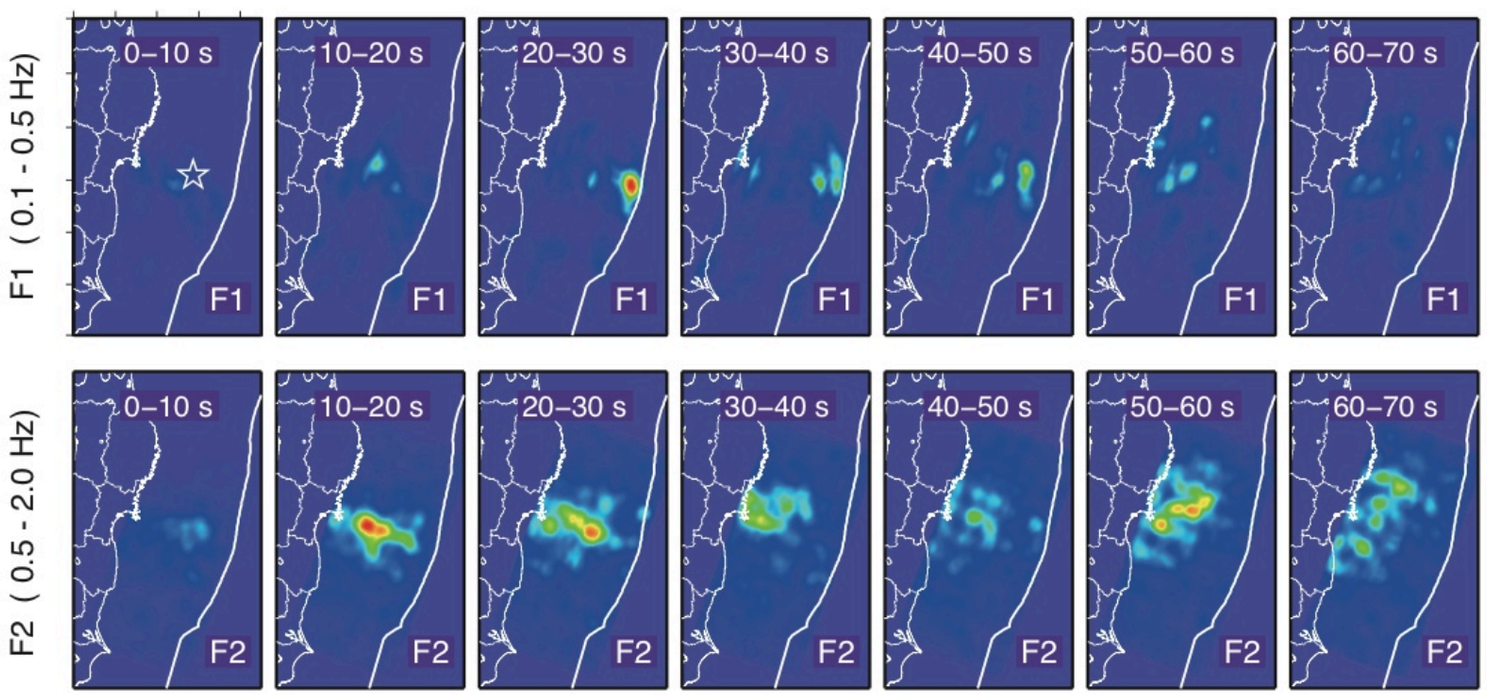

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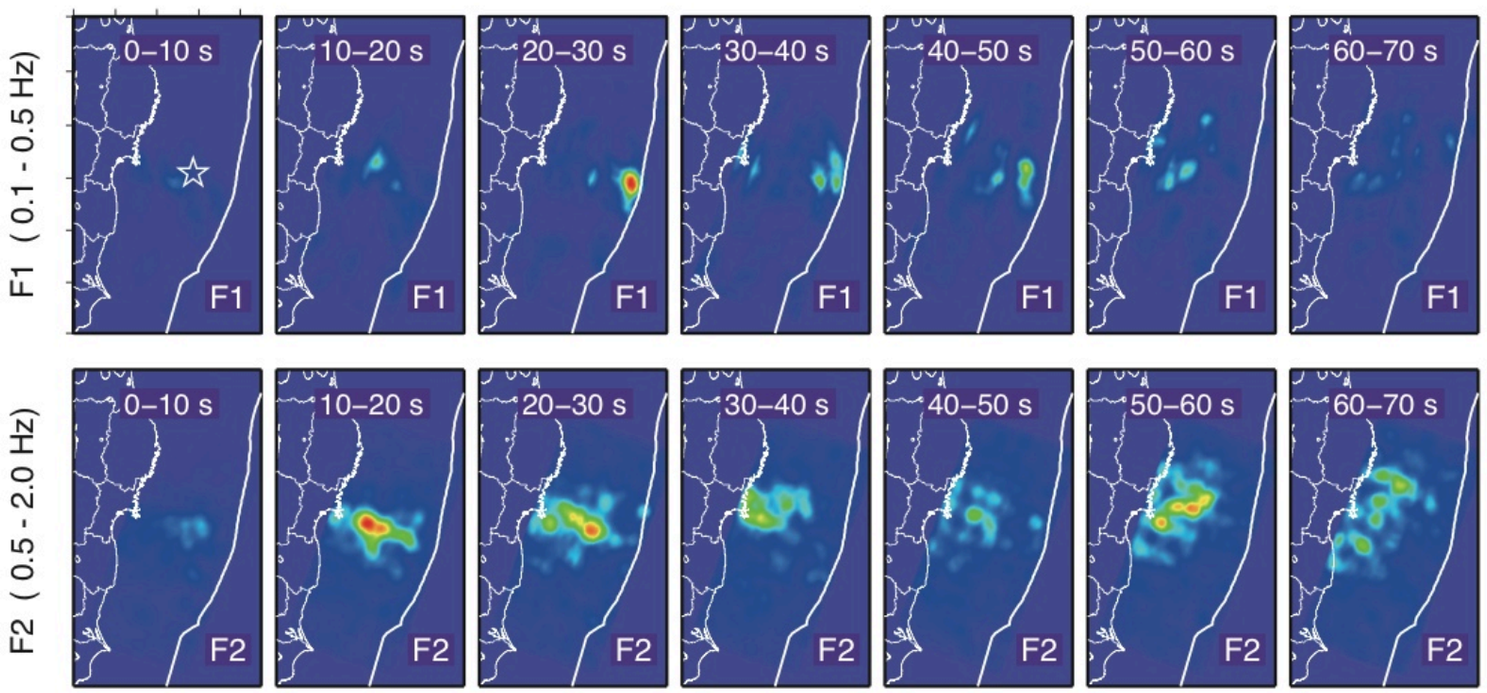

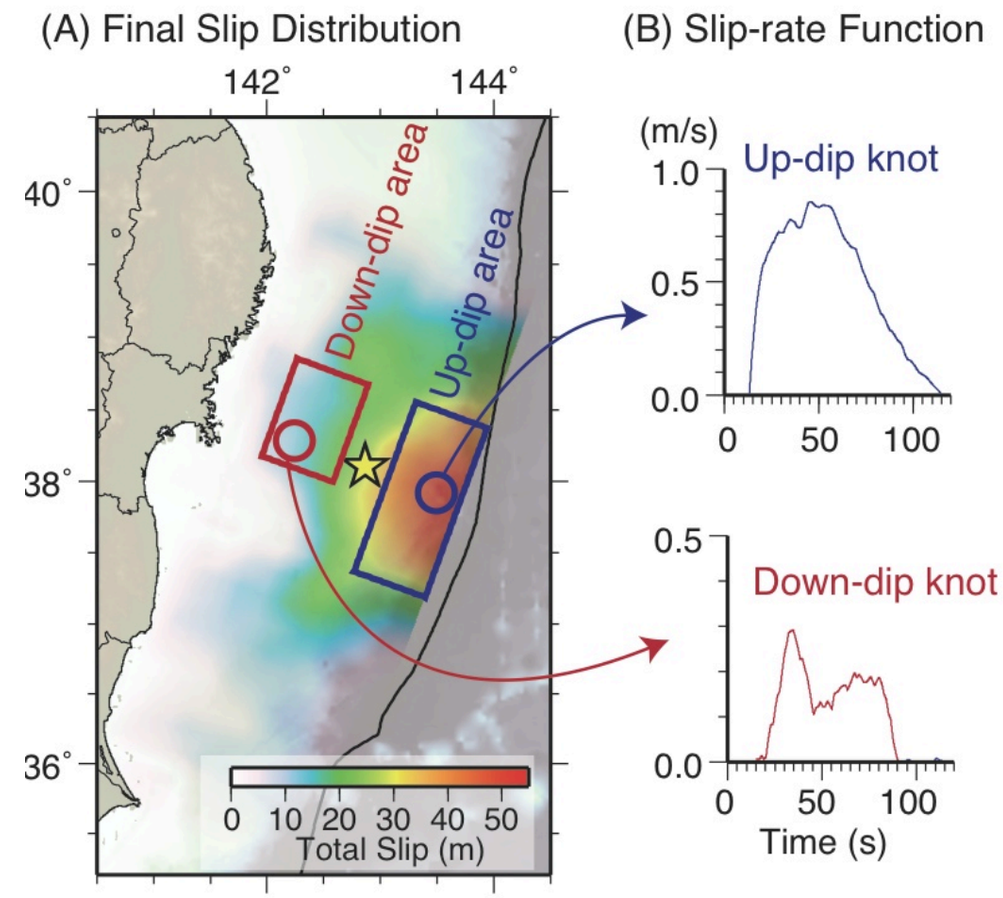

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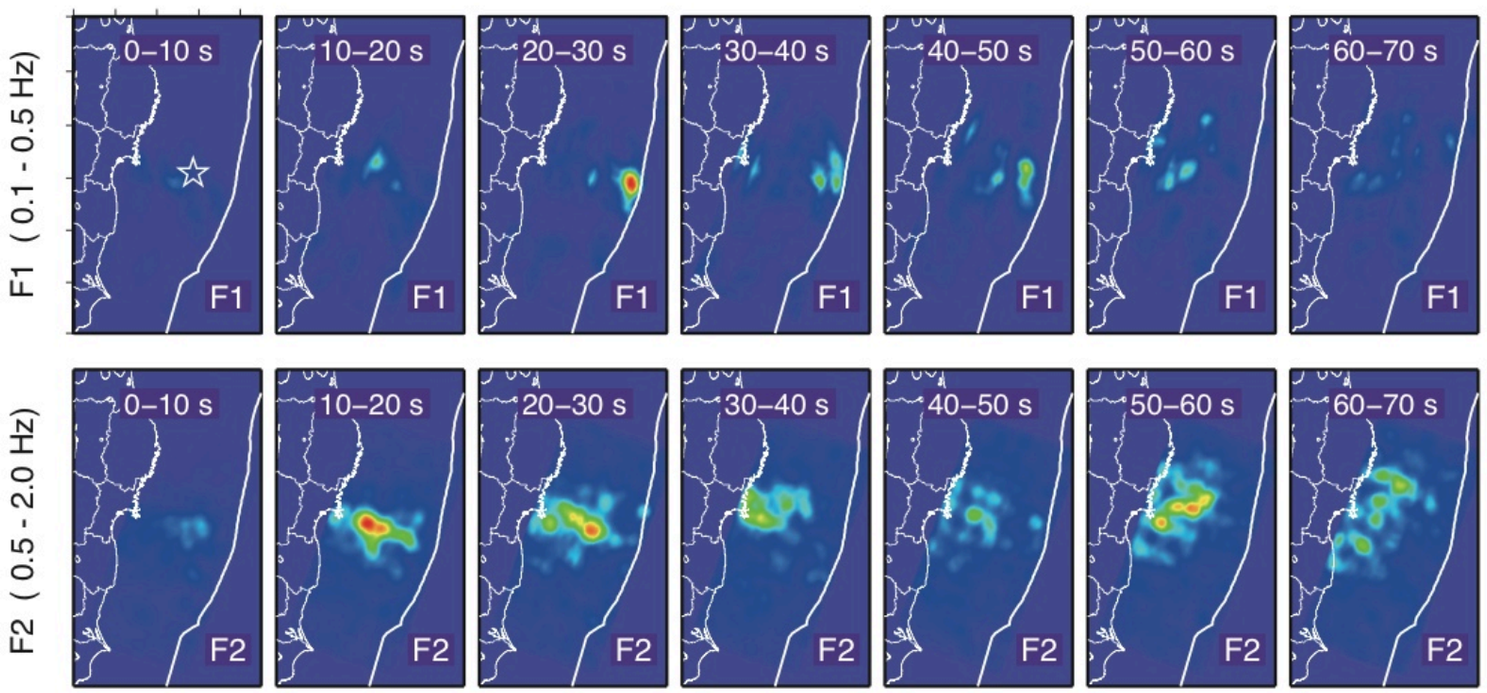

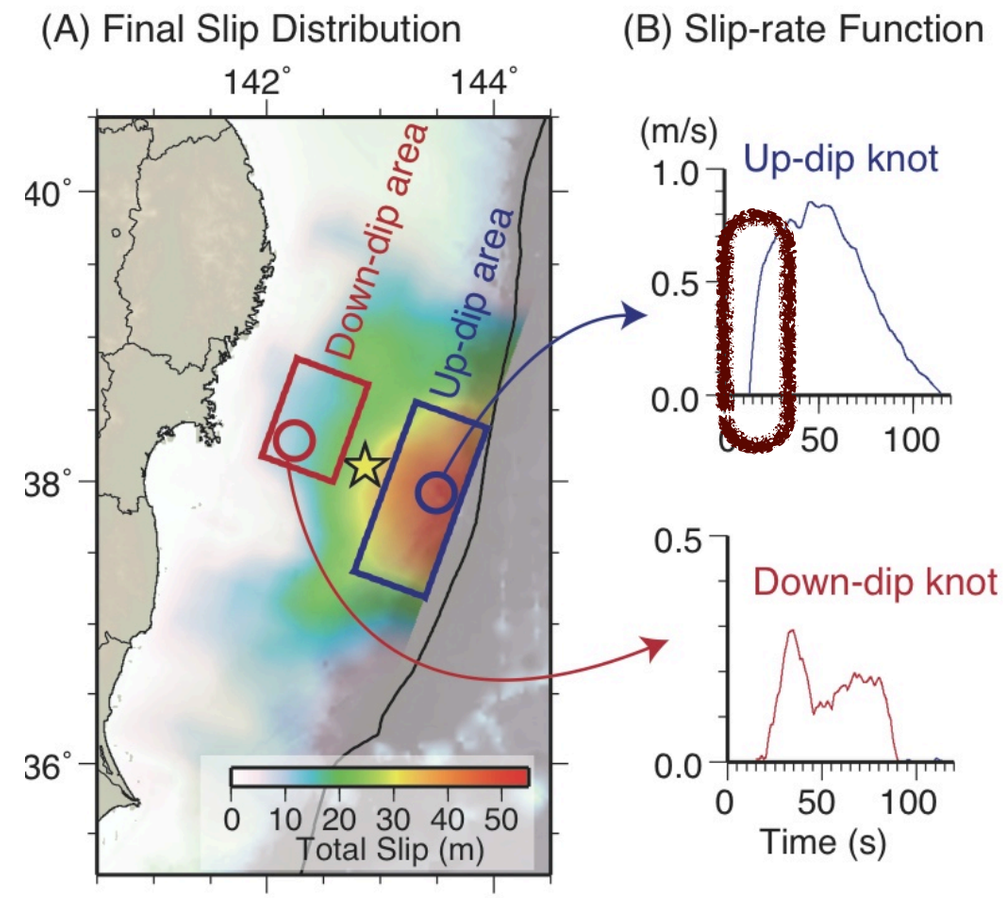

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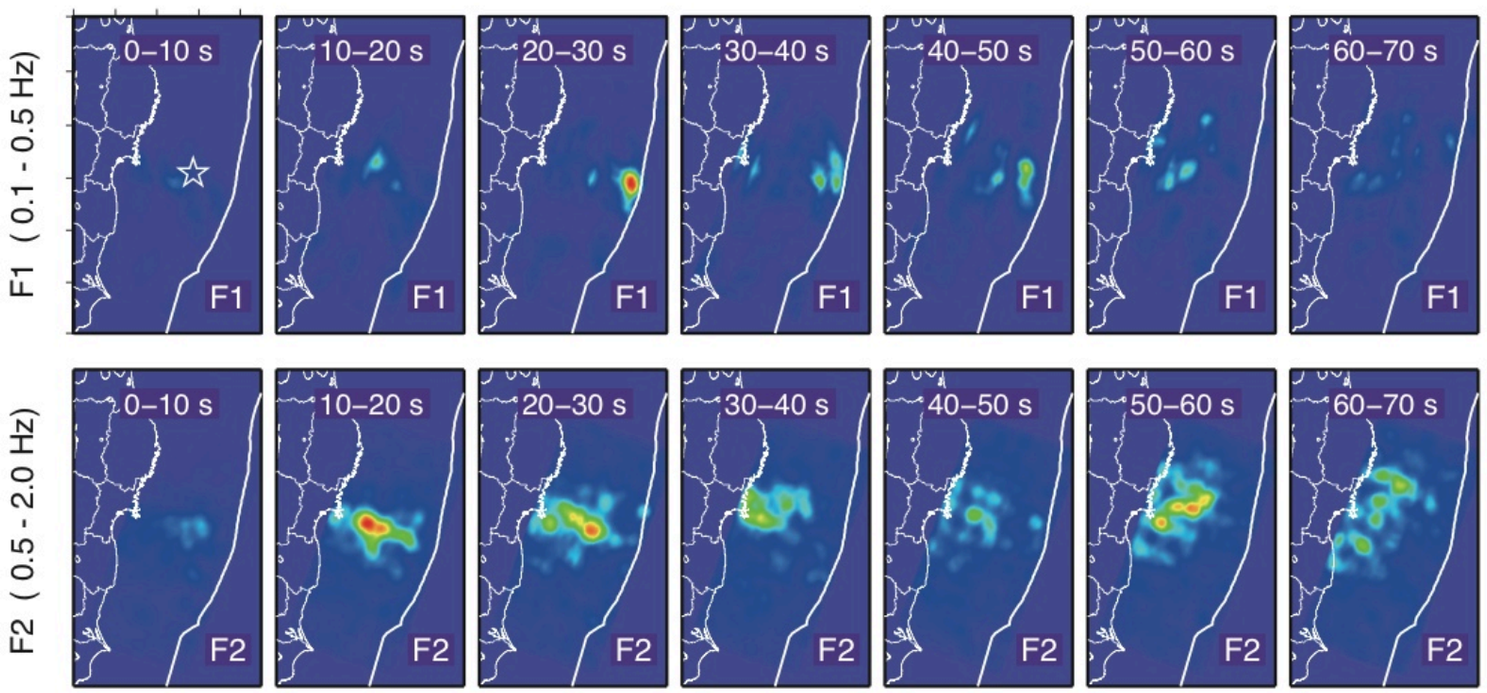

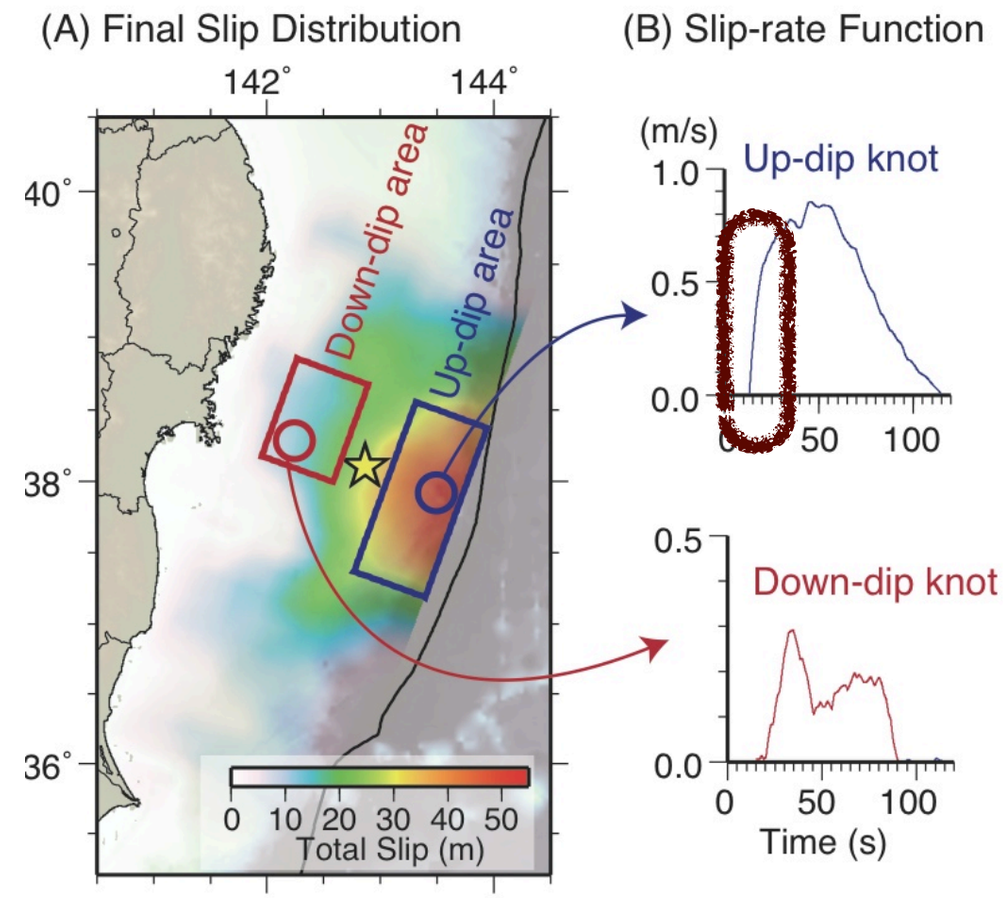

The shallow event corresponds to the rapid and smooth acceleration of the slip-rate function near the trench.

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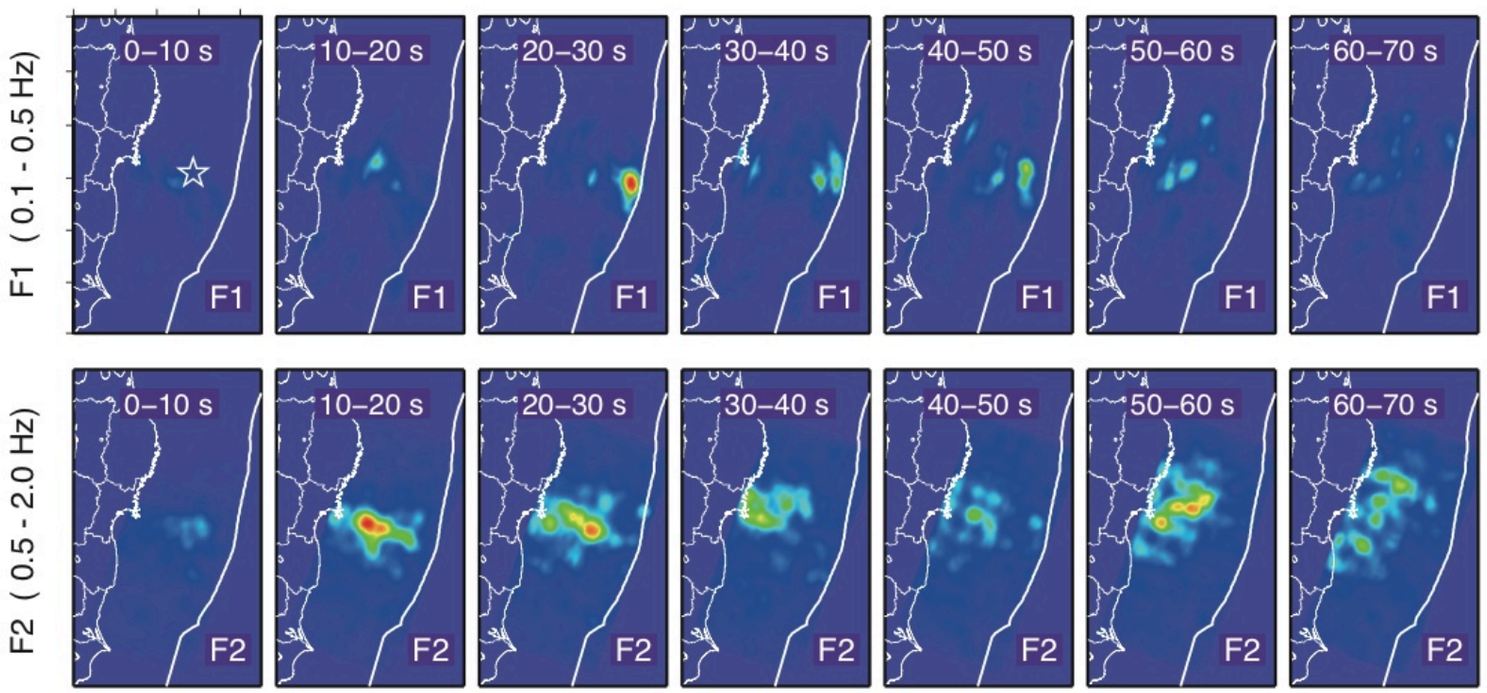

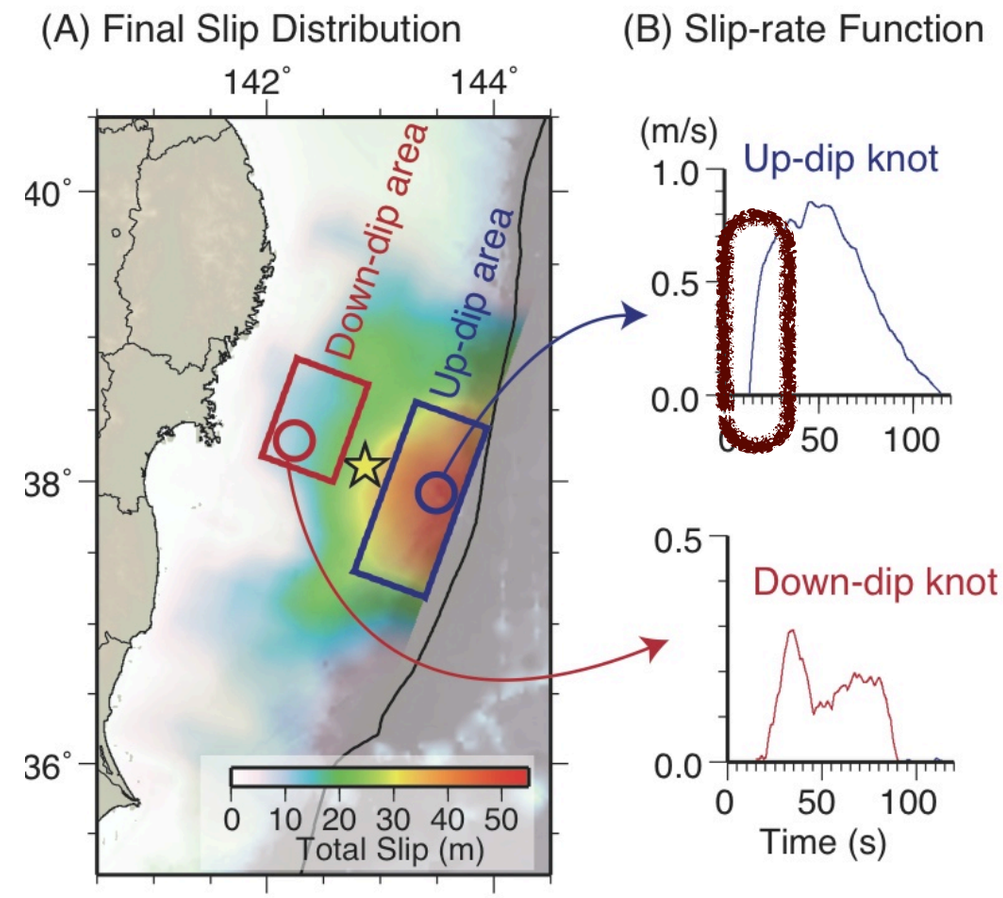

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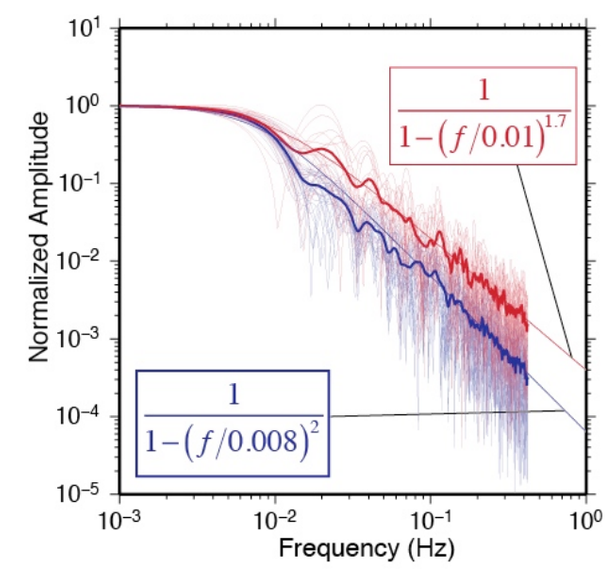

Source model from
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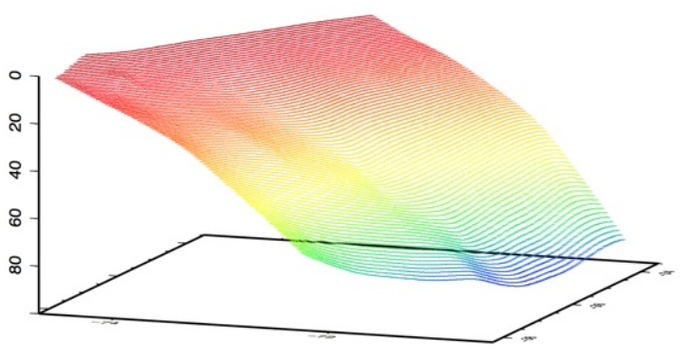
HBP with 3D fault model and Inversion

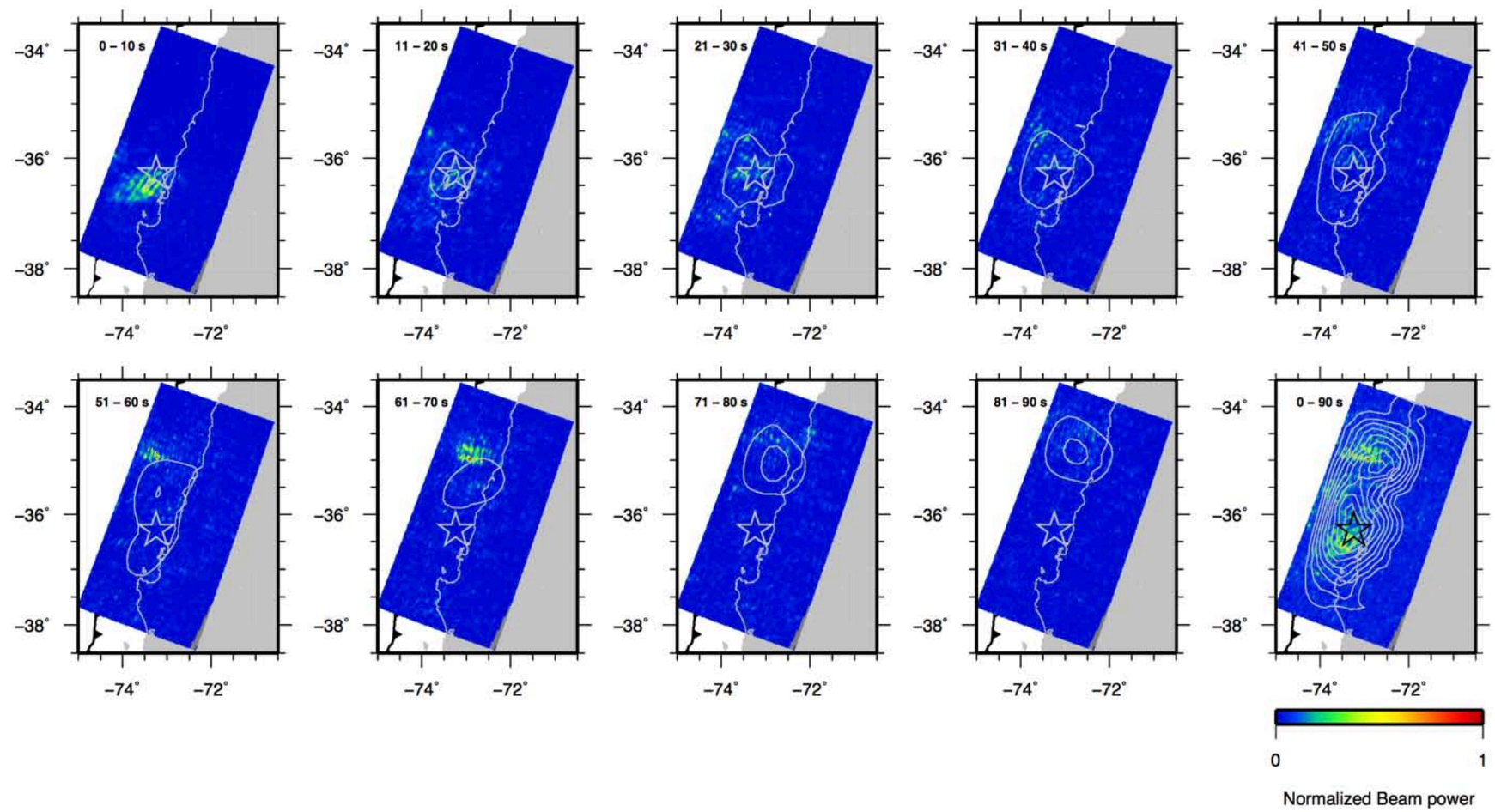
By Okuwaki & Yagi (2013)

2010 Maule Chile earthquake

3D fault model



Tassara & Echaurren (2012, GJI)



The high-frequency events correspond with the sudden rupture velocity; slip-rate change.
We can trace detailed motion of rupture front.

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HBP is better than BP

HBP corresponds to damped LSS with $\varepsilon^2 \to \infty$

Summary II-b

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Weak point of (dumped) LSS

1. High computational power is needed, which gives the limitation results.
   (*cf.* in BP and HBP, resolution is prescribed by the correlation distance of Green's function and/or slip-rate function.)
2. Information on Green's function is needed (*cf.* BP)