Seismic Source Inversion and Back Projection: (1) Introduction of uncertainty of Green's function into waveform inversion for seismic source processes

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Fukahata, Y. (DPRI, Kyoto University)
To Estimate the seismic source process:

Yagi & Fukahata (2011, GJI)
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We calculate a Green’s function under the assumption of a simplified earth structure model.

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**Our Solution:** Introducing “Uncertainty of a Green’s function” into waveform inversion.

Yagi & Fukahata (2011, GJI)
Observation Equation
Observation Equation

\[ u(x,t) = \sum_{q=1}^{2} \int_{\Sigma} G_{q}(x,\xi,t) \ast \dot{D}_{q}(\xi,t) \, dS + F(t) \ast e_{0}(t) \]
Observation Equation

Seismic wave \[ u(x,t) = \sum_{q=1}^{2} \int_{\Sigma} G_q(x,\xi,t) \ast \dot{D}_q(\xi,t) \ dS + F(t) \ast e_0(t) \]

Slip-rate

Green’s function
Observation Equation

Seismic wave \( u(x,t) = \sum_{q=1}^{2} \int_{\Sigma} G_q(x,\xi,t) * \dot{D}_q(\xi,t) \, dS + F(t) * e_0(t) \)

Slip-rate

Green’s function

Uncertainty of Green’s function
Observation Equation

Seismic wave \( u(x,t) = \sum_{q=1}^{2} \int_{\Sigma} G_q(x,\xi,t) \ast D_q(\xi,t) \, dS + F(t) \ast e_0(t) \)

Green’s function

Uncertainty of Green’s function
\( G_q(x,\xi,t) = F(t) \ast Q(t) \ast [\hat{g}_q(x,\xi,t) + \delta g_q(x,\xi,t)] \)
Observation Equation

Seismic wave

\[ u(x,t) = \sum_{q=1}^{2} \int_{\Sigma} G_q(x, \xi, t) \ast \dot{D}_q(\xi, t) \, dS + F(t) \ast e_0(t) \]

Green’s function

Uncertainty of Green’s function

\[ G_q(x, \xi, t) = F(t) \ast Q(t) \ast \left[ \hat{g}_q(x, \xi, t) + \delta g_q(x, \xi, t) \right] \]

Error term of Green’s function
Observation Equation

Seismic wave

\[ u(x,t) = \sum_{q=1}^{2} \int_{\Sigma} G_q(x,\xi,t) \ast \dot{D}_q(\xi,t) \ dS + F(t) \ast e_0(t) \]

Green's function

Uncertainty of Green's function

\[ G_q(x,\xi,t) = F(t) \ast Q(t) \ast \left[ \hat{g}_q(x,\xi,t) + \delta g_q(x,\xi,t) \right] \]

Error term of Green's function

\[ \sum_{q=1}^{2} \sum_{k=1}^{K} \dot{D}_q(\xi_k,t) \ast \delta g'_{qk}(x,t) \ast F(t) \ast Q(t) \]
Point of our formulation 1
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If we introduce uncertainty of Green’s function into Kernel matrix.
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If we introduce uncertainty of Green’s function into Kernel matrix.

\[ d = (H + \delta H)(a + \delta a) + F_0 \]

\[ = Ha + \delta Ha + H\delta a + \delta H\delta a + F_0 \]

\[ \approx Ha + \delta Ha + F_0 \]
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Impossible?
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Impossible?

We divide term of uncertainty of Green’s function into slip-rate function (matrix) and Gaussian error term (vector)
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Impossible?

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\[ d = Ha + P(a)\delta g + F_0 \]

Possible!

Covariance matrix

\[ C_d(a, \sigma^2_g, \chi^2) = \sigma^2_g [P(a)P^t(a) + \chi^2 FF^t] \]
Point of our formulation II
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Two hyper parameters: smoothing and modeling error
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We can estimate hyper parameter using Akaike Bayesian Information Criteria (ABIC).
Point of our formulation II

Two hyper parameters: smoothing and modeling error

We can estimate hyper parameter using Akaike Bayesian Information Criteria (ABIC).

\[
\text{ABIC}\left(\alpha^2, \gamma^2\right) = N \log s\left(\mathbf{a}^*\right) - \log \alpha^2 \left| G_1 + \chi^2 G_2 \right| \\
+ \log \left| \mathbf{H}^T \mathbf{C}_d^{-1} \left(\alpha^2, \mathbf{a}^i\right) \mathbf{H} + \alpha^2 \left( G_1 + \chi^2 G_2 \right) \right| \\
+ \log \left| \mathbf{C}_d^{-1} \left(\alpha^2, \mathbf{a}^i\right) \right| + C
\]
Importance of Error Structure

Yagi & Fukahata (2011, GRL)
Importance of Error Structure

[ The modeling error ] = 
[ The convolution of a slip-rate function and a random error of Green’s function ]

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<table>
<thead>
<tr>
<th>Location</th>
<th>Amplitude</th>
<th>Azimuth</th>
<th>Dip</th>
<th>Misfit of New Formulation</th>
<th>Misfit of Conventional Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>WCIBHZ</td>
<td>9.50E+02 μm/s</td>
<td>36.°</td>
<td>92.°</td>
<td>Large L2 norm</td>
<td>Small L2 norm</td>
</tr>
<tr>
<td>TUCBHZ</td>
<td>1.39E+03 μm/s</td>
<td>55.°</td>
<td>82.°</td>
<td></td>
<td></td>
</tr>
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Yagi & Fukahata (2011, GRL)
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2011 Tohoku-oki Earthquake

New Formulation

Conventional Formulation

Large L2 norm

Small L2 norm

Yagi & Fukahata (2011, GRL)
Spin-off

Yagi et al., (2012, EPSL)
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We can get a plausible solution without the Non-Negative constraint.

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Using fast math library (e.g. MKL), we can get a high resolution solution!

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Number of model parameters: over 40,000

Yagi et al., (2012, EPSL)
Numerical Test
Numerical Test

**INPUT**

\[ \sigma_g = 2\% \]
\[ \sigma_0 = 3.8 \, \mu m \]
Numerical Test

**INPUT**

\[ \sigma_g = 2\% \]
\[ \sigma_0 = 3.8 \ \mu m \]

**OUTPUT**

\[ \sigma_g = 1.9\% \]
\[ \sigma_0 = 3.7 \ \mu m \]
Application to real data set

**Check Point:**

1. Convergence
2. Dependence on initial value
3. Sampling and resolution
4. Critical Case
I. Convergence

2006 Tonga Earthquake

\[ \text{Diff} = \frac{\sum (a_m^k - a_m^{k-1})^2}{\sum (a_m^{k-1})^2} \]

\( k \): step number
2. Dependence on Initial Value

2008 Iwage-Miyagi, Japan earthquake (Mw 6.9)

<table>
<thead>
<tr>
<th>Initial Model</th>
<th>Vertical-striped Slip Model</th>
<th>Uniform Slip Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td><img src="image1.png" alt="Slide 1" /></td>
<td><img src="image2.png" alt="Slide 2" /></td>
</tr>
<tr>
<td>Step 3</td>
<td><img src="image3.png" alt="Slide 3" /></td>
<td><img src="image4.png" alt="Slide 4" /></td>
</tr>
<tr>
<td>Step 6</td>
<td><img src="image5.png" alt="Slide 5" /></td>
<td><img src="image6.png" alt="Slide 6" /></td>
</tr>
</tbody>
</table>
3. Sampling interval and resolution

2008 Iwage-Miyagi, Japan earthquake (Mw 6.9)
3. Sampling interval and resolution

2008 Iwage-Miyagi, Japan earthquake (Mw 6.9)
4. Critical Case

Long source time function

E.g.) 2006 Java Tsunami earthquake

(a) New formulation

(b) Conventional formulation

Unrealistic Slip Direction

Smooth Moment-rate function

Diagonal data covariance matrix
### Observation and Calculation

<table>
<thead>
<tr>
<th>Station</th>
<th>Amplitude (μm)</th>
<th>Azimuth (°)</th>
<th>Delay (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA2BHZ</td>
<td>2.59E+02</td>
<td>21</td>
<td>77</td>
</tr>
<tr>
<td>SURBHZ</td>
<td>1.60E+02</td>
<td>238</td>
<td>82</td>
</tr>
<tr>
<td>TSUMBHZ</td>
<td>1.03E+02</td>
<td>251</td>
<td>87</td>
</tr>
<tr>
<td>OBNBHZ</td>
<td>2.01E+02</td>
<td>327</td>
<td>87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>New formulation</th>
<th>L2 norm</th>
<th>High freq.</th>
<th>Good fitting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>large</td>
<td>no good fitting</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>small</td>
<td>no good fitting</td>
<td></td>
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**Graph:**

- **MA2BHZ:** Amplitude = 2.59E+02, Azimuth = 21, Delay = 77.
- **SURBHZ:** Amplitude = 1.60E+02, Azimuth = 238, Delay = 82.
- **TSUMBHZ:** Amplitude = 1.03E+02, Azimuth = 251, Delay = 87.
- **OBNBHZ:** Amplitude = 2.01E+02, Azimuth = 327, Delay = 87.

**New formulation**
- Large freq.
- Good fitting

**Conventional formulation**
- Small freq.
- No good fitting
Conclusion I

We introduced the uncertainty of Green’s function into waveform inversion analyses.

It well work for real tele-seismic data set.

We can get a plausible solution without the Non-Negative constraint. ( => high resolution)

L2-norm is not good criterion of solution.

It’s critical in large earthquake analysis such 2011 Tohoku-oki earthquake.
Seismic Source Inversion and Back Projection:
(2) Theoretical relationship between back-projection imaging and inverse solutions

Fukahata, Yagi & Rivera (submitted to GJI)
Yagi et al. (2012, EPSL)
Theoretical Background of BP

By Fukahata; Yagi; Rivera (2013, submitted to GJI)
Theoretical Background of BP

Basic Equation of the BP

\[ s_l^{BP}(t) = \sum_j c_j \dot{d}_j(t + t^P_{lj}) \]

\[ = \sum_j c_j \sum_i \left( \ddot{a}_i * G_{ij} \right)(t + t^P_{lj}) \]

- \( d_j \): displacement for station “j”
- \( t^P_{lj} \): Travel time from source grid “l” to station “j”
- \( a_i \): Slip function on grid “l”
- \( G_{ij} \): Green’s function from grid “l” to station “j”
Theoretical Background of BP

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Basic Equation of the BP

\[ s_l^{BP}(t) = \sum_j c_j \dot{d}_j (t + t^P_{lj}) \]

\[ = \sum_j c_j \sum_i (\ddot{a}_i \times G_{ij})(t + t^P_{lj}) \]

Important assumption in BP:
Waveform due to slips on the grid except for the grid “l” are cancelled out earth other (Ishii et al., 2005).

\[ s_l^{BP}(t) \approx \sum_j c_j (\dddot{a}_l \times G_{lj})(t + t^P_{lj}) \text{ or } \approx \sum_j c_j (\dddot{a}_l \times \dot{G}_{lj})(t + t^P_{lj}) \]
Theoretical Background of BP
By Fukahata; Yagi; Rivera (2013, submitted to GJI)
Theoretical Background of BP

Implicit Assumption in BP:
Green’s function is assumed to be like the delta function, since we only use predicted travel time in BP.

\[ s_{l}^{BP}(t) \sim \sum_{j} c_{j}\ddot{a}_{l}(t) \quad \text{or} \quad \sim \sum_{j} c_{j}\dot{a}_{l}(t) \]

By Fukahata; Yagi; Rivera (2013, submitted to GJI)
Theoretical Background of BP
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Implicit Assumption in BP:
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\[ s_l^{BP}(t) \sim \sum_j c_j \dot{a}_l(t) \quad \text{or} \quad \sim \sum_j c_j \ddot{a}_l(t) \]

BP is directly related not to seismic energy release but to slip-acceleration (or slip-rate) on fault plane!

\sim Yao et al. (2012 GJI)
BP well work for deep earthquake

Green’s function can be assumed to be like the delta function (Suzuki & Yagi, 2011, GRL)
Hybrid Back-Projection (HBP)

By Yagi; Nakao; Kasahara (2012, EPSL)
Hybrid Back-Projection (HBP)

Problem:
P-waveform trains contains pP and sP phases; Green’s function can not be approximated to Delta function.

By Yagi; Nakao; Kasahara (2012, EPSL)
Hybrid Back-Projection (HBP)

By Yagi; Nakao; Kasahara (2012, EPSL)

Problem:
P-waveform trains contains pP and sP phases; Green’s function can not be approximated to Delta function.

Solution:
Cross-correlation of an observed waveform with the theoretical Green’s function
Theoretical Background of HBP

By Fukahata; Yagi; Rivera (2013, submitted to GJI)
Theoretical Background of HBP

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Basic Equation of the HBP

\[ s_i^{HBP}(t) = \sum_j c_j (\hat{u}_j \hat{\cdot} \hat{G}_{lj})(t) = \sum_j c_j \sum_i \left[(\hat{a}_i \hat{\cdot} \hat{G}_{ij}) \hat{\cdot} \hat{G}_{lj}\right](t) \]
Theoretical Background of HBP
By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Basic Equation of the HBP

\[ s_l^{HBP} (t) = \sum_j c_j \left( \dot{u}_j \ast \hat{G}_{lj} \right)(t) = \sum_j c_j \sum_i \left[ (\dot{a}_i \ast \hat{G}_{ij}) \ast \hat{G}_{lj} \right](t) \]

Important assumption in HBP:
Waveform due to slips on the grid except for the grid “l” are cancelled out earth other.

\[ s_l^{HBP} (t) \approx \dot{a}_l (t) \ast \sum_j c_j \left( \hat{G}_{lj} \ast \hat{G}_{lj} \right)(t) \]
Theoretical Background of HBP

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Basic Equation of the HBP

\[ s_t^{HBP}(t) = \sum_j c_j (\hat{u}_j \ast \hat{G}_{lj})(t) = \sum_j c_j \sum_i \left[ (\hat{a}_i \ast \hat{G}_{ij}) \ast \hat{G}_{lj} \right](t) \]

Important assumption in HBP:
Waveform due to slips on the grid except for the grid “l” are cancelled out earth other.

\[ s_t^{HBP}(t) \approx \hat{a}_l(t) \ast \sum_j c_j \left( \hat{G}_{lj} \ast \hat{G}_{lj} \right)(t) \]

Auto correlation of greens function is assumed to be the delta function.

\[ s_t^{HBP}(t) \sim \sum_j c_j \hat{a}_l(t) \]
In Vector Form

By Fukahata; Yagi; Rivera (2013, submitted to GJI)
In Vector Form

Relation of data to slip

\[ \dot{d}_j = \sum_i (\dot{G}_{ij} \ast \dot{a}_i)(t) \quad d_j = G_j a \]

\[ d = Ga \]
In Vector Form

Relation of data to slip

\[ \dot{d}_j = \sum_i (\dot{G}_{ij} \ast \dot{a}_i)(t) \]
\[ d_j = G_ja \]
\[ d = Ga \]

HBP: \[ s_{lHBP}^H(t) = \sum_j c_j (\dot{d}_j \ast \dot{G}_{lj})(t) \]
\[ \mathbf{s}_{lHBP} = \sum_j c_j \mathbf{G}_{lj}^T \mathbf{d}_j \]
\[ \mathbf{s}_{HBP} = \mathbf{G}^T \mathbf{Cd} \]
\[ \mathbf{C} = \begin{pmatrix} c_1 \mathbf{I}_1 & 0 \\ \vdots & \ddots & \ddots \\ 0 & \cdots & c_J \mathbf{I}_J \end{pmatrix} \]
In Vector Form

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Relation of data to slip

\[ \dot{d}_j = \sum_i (\dot{G}_{ij} \ast \dot{a}_i)(t) \quad \text{d}_j = G_j \text{a} \]

**HBP:**

\[ s_{l_HBP}^H(t) = \sum_j c_j (\dot{d}_j \hat{\ast} \dot{G}_{lj})(t) \]

\[ s_{l_HBP} = \sum_j c_j G_{lj}^T d_j \]

\[ s_{l_HBP}^H = G^T \text{Cd} \]

\[ \text{C} = \begin{pmatrix} c_1 \text{I}_1 & 0 \\ 0 & c_J \text{I}_J \end{pmatrix} \]

**BP:**

\[ s_{lBP}^P(t) = \sum_j c_j \dot{d}_j (t + t_{jlp}) \]

\[ s_{BP} = B^T \text{Cd} \]

\[ B_{ijkl} = \begin{cases} 1 & t_i - \tau_j = t_{kl}^p \\ 0 & t_i - \tau_j \neq t_{kl}^p \end{cases} \]
Inverse Solutions

By Fukahata; Yagi; Rivera (2013, submitted to GJI)
Inverse Solutions

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Observation eq. $d = Ga$
Inverse Solutions

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Observation eq.

\[ d = Ga \]

Simplest practically used
(Claerbout, 2001)

\[ \hat{a} = G^T d \]
Inverse Solutions

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Observation eq. \( \mathbf{d} = \mathbf{G}\mathbf{a} \)

Simplest practically used (Claerbout, 2001) \( \hat{\mathbf{a}} = \mathbf{G}^T \mathbf{d} \)

Least Squares Solution (LSS) \( \hat{\mathbf{a}} = \left( \mathbf{G}^T \mathbf{E}^{-1} \mathbf{G} \right)^{-1} \mathbf{G}^T \mathbf{E}^{-1} \mathbf{d} \)
Inverse Solutions

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Observation eq. \( d = Ga \)

Simplest practically used (Claerbout, 2001)
\( \hat{a} = G^T d \)

Least Squares Solution (LSS)
\( \hat{a} = (G^T E^{-1}G)^{-1} G^T E^{-1} d \)

Dumped LSS
\( \hat{a} = (G^T E^{-1}G + \varepsilon^2 I)^{-1} G^T E^{-1} d \)
Model Resolution Matrix

By Fukahata; Yagi; Rivera (2013, submitted to GJI)
Model Resolution Matrix

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

\[
\hat{a} = \left( G^T E^{-1} G \right)^{-1} G^T E^{-1} (G \hat{a}) = a \\
\therefore R^{LLS} = I
\]
Model Resolution Matrix

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

LSS: \[ \hat{a} = \left( G^T E^{-1} G \right)^{-1} G^T E^{-1} (Ga) = a \]
\[ \therefore \, R^{LLS} = I \]

HBP: \[ s^{HBP} = G^T Cd = \left( G^T CG \right) a = R^{HBP} a \]
\[ \therefore \, R^{HBP} = G^T CG \]
Model Resolution Matrix
By Fukahata; Yagi; Rivera (2013, submitted to GJI)

LSS:
\[ \hat{a} = \left( G^T E^{-1} G \right)^{-1} G^T E^{-1} (Ga) = a \]
\[ \therefore R_{LLS} = I \]

HBP:
\[ s_{HBP} = G^T Cd = \left( G^T CG \right) a = R_{HBP} a \]
\[ \therefore R_{HBP} = G^T CG \]

BP:
\[ s_{BP} = B^T Cd = \left( B^T CG \right) a = R_{BP} a \]
\[ \therefore R_{HBP} = B^T CG \]
Model Resolution Matrix

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

LSS:
\[ \hat{a} = \left( G^T E^{-1} G \right)^{-1} G^T E^{-1} (Ga) = a \]
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HBP:
\[ s^{HBP} = G^T Cd = \left( G^T CG \right) a = R^{HBP} a \]
\[ \therefore R^{HBP} = G^T CG \]

BP:
\[ s^{BP} = B^T Cd = \left( B^T CG \right) a = R^{BP} a \]
\[ \therefore R^{HBP} = B^T CG \]

In terms of model resolution matrix, BP and HBP are inferior to LSS.
HBP and Damped LSS
By Fukahata; Yagi; Rivera (2013, submitted to GJI)
HBP and Damped LSS

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Dumped LSS: \[ \hat{a} = \left( G^T E^{-1} G + \varepsilon^2 I \right)^{-1} G^T E^{-1} d \]
HBP and Damped LSS

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Dumped LSS: \[ \hat{a} = \left( G^T E^{-1} G + \varepsilon^2 I \right)^{-1} G^T E^{-1} d \]

\[ \varepsilon^2 \to \infty \]

\[ \hat{a} = \beta^2 G^T E^{-1} d \]
HBP and Damped LSS
By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Dumped LSS: \( \hat{a} = \left( G^T E^{-1} G + \varepsilon^2 I \right)^{-1} G^T E^{-1} d \)

\[ \varepsilon^2 \rightarrow \infty \]

\( \hat{a} = \beta^2 G^T E^{-1} d \)

HBP: \( s^{HBP} = G^T Cd \)

HBP solution corresponds to a damped least squares solution with an extremely large damping parameter
HBP and Damped LSS

By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Dumped LSS: \( \hat{a} = \left( G^T E^{-1} G + \varepsilon^2 I \right)^{-1} G^T E^{-1} d \)

\[ \varepsilon^2 \rightarrow \infty \]

\( \hat{a} = \beta^2 G^T E^{-1} d \)

HBP:

\( s^{\text{HBP}} = G^T C d \)

HBP solution corresponds to a damped least squares solution with an extremely large damping parameter

BP:

\( s^{\text{BP}} = B^T C d \)

In BP, the Green’s function in “\( G \)“ is further approximated by the delta function.
Simple Numerical Examples

By Fukahata; Yagi; Rivera (2013, submitted to GJI)
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Condition: \( d = G \)
Simple Numerical Examples
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Condition: \[ d = G \]
Simple Numerical Examples  
By Fukahata; Yagi; Rivera (2013, submitted to GJI)

Condition: \( d = G \)

BP: \[
\sum_{ij} \hat{G}_{ij}(t - t_{ij}^P) = B^T G = R^{BP}
\]

HBP: \[
\sum_{ij} \left( \hat{G}_{ij} \ast \hat{G}_{ij} \right)(t) = G^T G = R^{HBP}
\]

Dumped LSS: \[
\hat{a} = \left( G^T G + \varepsilon^2 I \right)^{-1} G^T G = R^{DLSS}
\]
HBP and Inversion
By Yagi et al. (2012, EPSL)

Source model from
Yagi and Fukahata (2011, GRL)
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The shallow event corresponds to the rapid and smooth acceleration of the slip-rate function near the trench.

Source model from Yagi and Fukahata (2011, GRL)
HBP and Inversion
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The shallow event corresponds to the rapid and smooth acceleration of the slip-rate function near the trench.

Source model from Yagi and Fukahata (2011, GRL)
HBP with 3D fault model and Inversion
By Okuwaki & Yagi (2013)

2010 Maule Chile earthquake

The high-frequency events correspond with the sudden rupture velocity; slip-rate change.
We can trace detailed motion of rupture front.
Summary II-a
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Both BP and HBP intend to estimate slip motion on the fault under the following 2 key assumption:
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1. All contribution other than the corresponding grid cell “l” are cancelled out each other
2. Green’s function (BP) or auto-correlation of Green’s function is like the delta function
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In estimating slip motion, the following expressions are used:

BP: \[ \hat{a} = B^T C_d \]

HBP: \[ \hat{a} = G^T C_d \]

HBP is better than BP
HBP corresponds to damped LSS with \( \epsilon^2 \rightarrow \infty \)
Summary II-b
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Dumped LSS is superior to BP and HBP solution.
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We should verify image of HBP and BP by using result of seismic source inversion.
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Weak point of (dumped) LSS
Summary II-b

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Weak point of (dumped) LSS

1. High computational power is needed, which gives the limitation results.
   (cf. in BP and HBP, resolution is prescribed by the correlation distance of Green’s function and/or slip-rate function.)
2. Information on Green’s function is needed (cf. BP)