Toward accounting for prediction uncertainty when inferring subsurface fault slip

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Acknowledgements: Mark Simons, Pablo Ampuero, Piyush Agram, Sarah Minson, Luis Rivera, James Beck, Michael Aivasis, Hailiang Zhang.

2013 SCEC ANNUAL MEETING: SOURCE INVERSION VALIDATION

September 2013

Project : Toward the next generation of source models including realistic statistics of uncertainties

SIV initiative

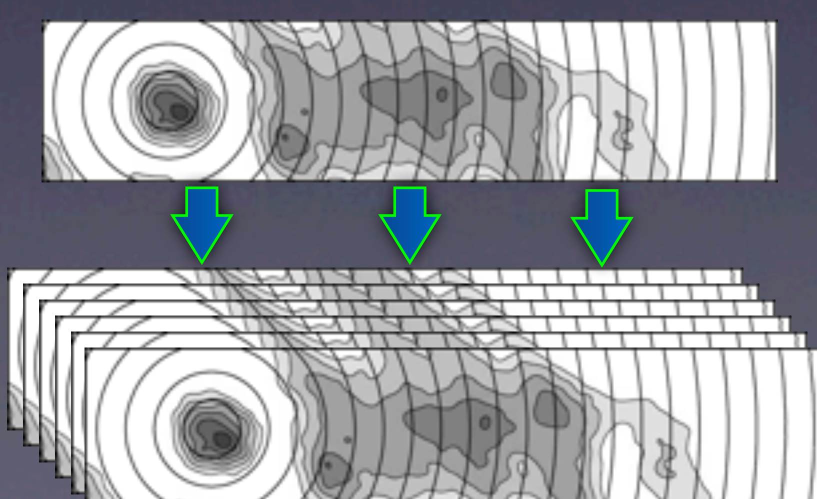
Modeling ingredients

- Data:
  - Field observations
  - Seismology
  - Geodesy
  - ...
- Theory:
  - Source geometry
  - Earth model
  - ...

Sources of uncertainty

- Observational uncertainty:
  - Instrumental noise
  - Ambient seismic noise
- Prediction uncertainty:
  - Fault geometry
  - Earth model

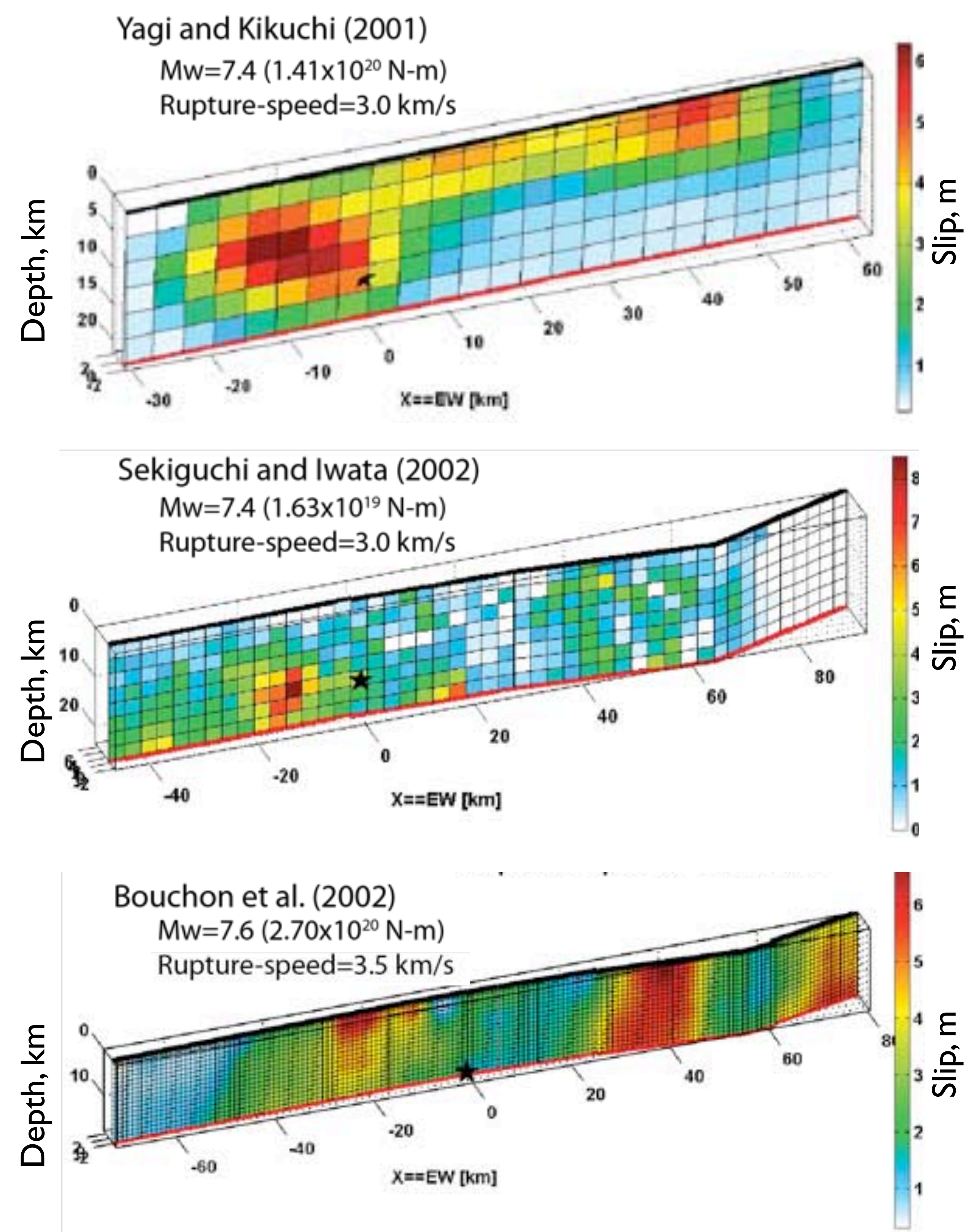
A posteriori distribution



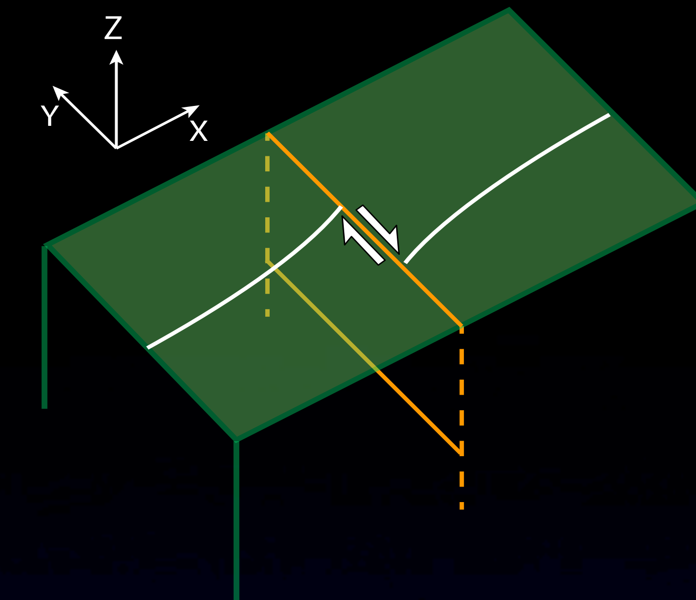
Single model

Ensemble of models

Izmit earthquake (1999)



A realistic statistical model for the prediction uncertainty



The forward problem

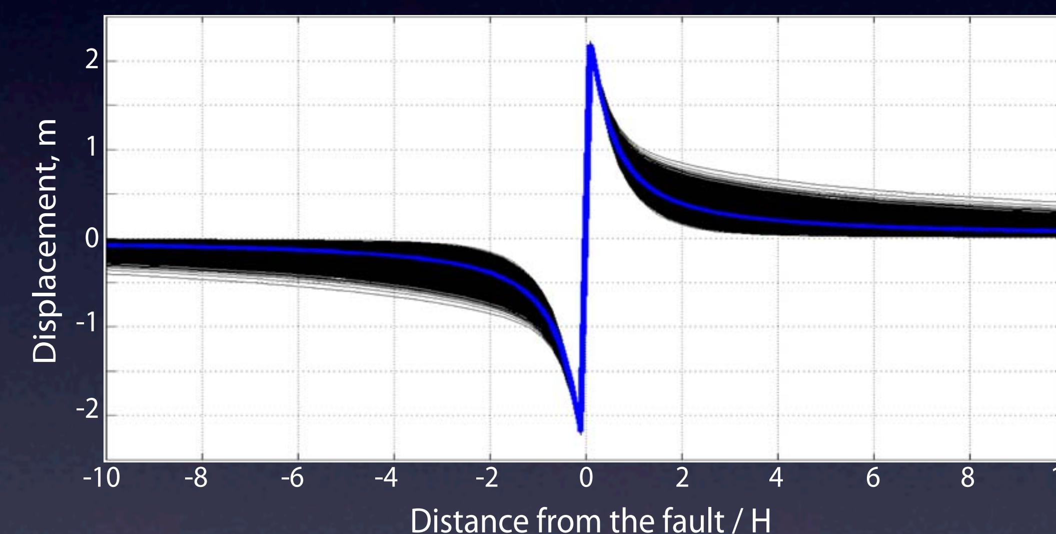
- posterior distribution: $p(\mathbf{m}|\mathbf{d}_{\text{obs}}) \propto p(\mathbf{m}) \int_D p(\mathbf{d}_{\text{obs}}|\mathbf{d})\, p(\mathbf{d}|\mathbf{m})\, d\mathbf{d}$

Exact theory

$$p(\mathbf{d}|\mathbf{m}) = \delta(\mathbf{d} - \mathbf{g}(\tilde{\Omega},\mathbf{m}))$$

Stochastic (non-deterministic) theory

$$p(\mathbf{d}|\mathbf{m}) = N(\mathbf{d} \mid \mathbf{g}(\tilde{\Omega},\mathbf{m}), \mathbf{C}_p)$$



Calculation of Cp based on the physics of the problem: A perturbation approach

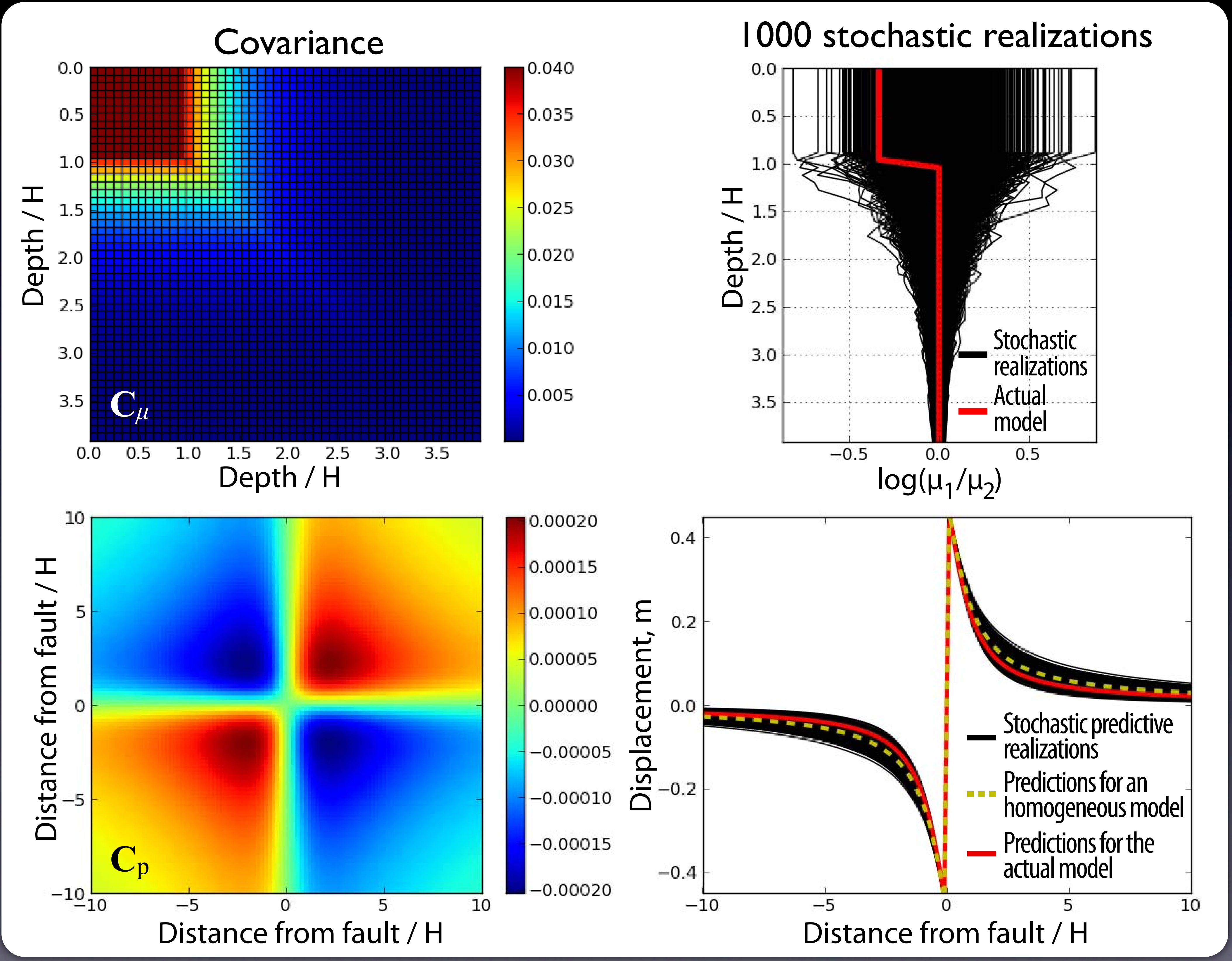
$$\boldsymbol{\delta}\mathbf{p} = \mathbf{K}_\mu \cdot \boldsymbol{\delta} \ln \boldsymbol{\mu} \quad \Rightarrow \quad \mathrm{C_p} = \mathbf{K}_\mu \cdot \mathbf{C}_\mu \cdot \mathbf{K}_\mu^T$$

Partial derivatives w.r.t. the elastic parameters (sensitivity kernel)

Covariance matrix describing uncertainty in the Earth model parameters

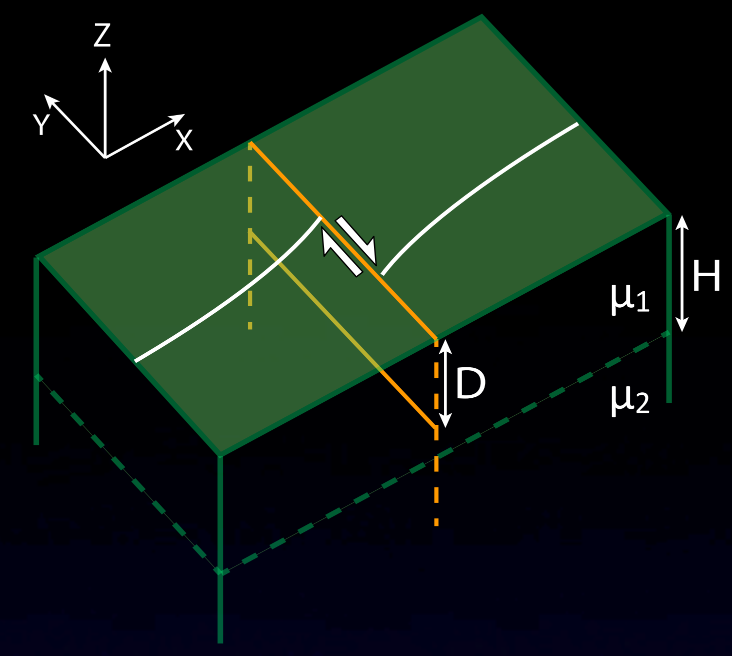
Prediction uncertainty due to the earth model

$$\mathbf{C}_\mathrm{p} = \mathbf{K}_\mu \cdot \mathbf{C}_\mu \cdot \mathbf{K}_\mu^T$$

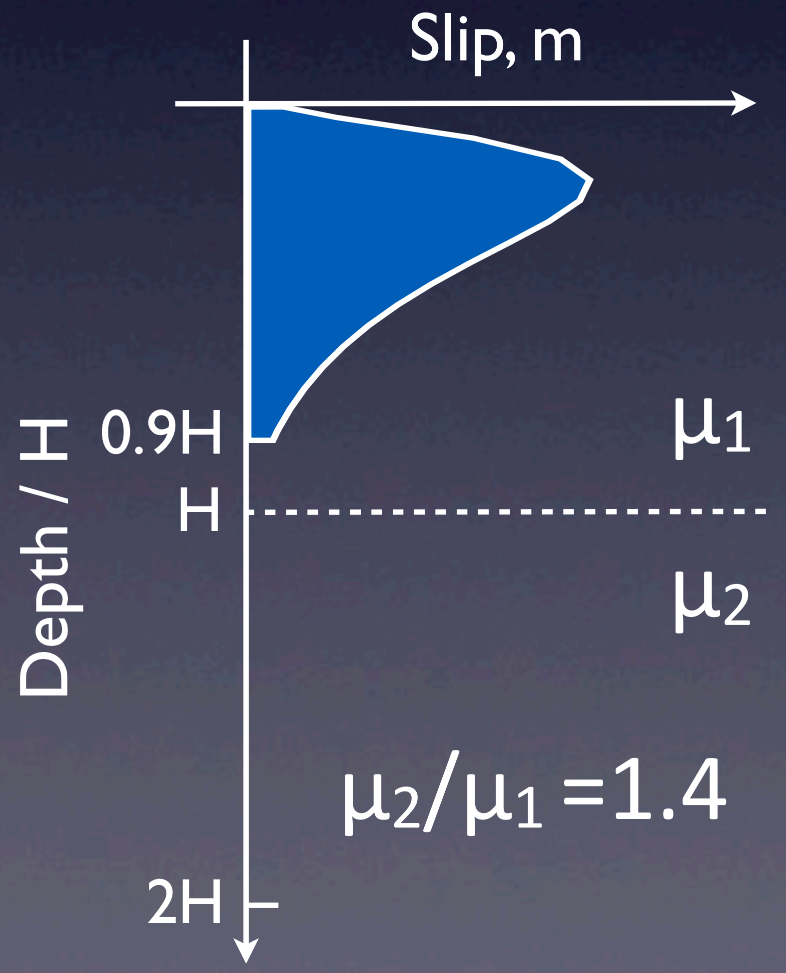


Toy model 1: Infinite strike-slip fault

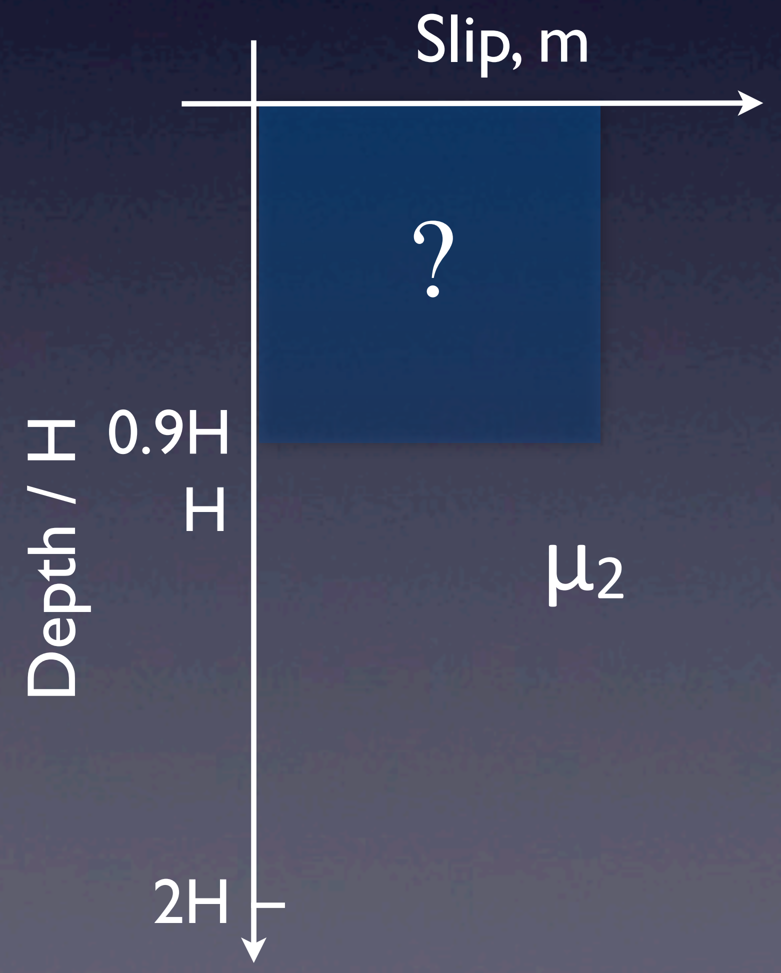
- Data generated for a layered half-space ($\mathbf{d}_{obs}$)
- 5mm uncorrelated observational noise ($\rightarrow \mathbf{C}_d$)
- GFs for an homogeneous half-space ($\rightarrow \mathbf{C}_p$)
- CATMIP bayesian sampler (Minson et al., GJI 2013):
    > 1,310,720 metropolis chains running in parallel



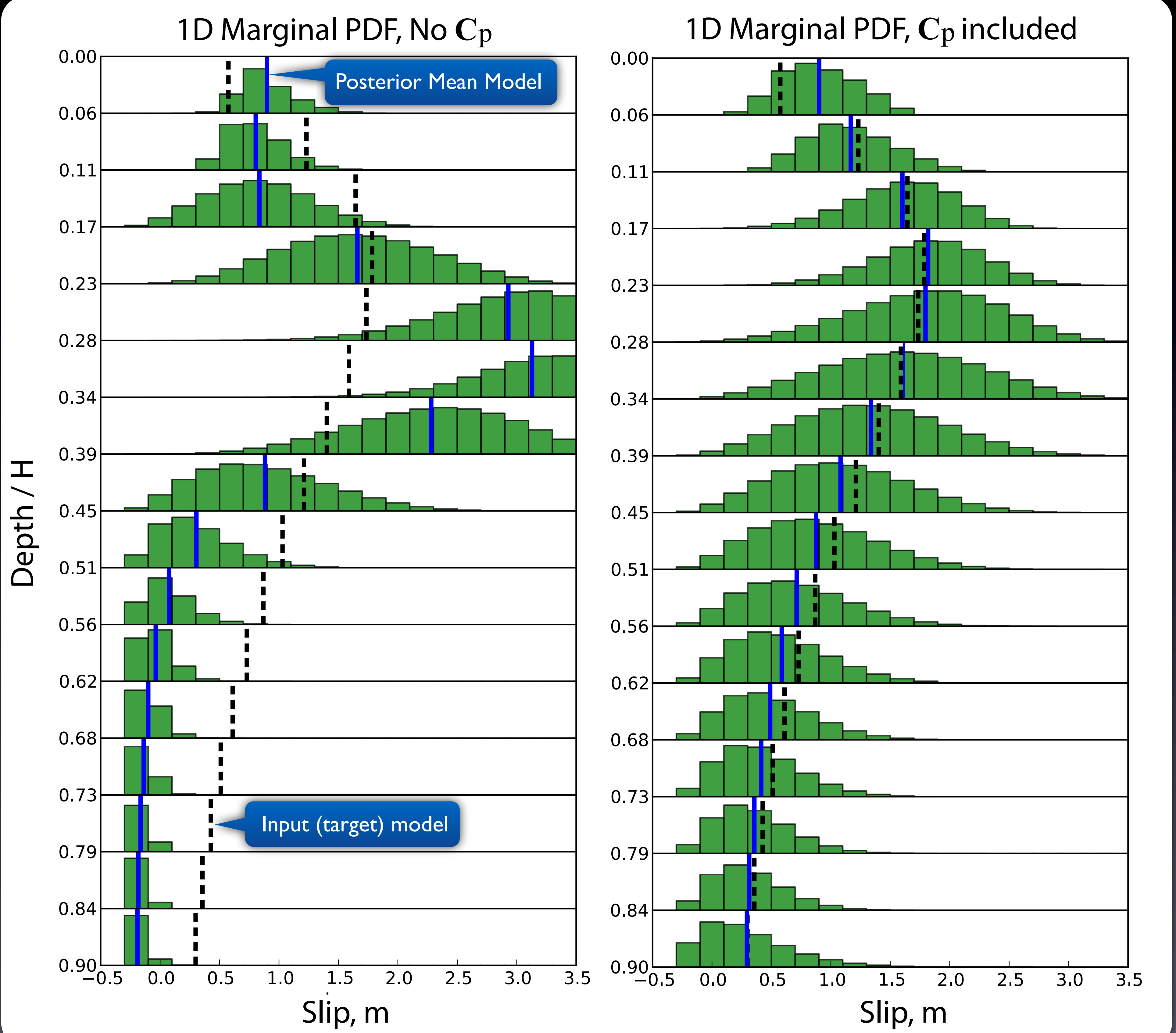
Synthetic Data + Noise
shallow fault + Layered half-space

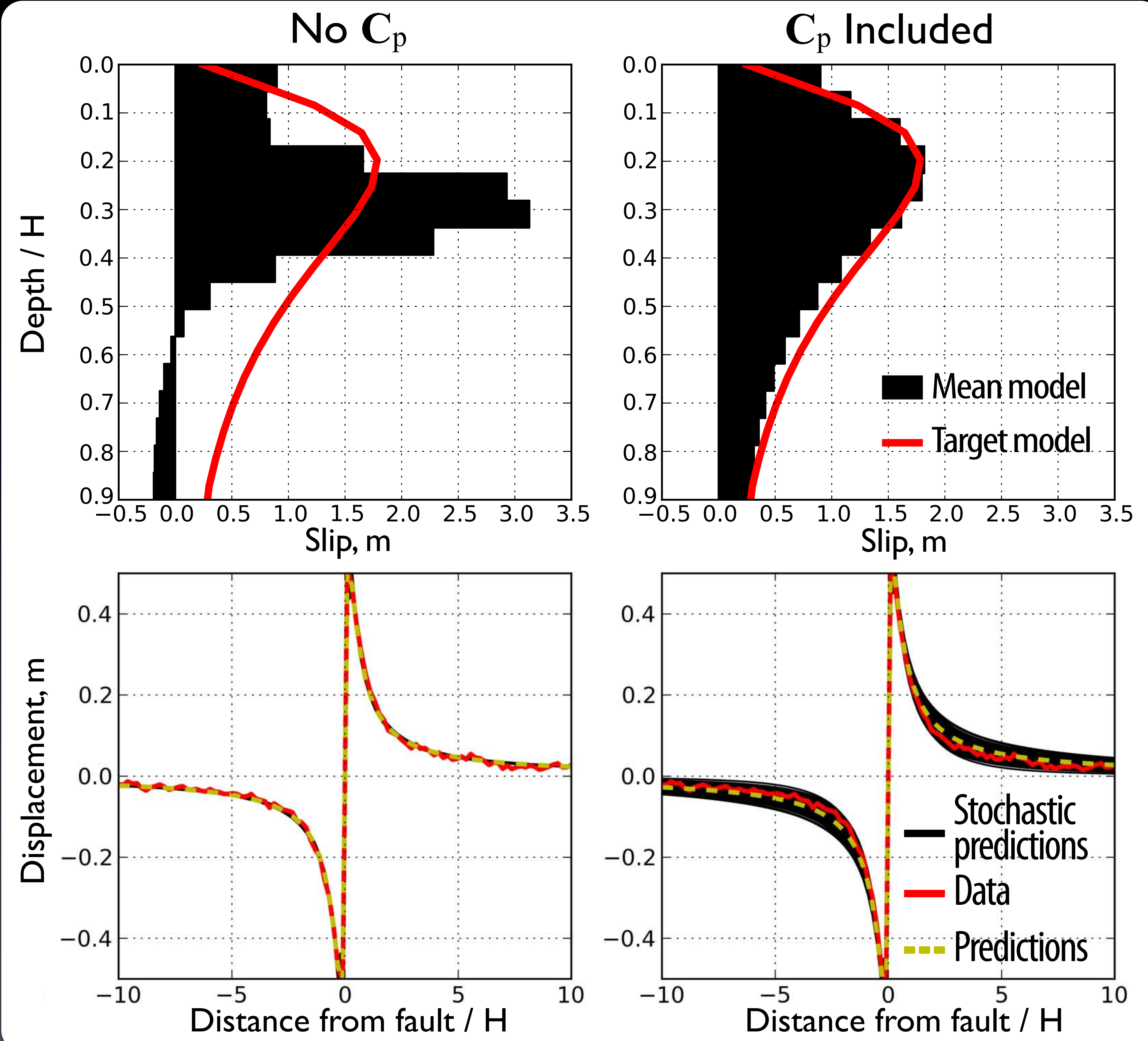


Inversion:
Homogeneous half-space



Toy model 1: Infinite strike-slip fault





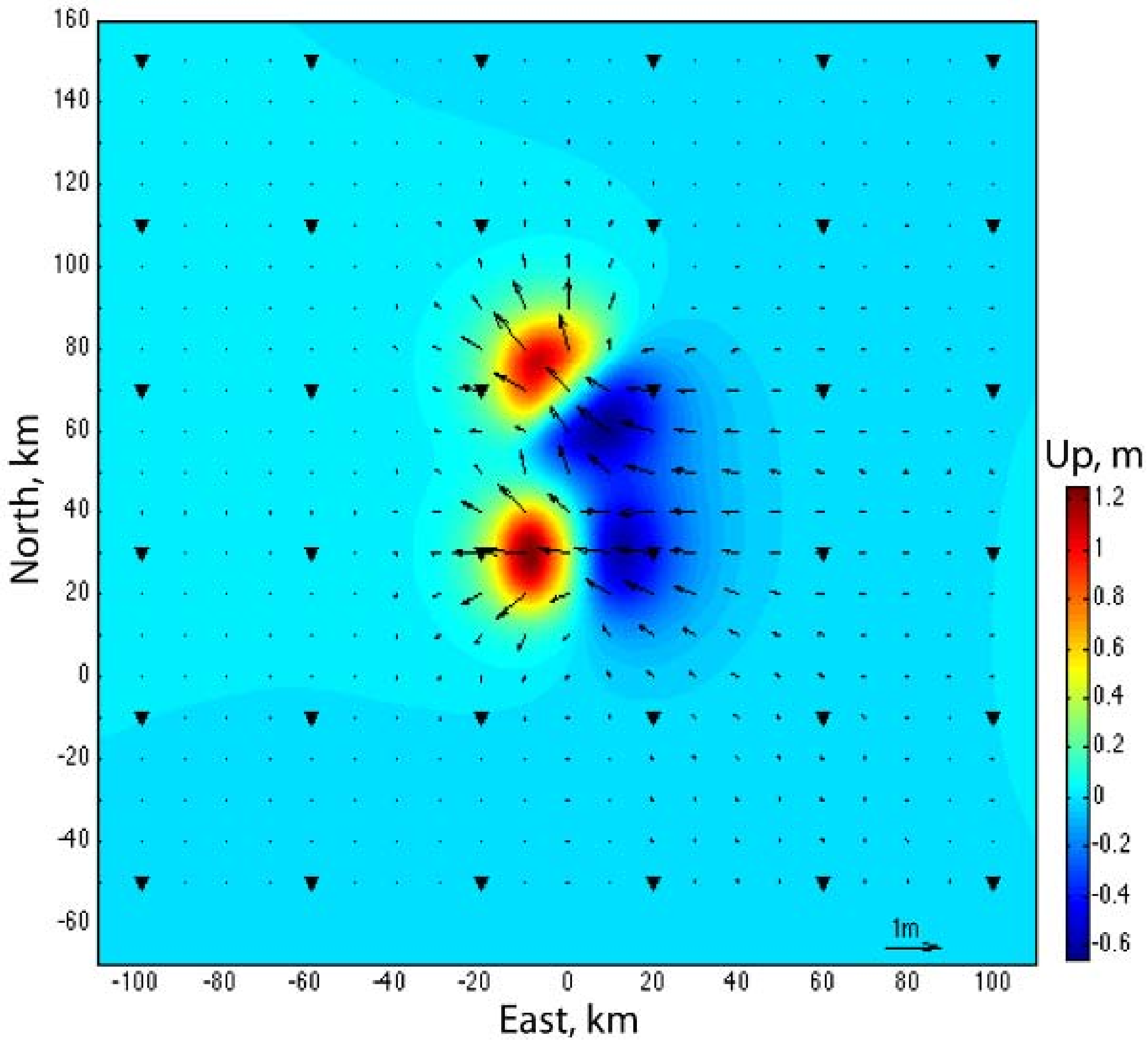
No C_p
C_p Included
Depth / H
Slip, m
Slip, m
Mean model
Target model
Displacement, m
Distance from fault / H
Distance from fault / H
Stochastic predictions
Data
Predictions

Toy Model 2: Static Finite-fault modeling

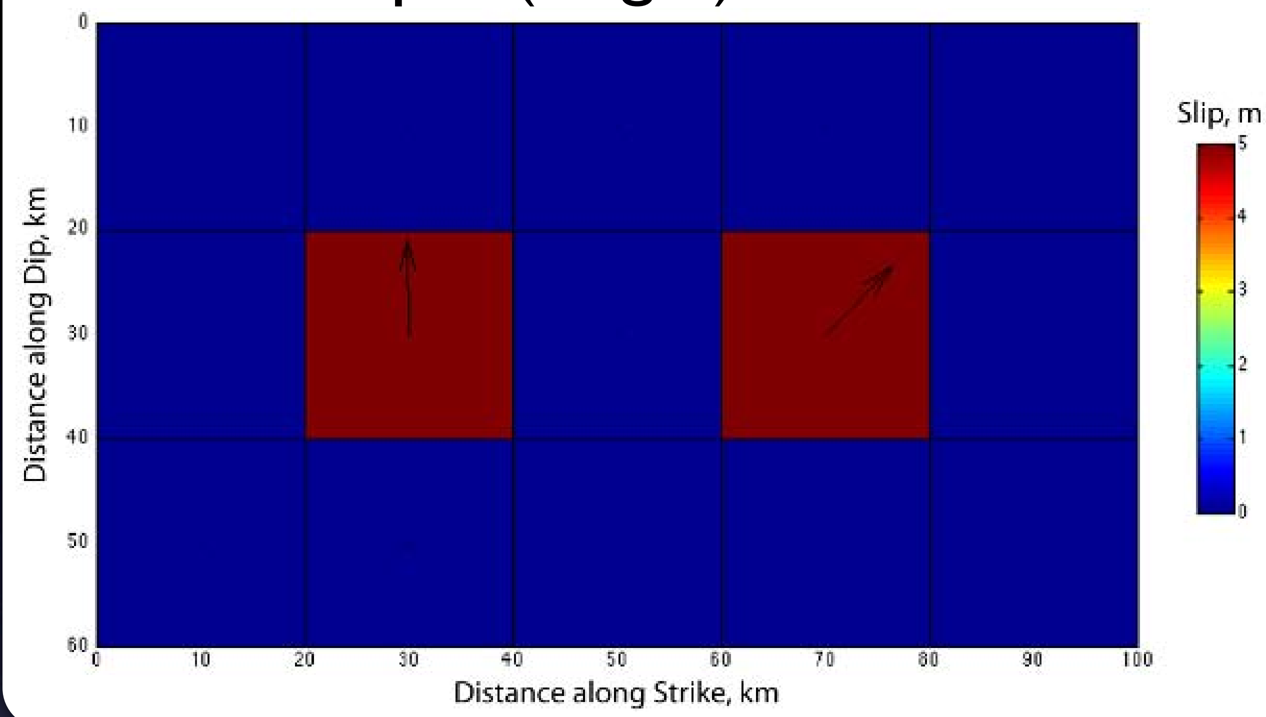
Finite shallow-dipping thrust fault

- 4km layer over an half-space (μ_2/μ_1 =2.0)
- Top of the fault at 6km

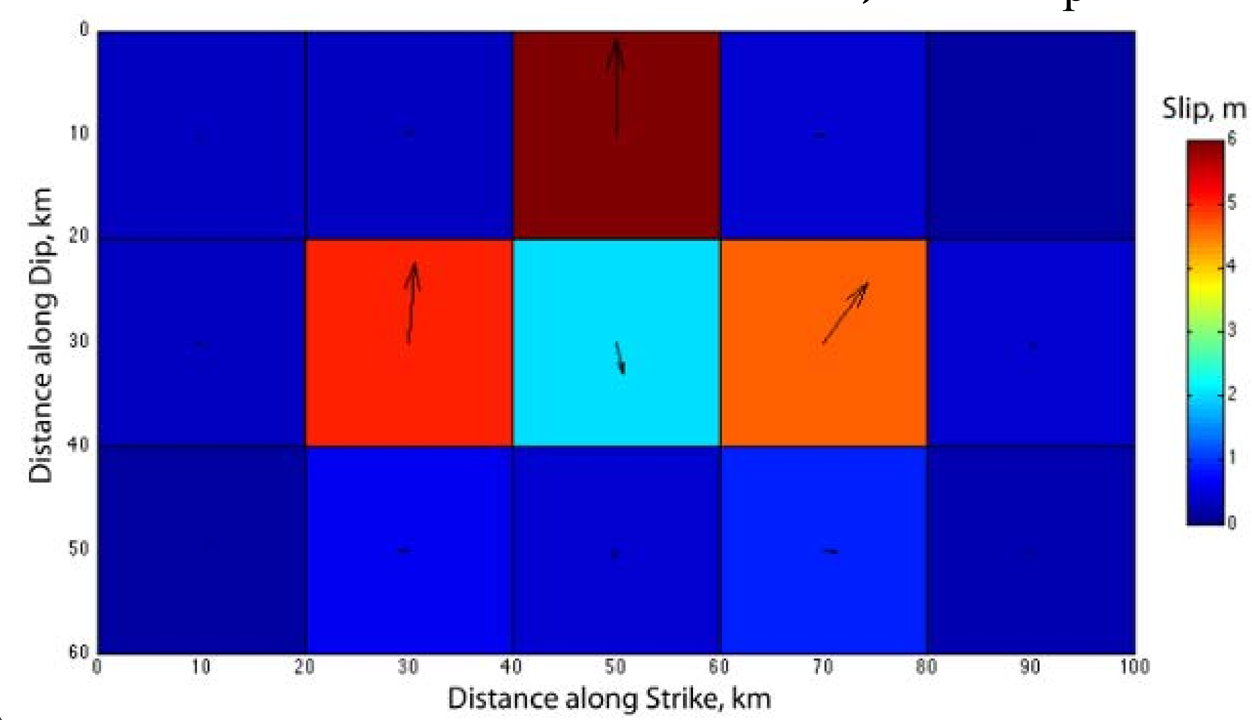
Synthetic displacement data



Input (target) model



Posterior mean model, No $\mathbf{C}_p$

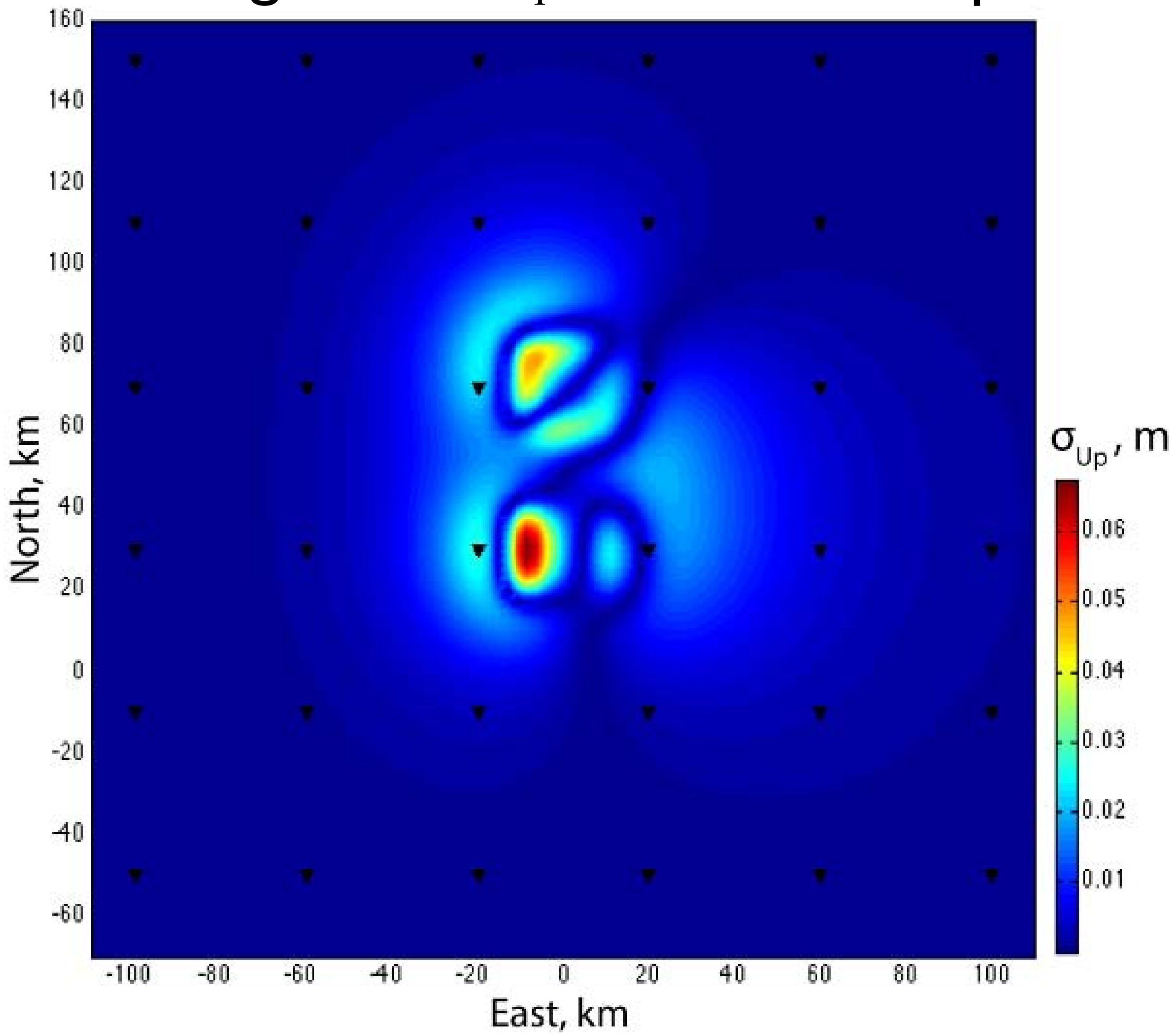


Toy Model 2: Static Finite-fault modeling

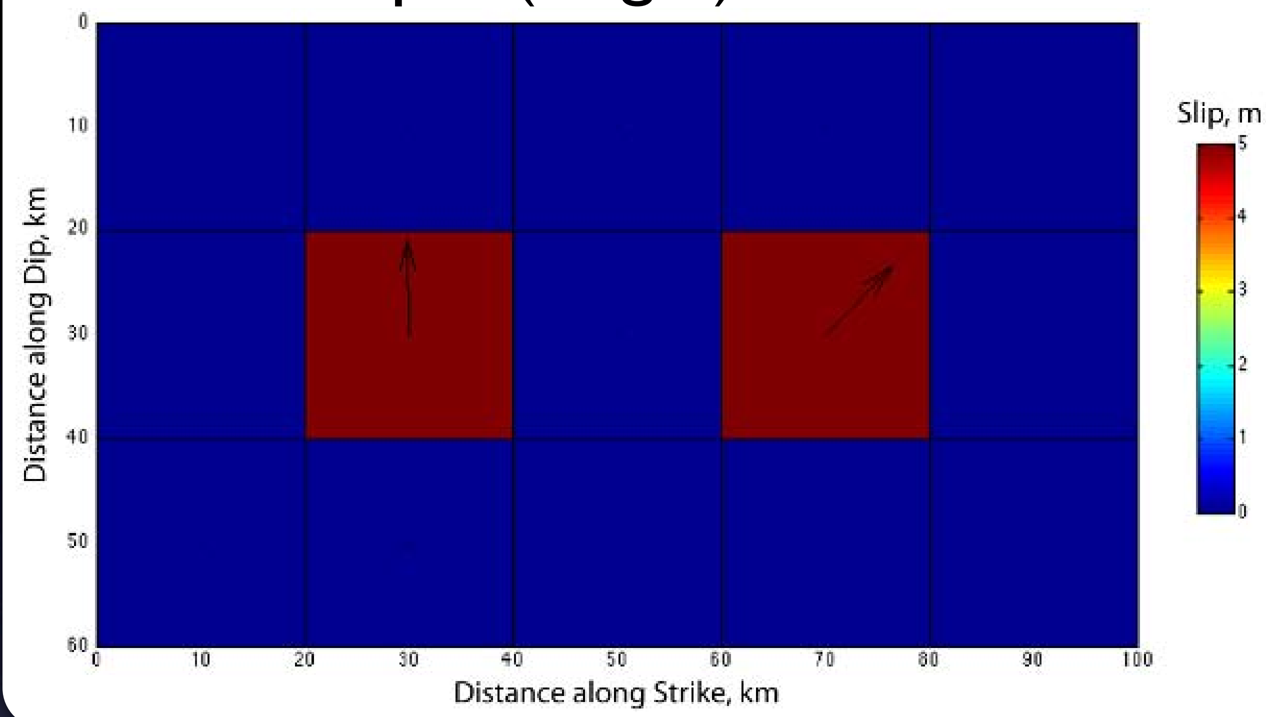
Finite shallow-dipping thrust fault

- 4km layer over an half-space (μ_2/μ_1 =2.0)
- Top of the fault at 6km

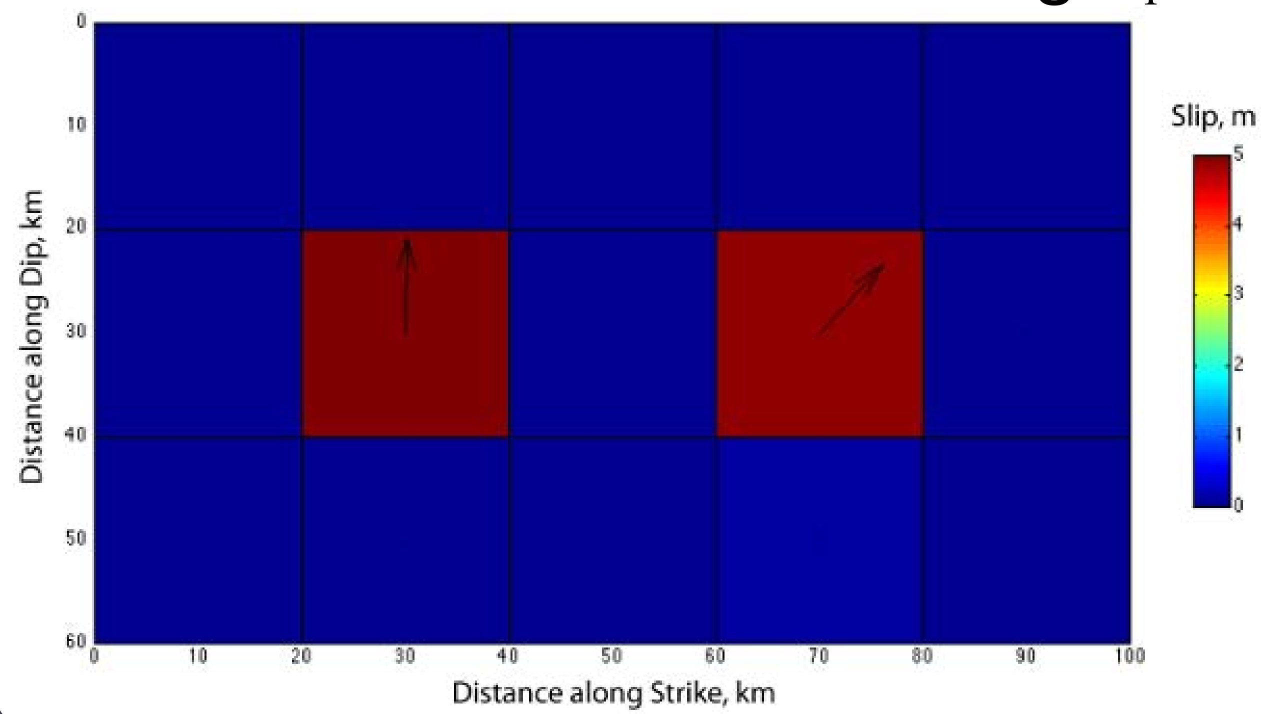
Diagonal of $\mathbf{C}_p$ for vertical disp.



Input (target) model



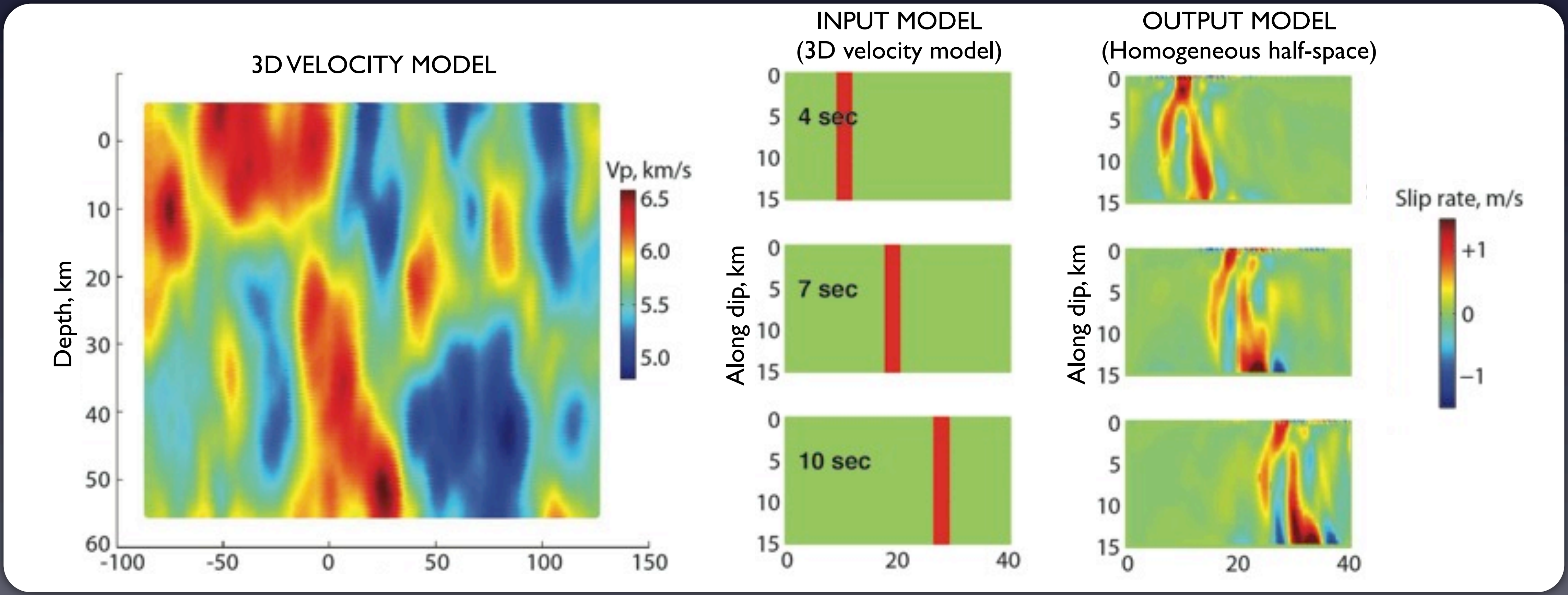
Posterior mean model, including $\mathbf{C}_p$



Conclusion and Perspectives

Improving source modeling by accounting for realistic uncertainties

- 2 sources of uncertainty
  - Observational error
  - Modeling uncertainty
- Importance of incorporating realistic covariance components
  - More realistic uncertainty estimations
  - Improvement of the solution itself
- Improving kinematic source models



Somala et al., submitted to GJI

References

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Z. Duputel, P. S. Agram, M. Simons and S. E. Minson, 2013. Accounting for prediction error when inferring subsurface fault slip. Submitted to Geophys. J. Int.

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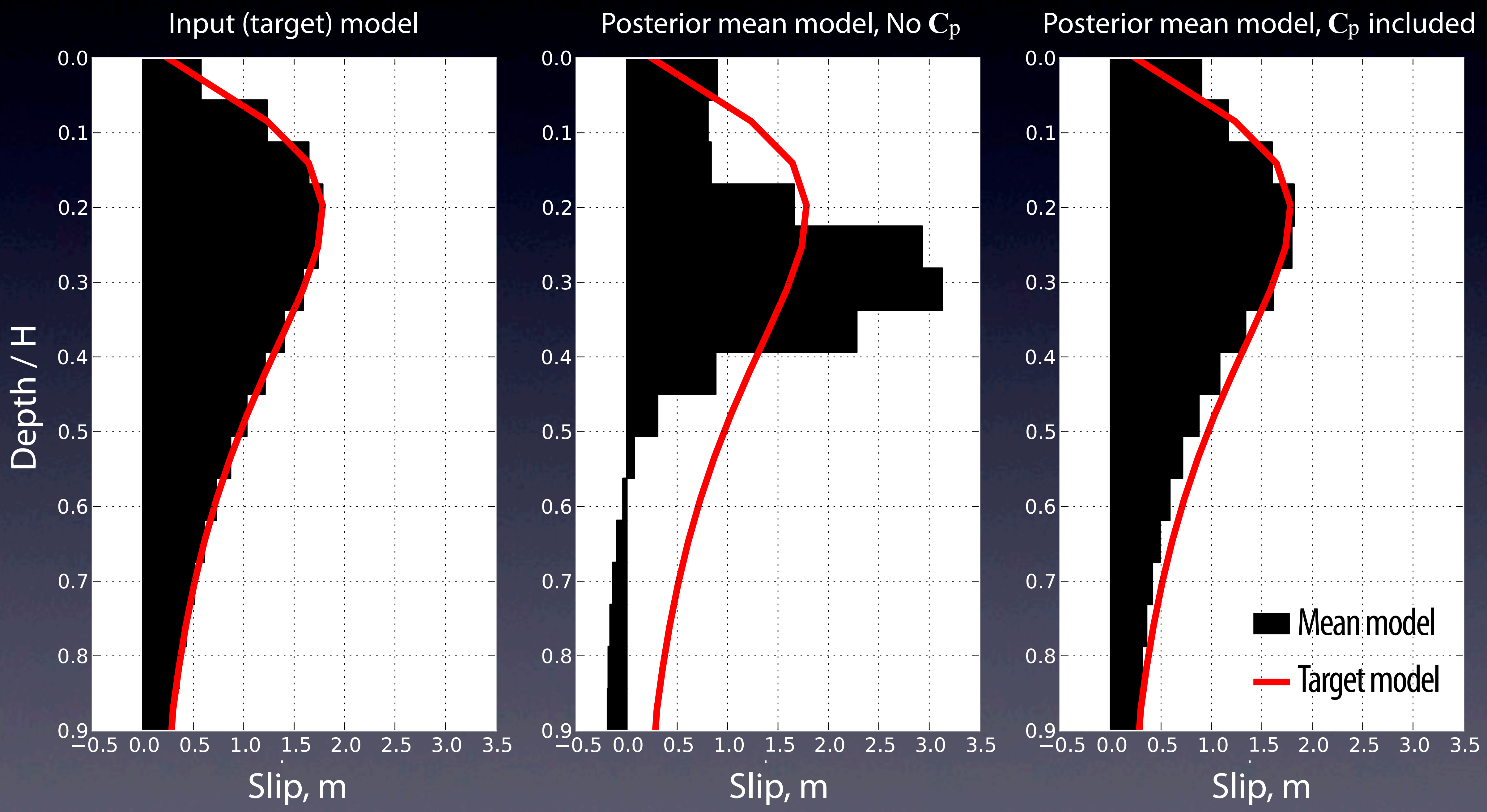
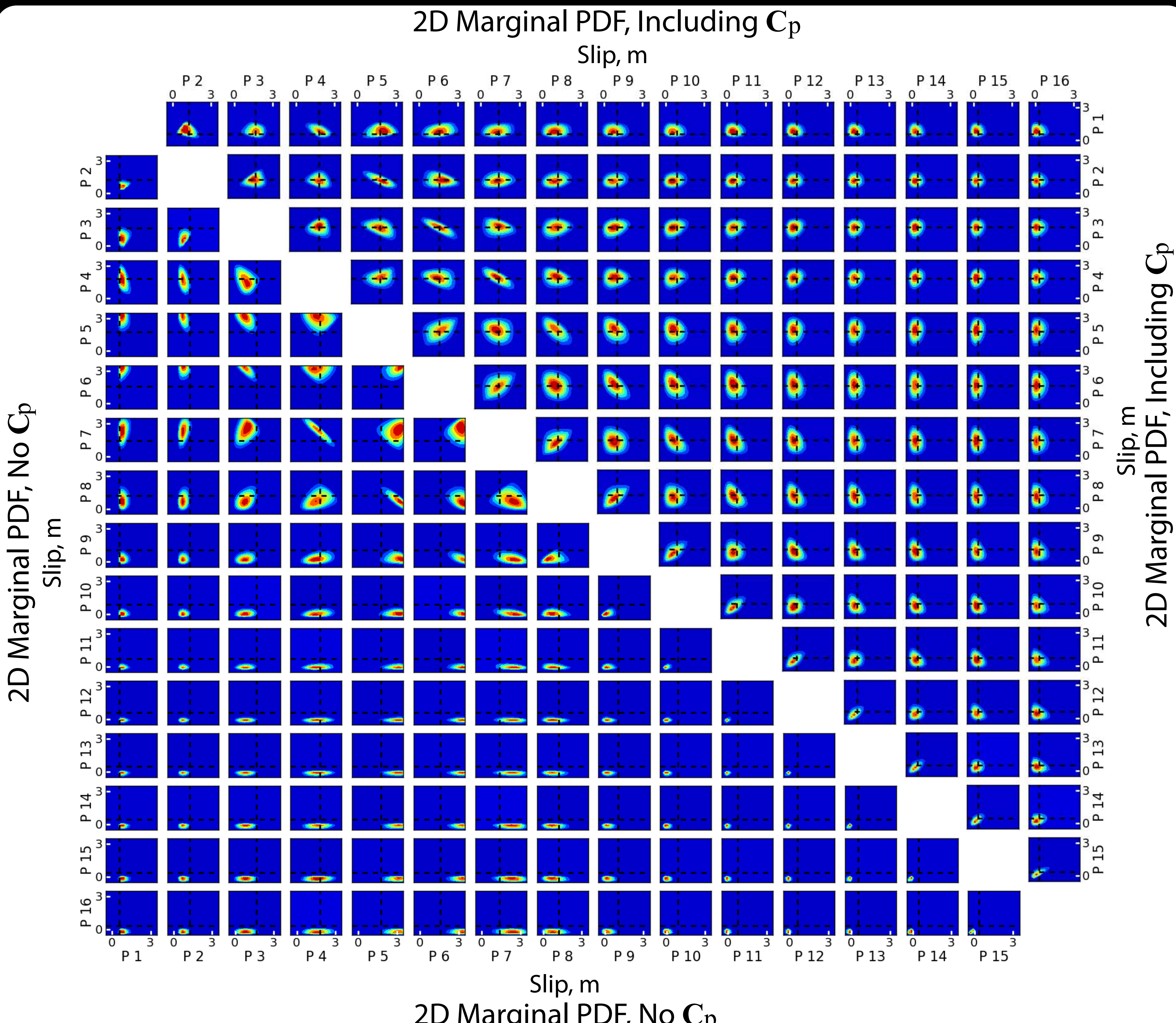
S. N. Somala, J.-P. Ampuero and N. Lapusta, 2013. Finite-fault source inversion using adjoint methods in 3D heterogeneous medium. Submitted to Geophys. J. Int.

Toy model 1: Infinite strike-slip fault

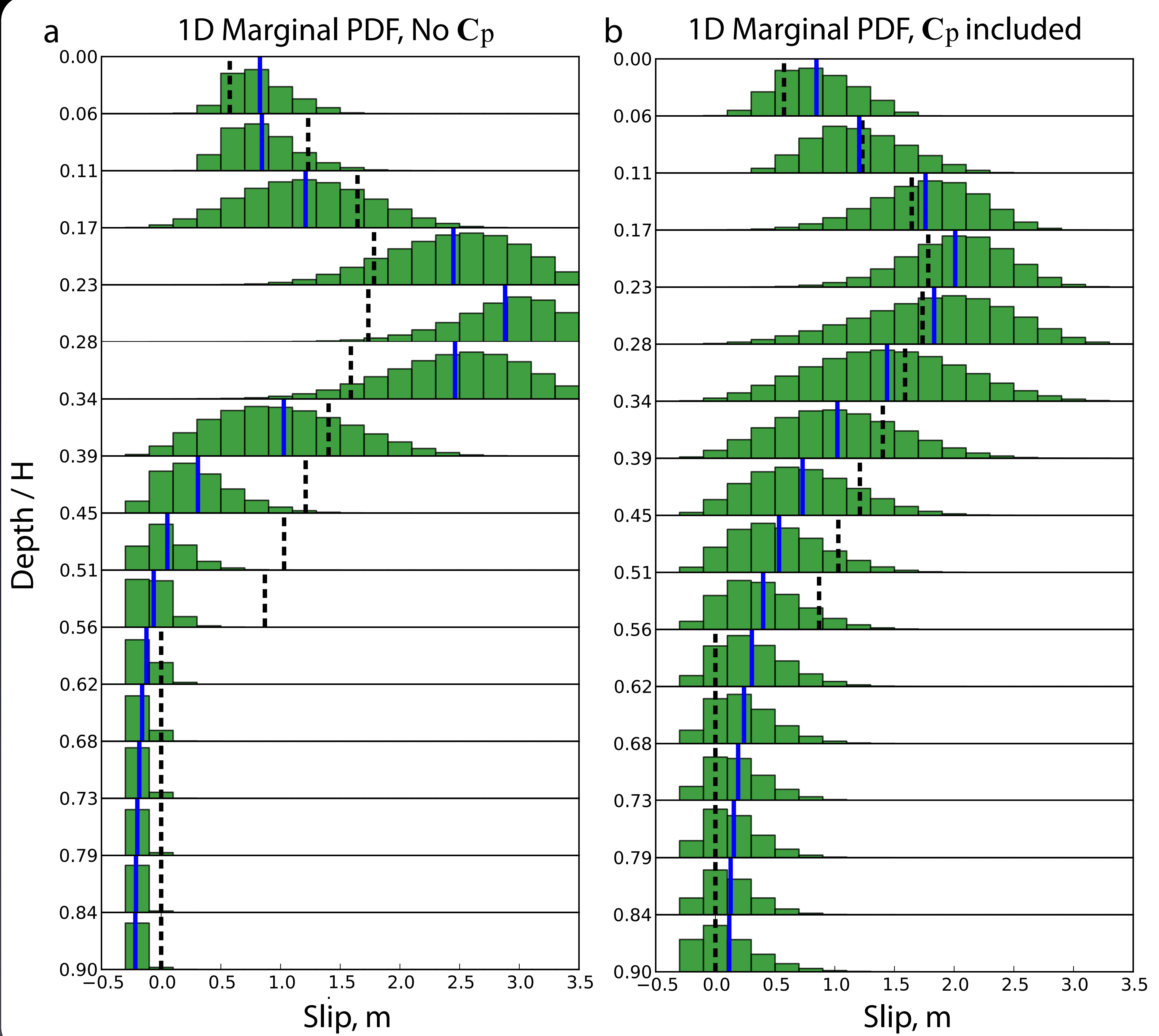
Comparison of inversion results with and without neglecting $\mathbf{C}_p$



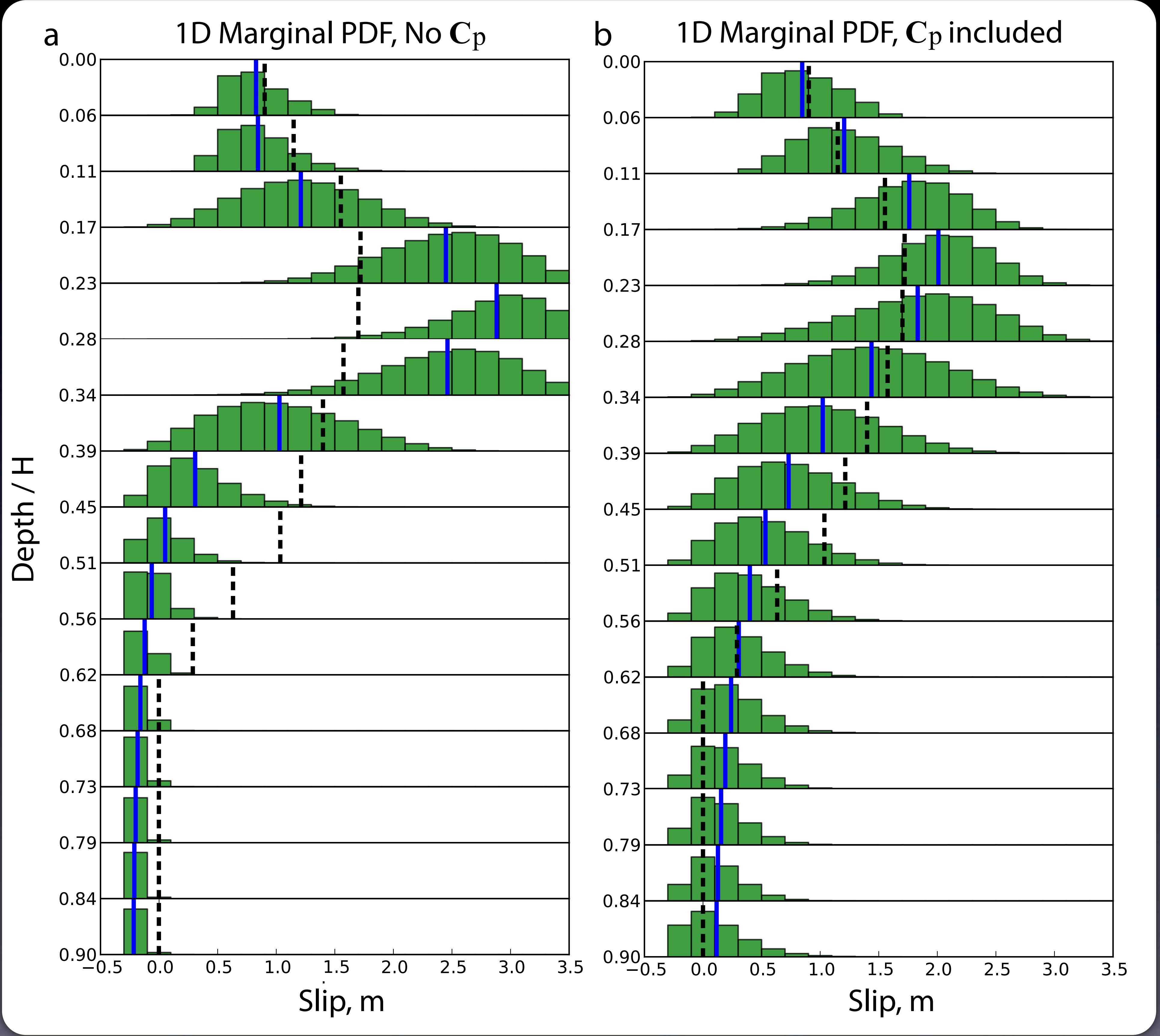


2D Marginal PDF, Including $\mathbf{C}_p$
Slip, m
2D Marginal PDF, No $\mathbf{C}_p$
Slip, m
P 1
P 2
P 3
P 4
P 5
P 6
P 7
P 8
P 9
P 10
P 11
P 12
P 13
P 14
P 15
P 16
0
3

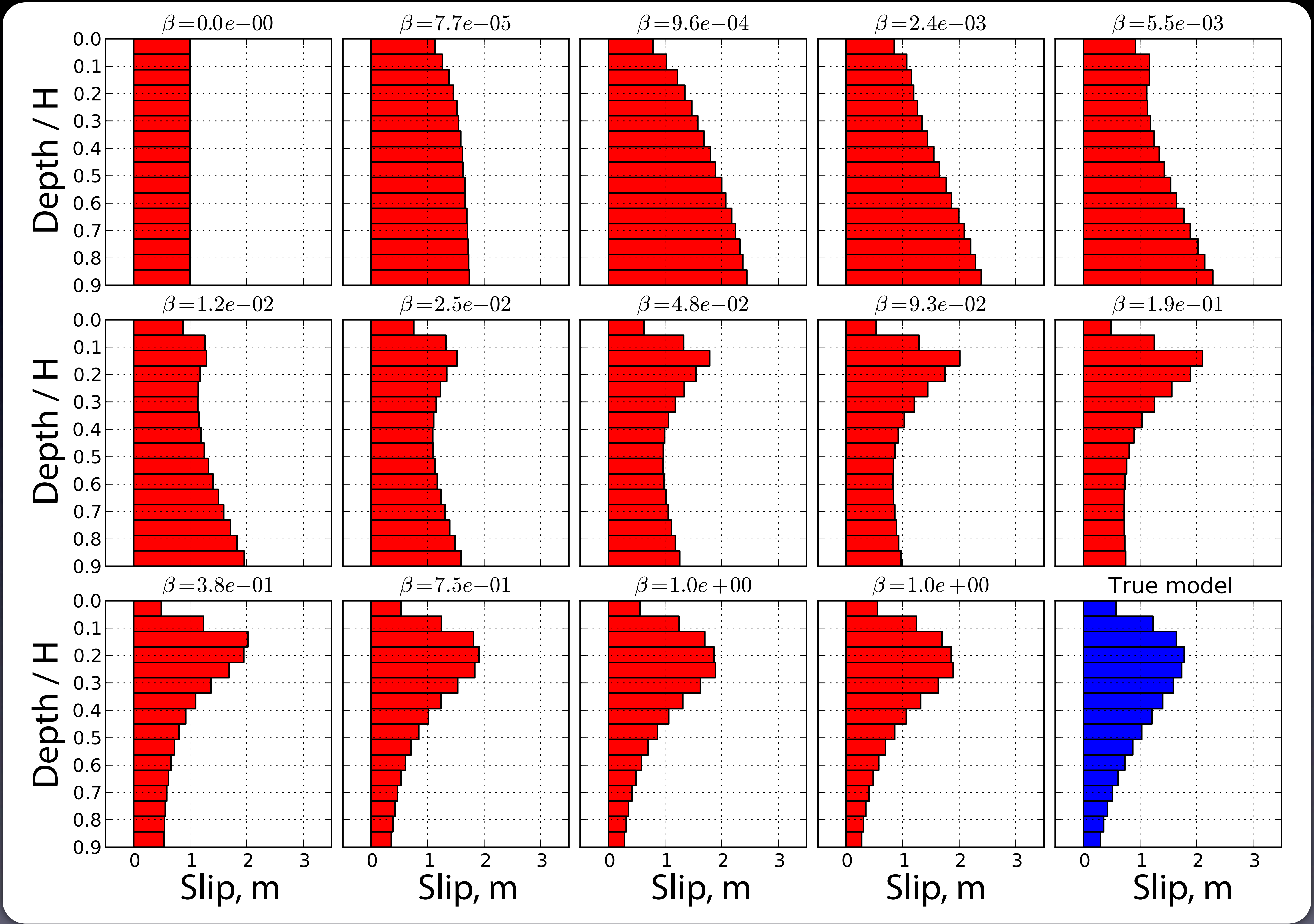
Toy model including a slip step



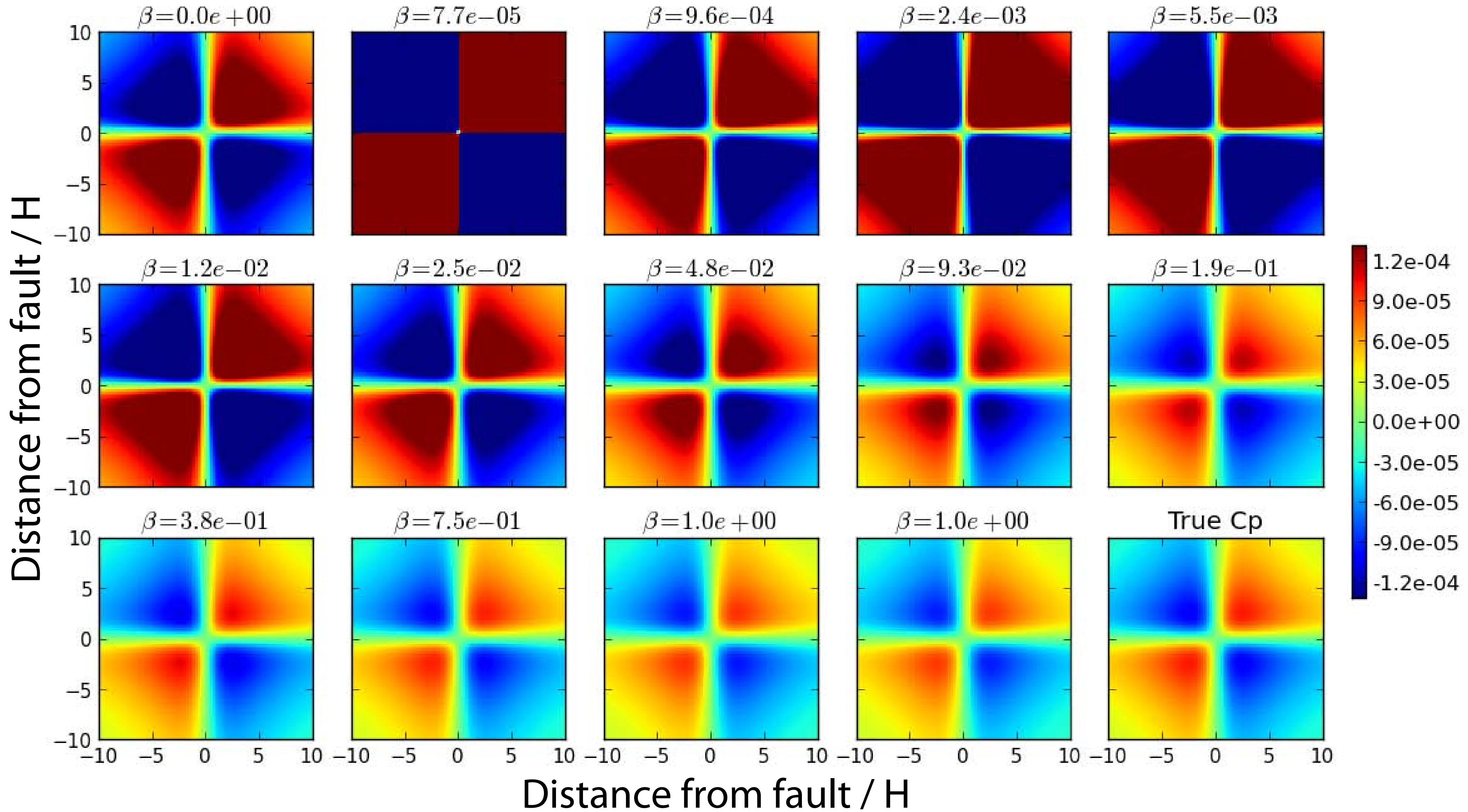
Toy model including a slip step

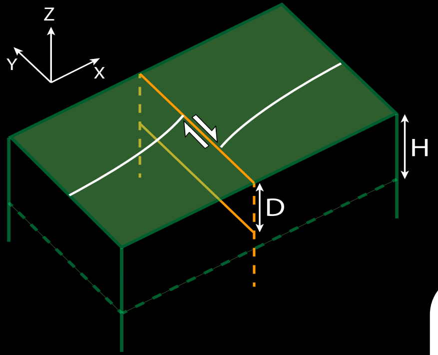


Evolution of m at each beta step

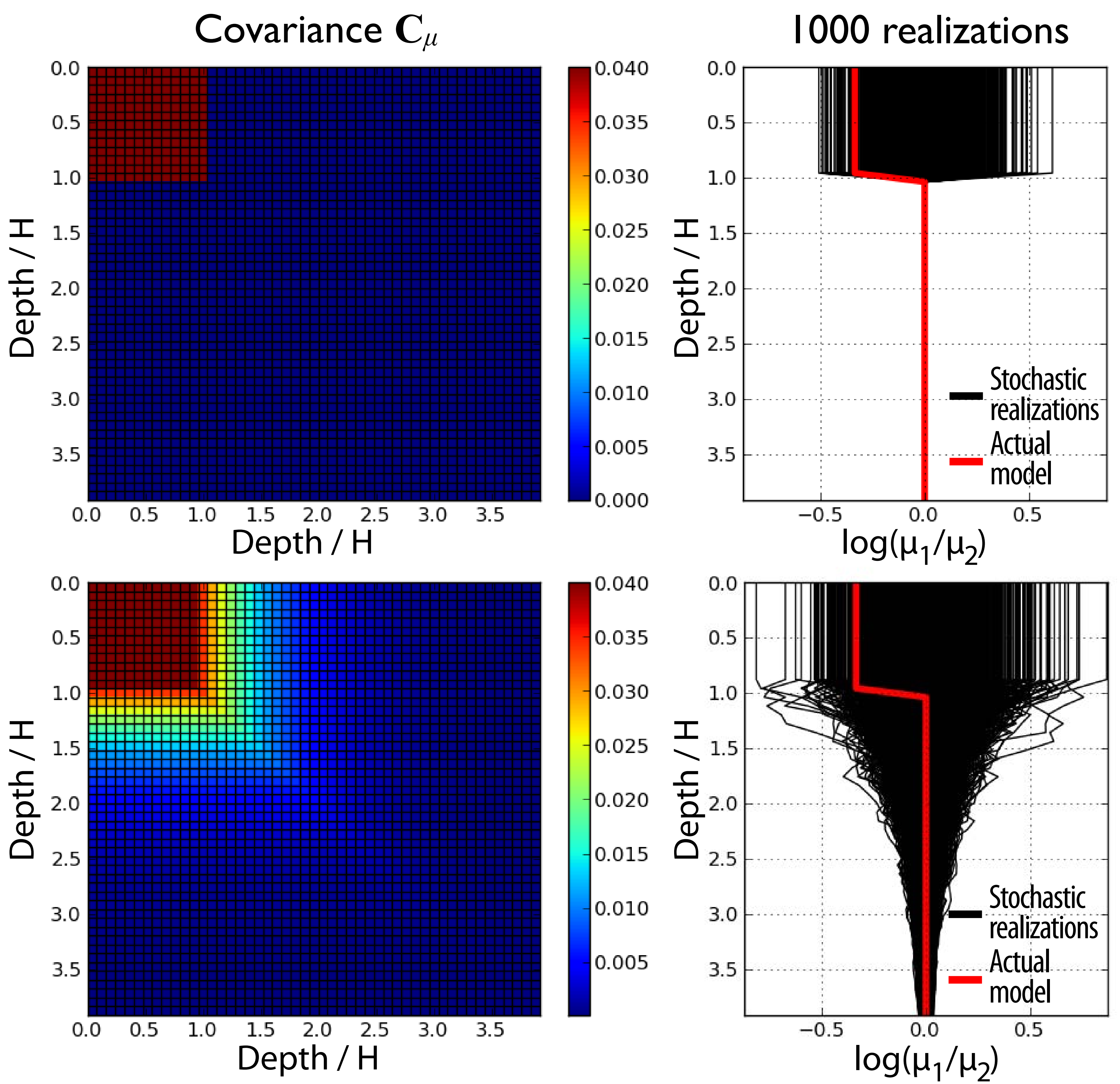


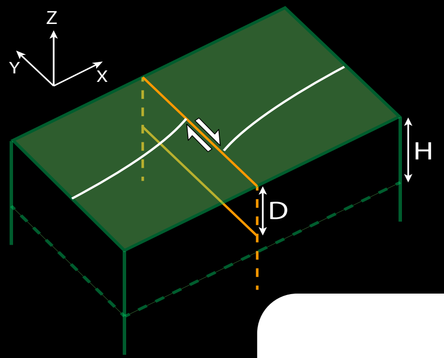
Evolution of C_p at each beta step



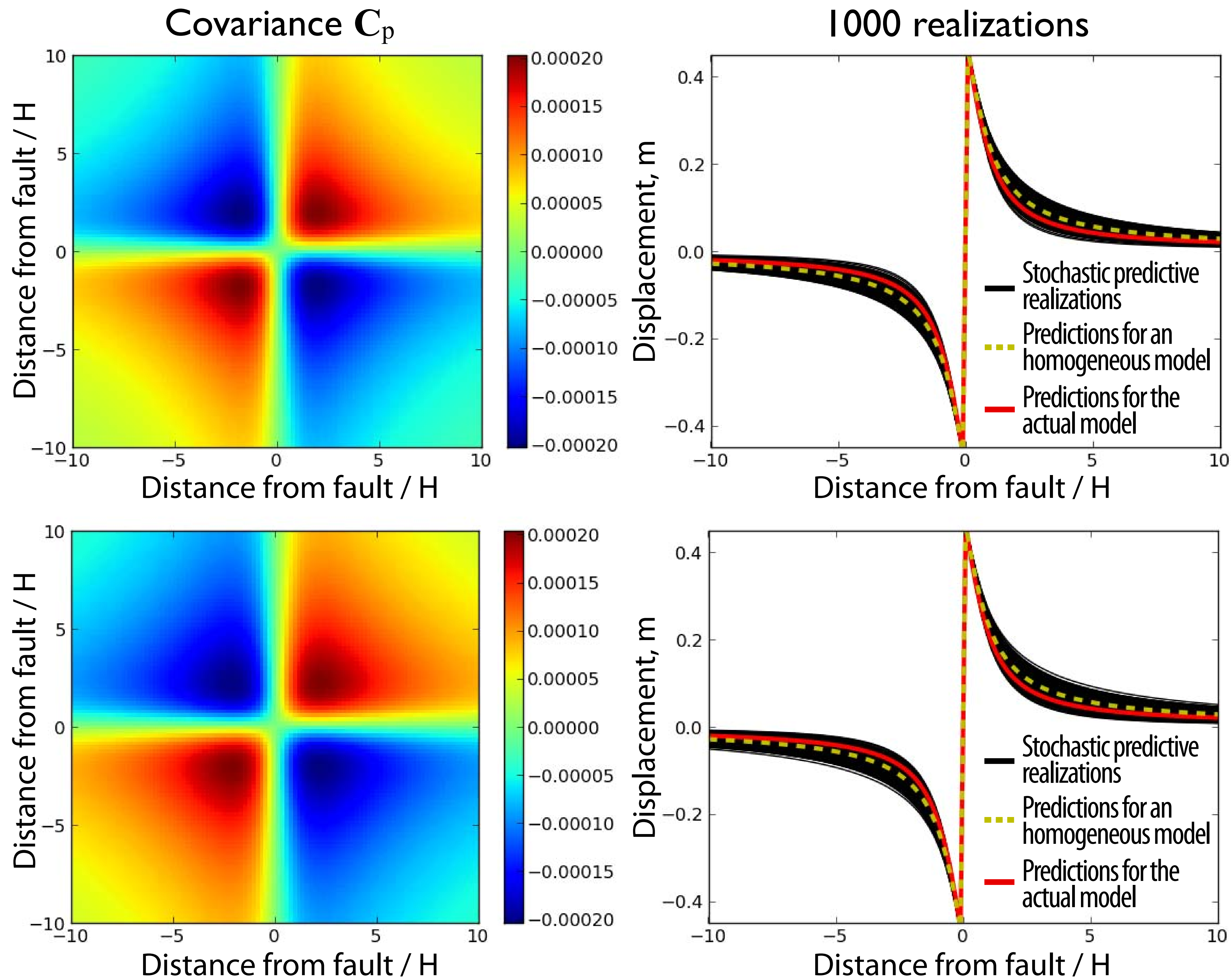


$$\mathbf{C}_\mathrm{p} = \mathbf{K}_\mu \cdot \mathbf{C}_\mu \cdot \mathbf{K}_\mu^T$$

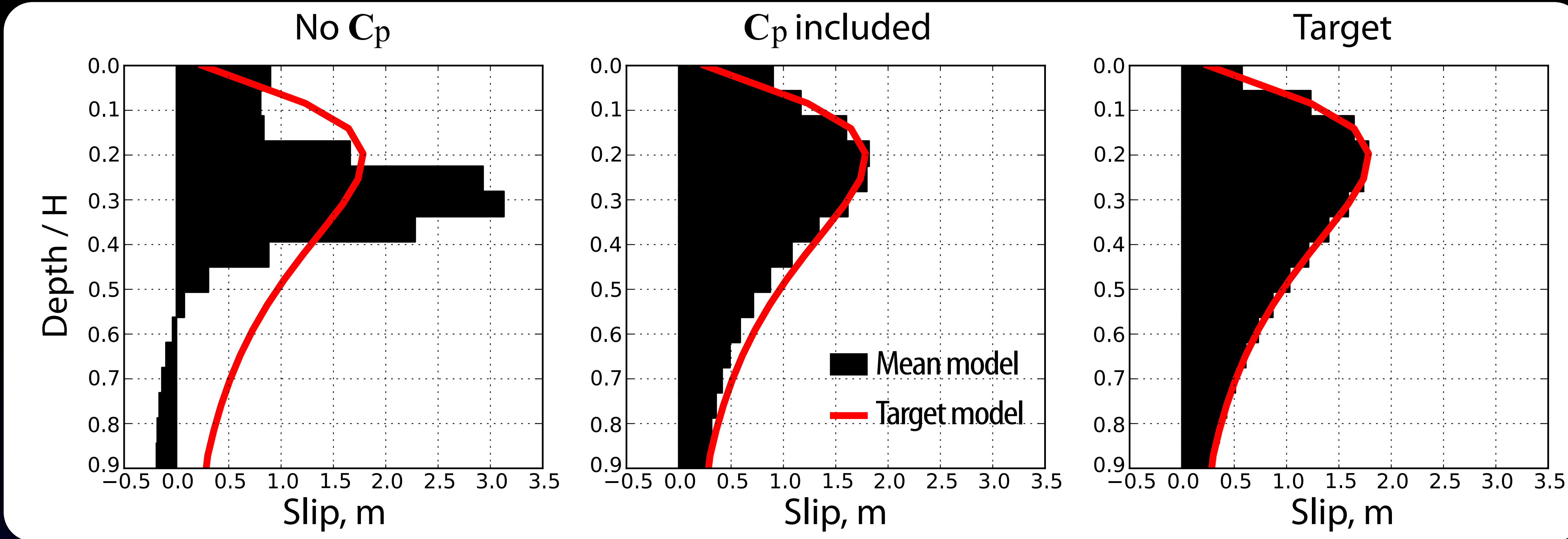




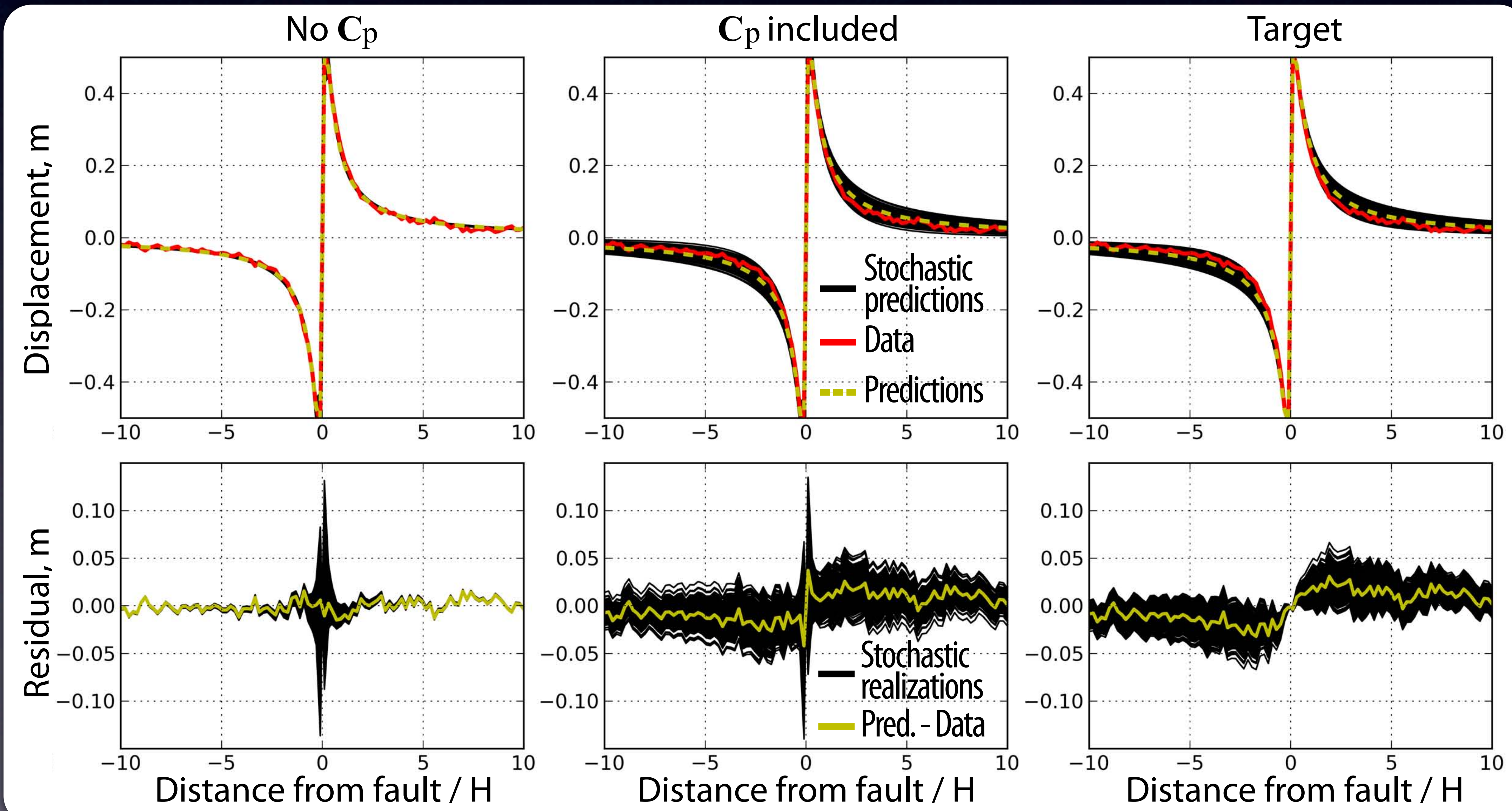
$$\mathbf{C}_{\mathrm{p}} = \mathbf{K}_{\mu} \cdot \mathbf{C}_{\mu} \cdot \mathbf{K}_{\mu}^{T}$$



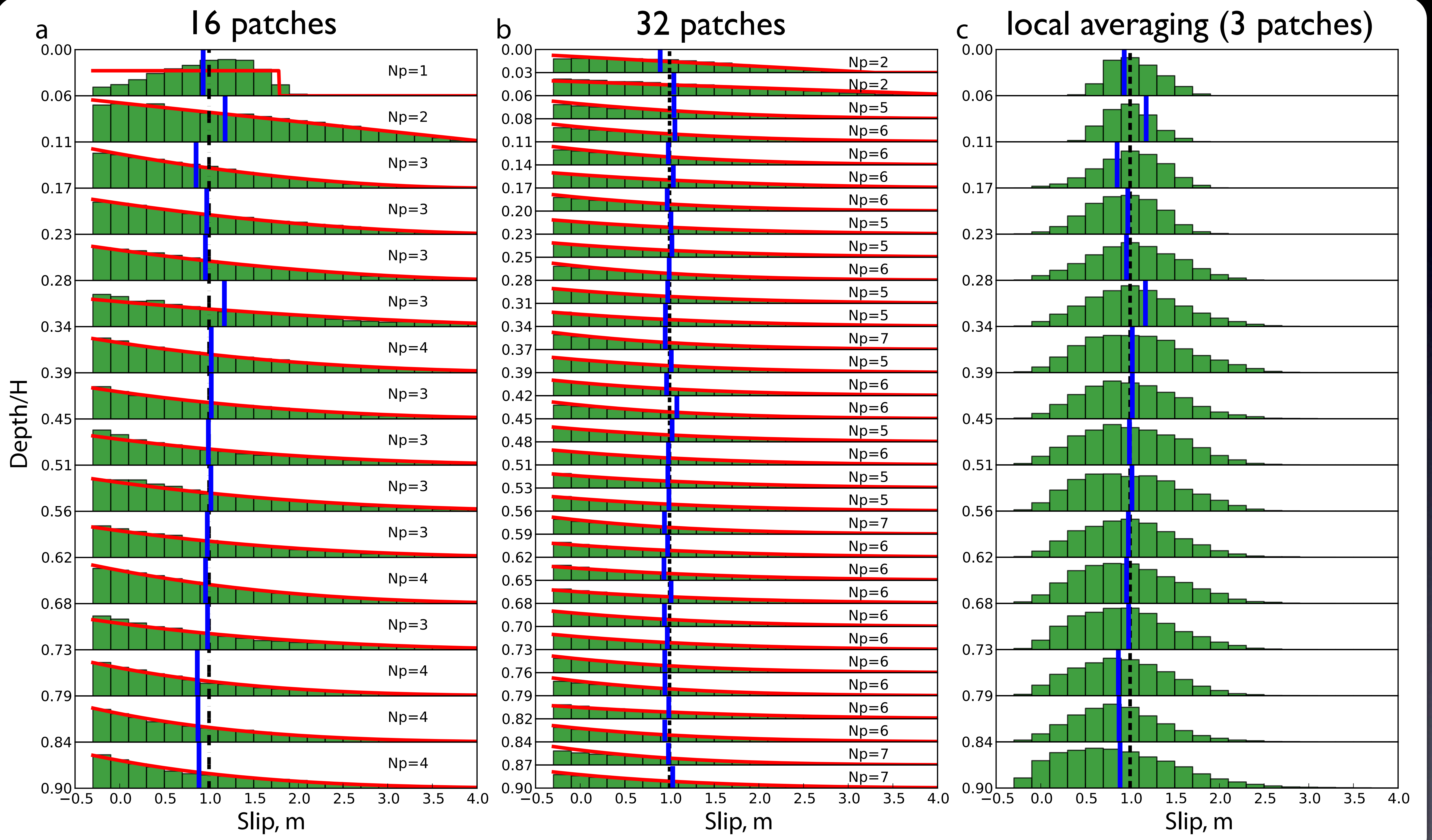
Source Models



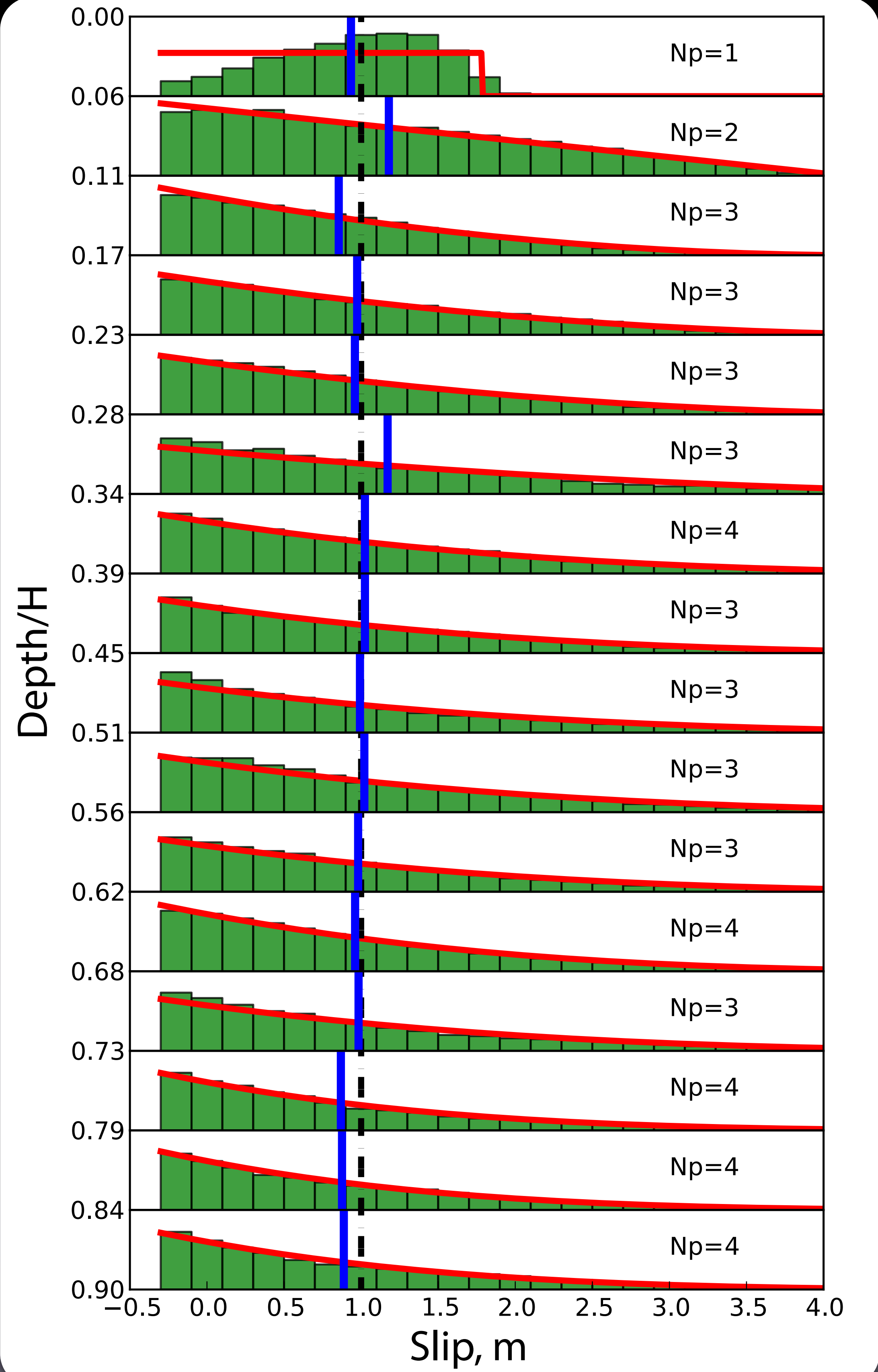
Data fit

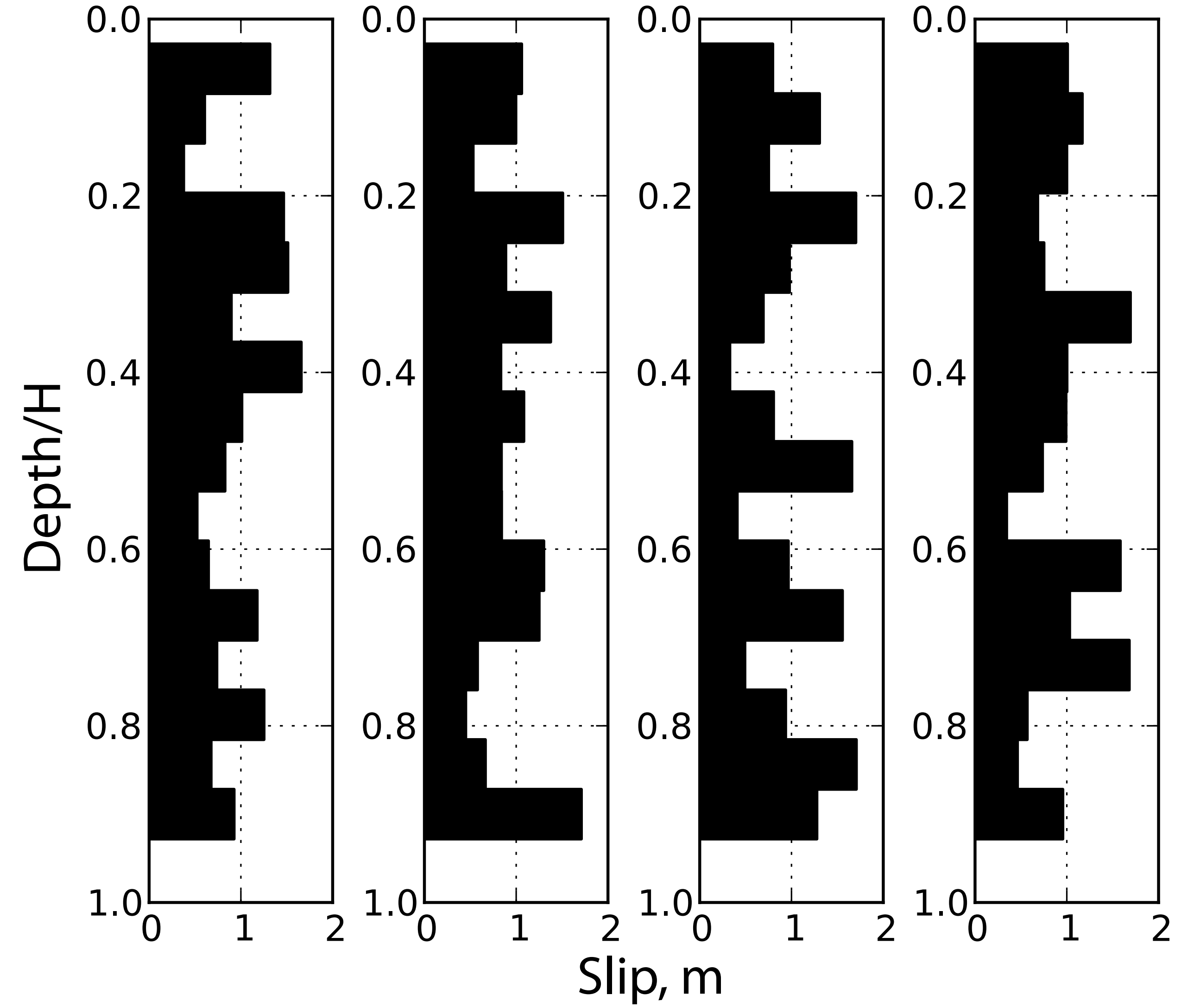


Spatial resolution



Spatial resolution





On the importance of Prediction uncertainty

Observational error: $\mathbf{e} = \mathbf{d}_{\text{obs}} - \mathbf{d}$

Measurements | Displacement field

- Measurements $\mathbf{d}_{\text{obs}}$: single realization of a stochastic variable $\mathbf{d}^*$ which can be described by a probability density $p(\mathbf{d}^*|\mathbf{d}) = N(\mathbf{d}^*|\mathbf{d}, \mathbf{C}_d)$

Prediction uncertainty: $\boldsymbol{\epsilon} = \mathbf{d} - \mathbf{d}_{\text{pred}} = \mathbf{d} - \mathbf{g}(\widetilde{\boldsymbol{\Omega}}, \mathbf{m})$, where $\boldsymbol{\Omega} = [\boldsymbol{\mu}^T, \boldsymbol{\varphi}^T]^T$

Earth model | Source geometry

- $\boldsymbol{\Omega}_{\text{true}}$ is not known and we work with an approximation $\widetilde{\boldsymbol{\Omega}}$
- The prediction uncertainty:
  - scales with the with the magnitude of $\mathbf{m}$
  - can be described by $p(\mathbf{d}|\mathbf{m}) = N(\mathbf{d} \mid \mathbf{g}(\widetilde{\boldsymbol{\Omega}},\mathbf{m}), \mathbf{C}_p)$

A posteriori distribution: $p(\mathbf{m}|\mathbf{d}_{\text{obs}}) \propto p(\mathbf{m}) \int_D p(\mathbf{d}_{\text{obs}}|\mathbf{d})\, p(\mathbf{d}|\mathbf{m})\, d\mathbf{d}$

Prior information

- In the Gaussian case, the solution of the problem is given by:

$$
\begin{aligned}
p(\mathbf{m}|\mathbf{d}_{\text{obs}}) &\propto p(\mathbf{m})\, \mathcal{N}\left(\mathbf{d}_{\text{obs}} \,|\, \mathbf{g}(\widetilde{\Omega}, \mathbf{m}), \mathbf{C}_\chi\right) \\
&\propto p(\mathbf{m}) \frac{1}{\sqrt{(2\pi)^N |\mathbf{C}_\chi|}} \exp\left(-\frac{1}{2}\left(\mathbf{d}_{\text{obs}} - \mathbf{g}(\widetilde{\Omega}, \mathbf{m})\right)^T \mathbf{C}_\chi^{-1} \left(\mathbf{d}_{\text{obs}} - \mathbf{g}(\widetilde{\Omega}, \mathbf{m})\right)\right)
\end{aligned}
$$

$$\mathbf{C}_\chi = \sigma^2 \mathbf{I}$$

$$\downarrow$$

$$\mathbf{C}_\chi = \mathbf{C}_d + \mathbf{C}_p$$

Measurement errors | Prediction errors

D: Prediction space

On the importance of Prediction uncertainty

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Measurements | Displacement field

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~~$\mathbf{C}_\chi = \sigma^2 \mathbf{I}$~~

↓

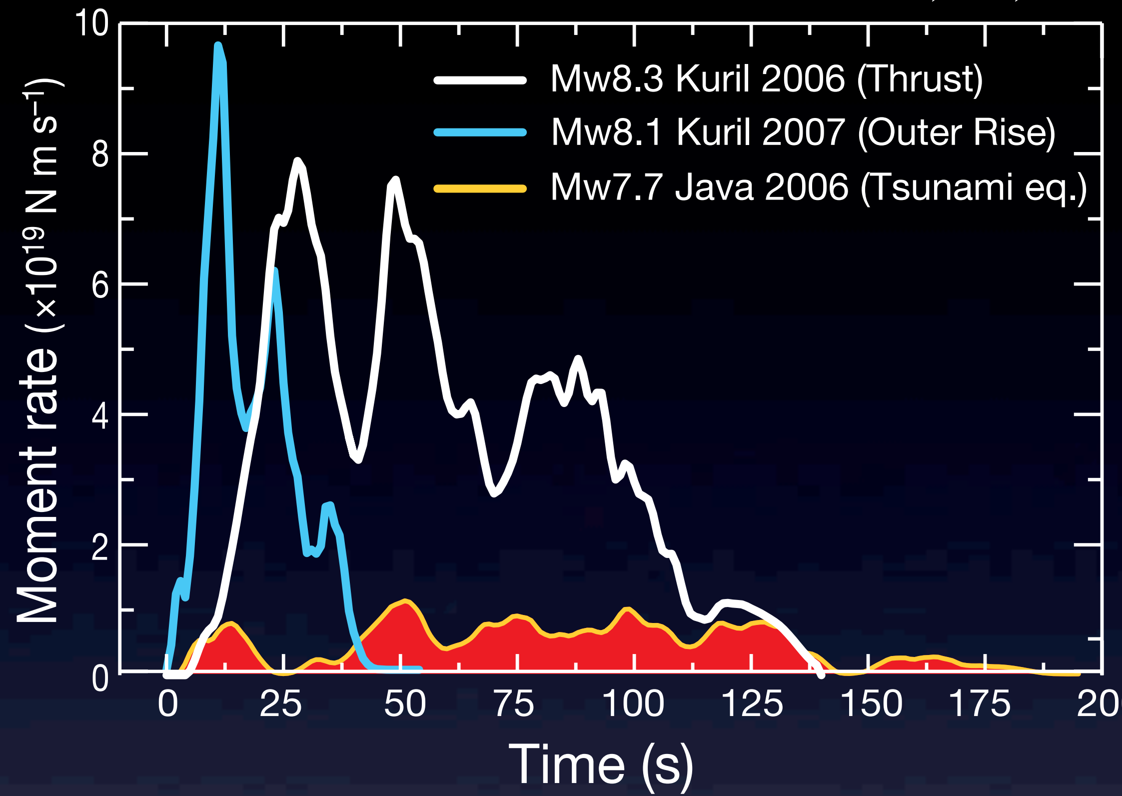
$$\mathbf{C}_\chi = \mathbf{C}_\text{d} + \mathbf{C}_\text{p}$$

Measurement errors | Prediction errors

D: Prediction space

Motivation : Complexity/Diversity

Ammon et al., 2006, 2008



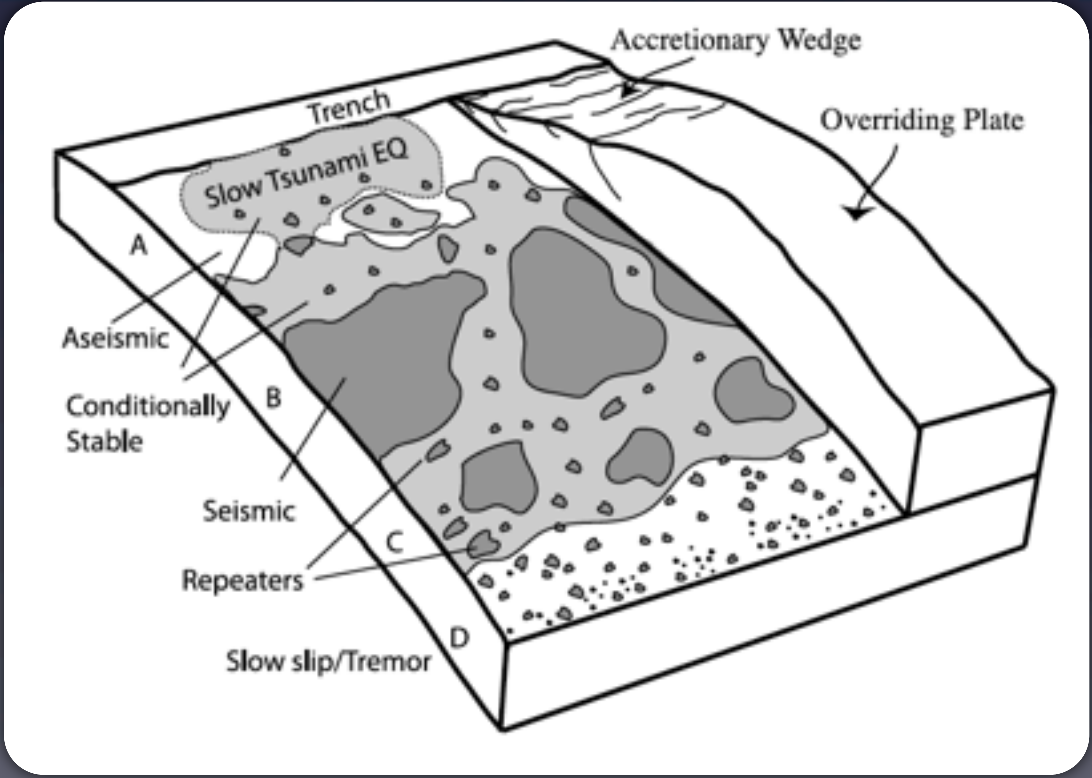
Similarity of earthquake rupture processes

- First order behavior of the source
- Scaling laws

Observational/Methodological Improvements

- Expansion and densification of observational networks
- Improvements in the fidelity of forward modeling capability

Lay et al. (2011)



Diversity/Complexity of the rupture

- Scaling laws are not always relevant
- Energy partitioning during an Earthquake :

➡ Variability in the radiation efficiency (e.g., slow slip)

- Rupture propagating over multiple segmented faults
- Fault interaction

Combining multiple data-types

Accounting for the complexity of the fault

Complex geometries :

- Non-planar faults,
- Multi-segmented faults

Using available information :

- Field observations
- Seismic reflection
- Earthquakes mechanism/location

Cascading integration of the information

CATMIP Bayesian sampler (Minson et al., GJI 2013):

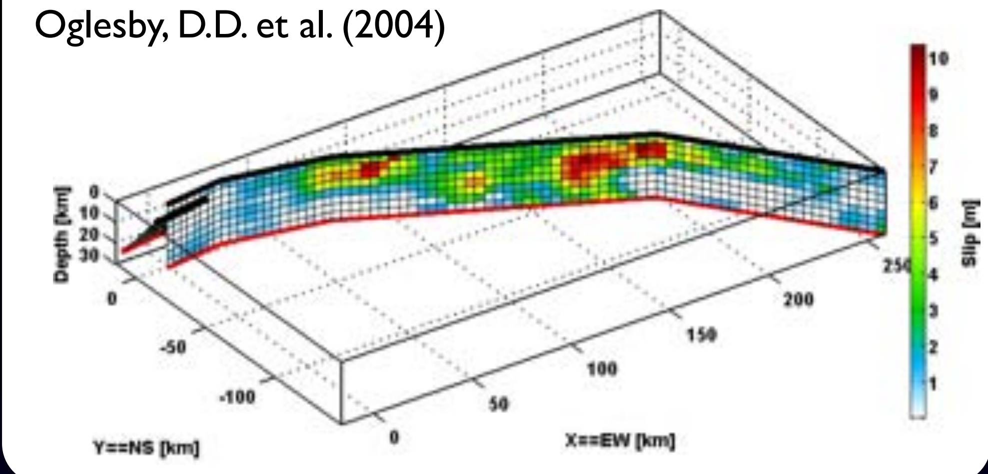
Current challenge

Static inversion (GPS, InSAR,...)

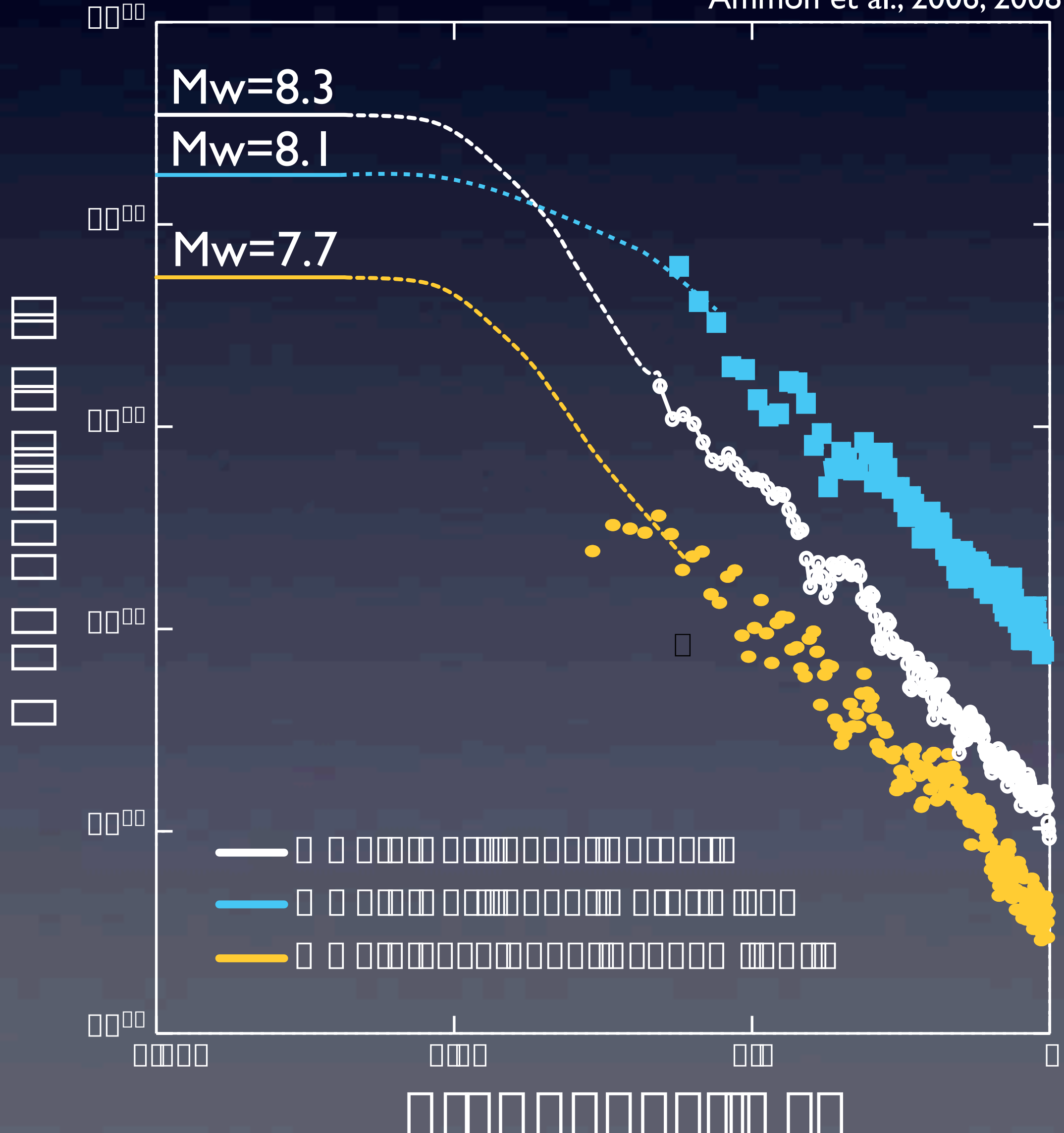
LP kinematic inversion (Broad-band, HR-GPS,...)

HF kinematic inversion (Strong-Motion, Broad-Band)

Denali earthquake (2002)
Oglesby, D.D. et al. (2004)



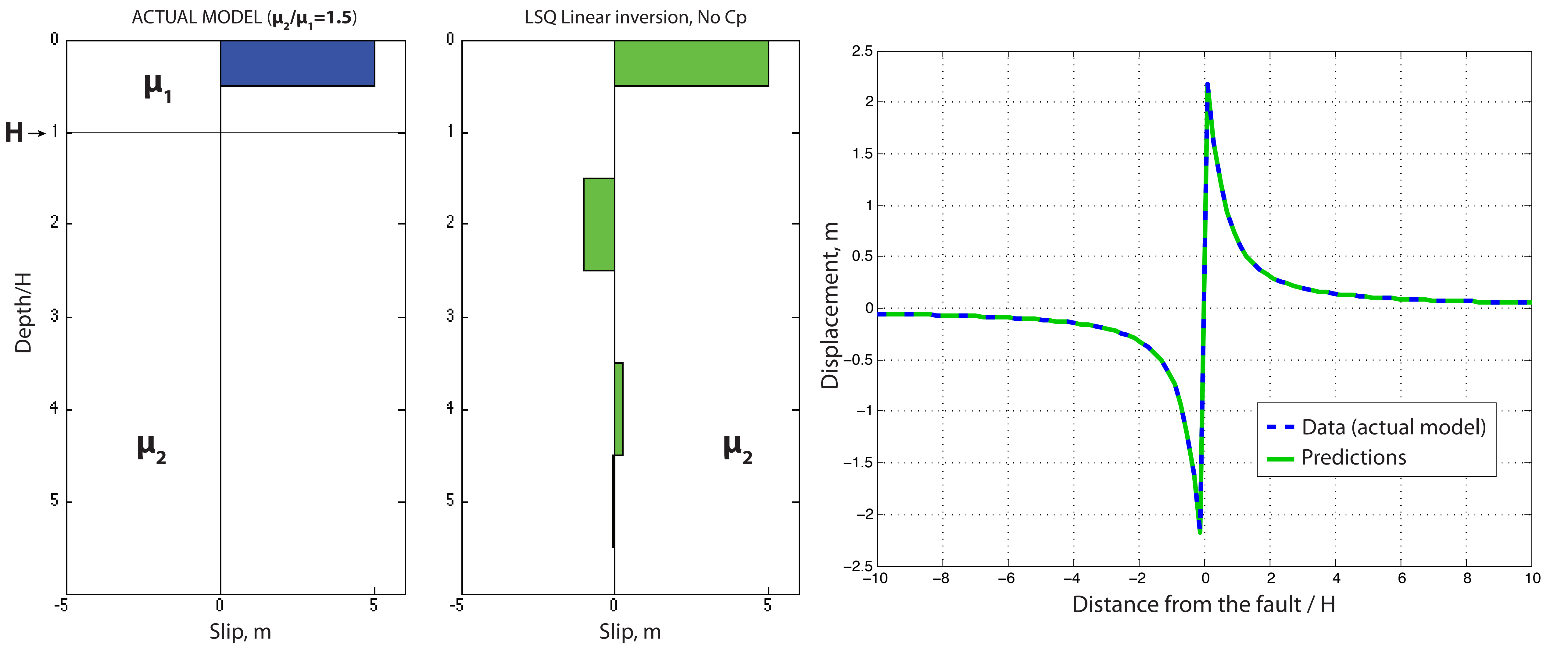
Ammon et al., 2006, 2008



Prediction uncertainty due to the earth model

Motivational example: A two-dimensional infinite strike-slip fault:

Assuming an extended fault



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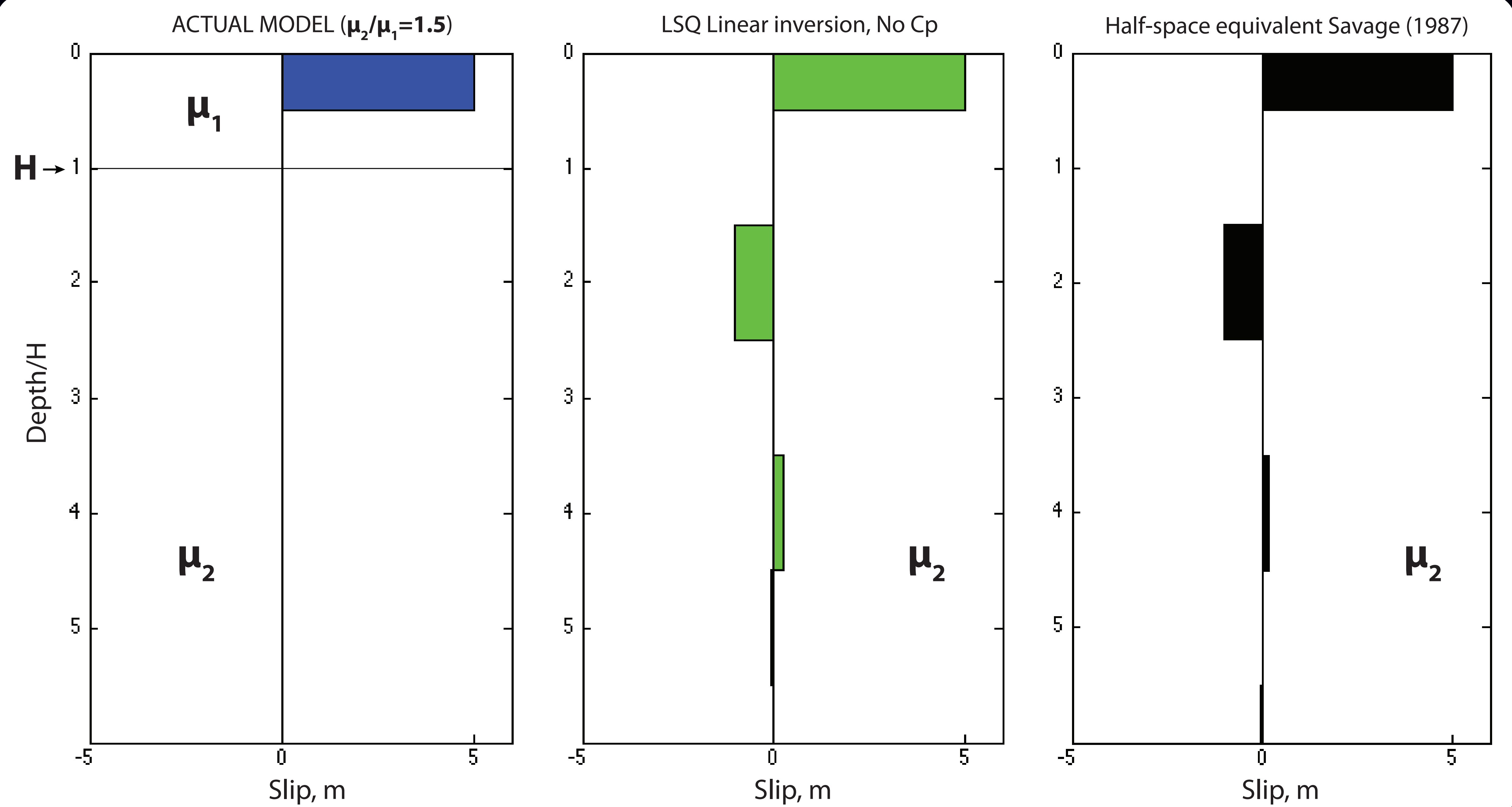
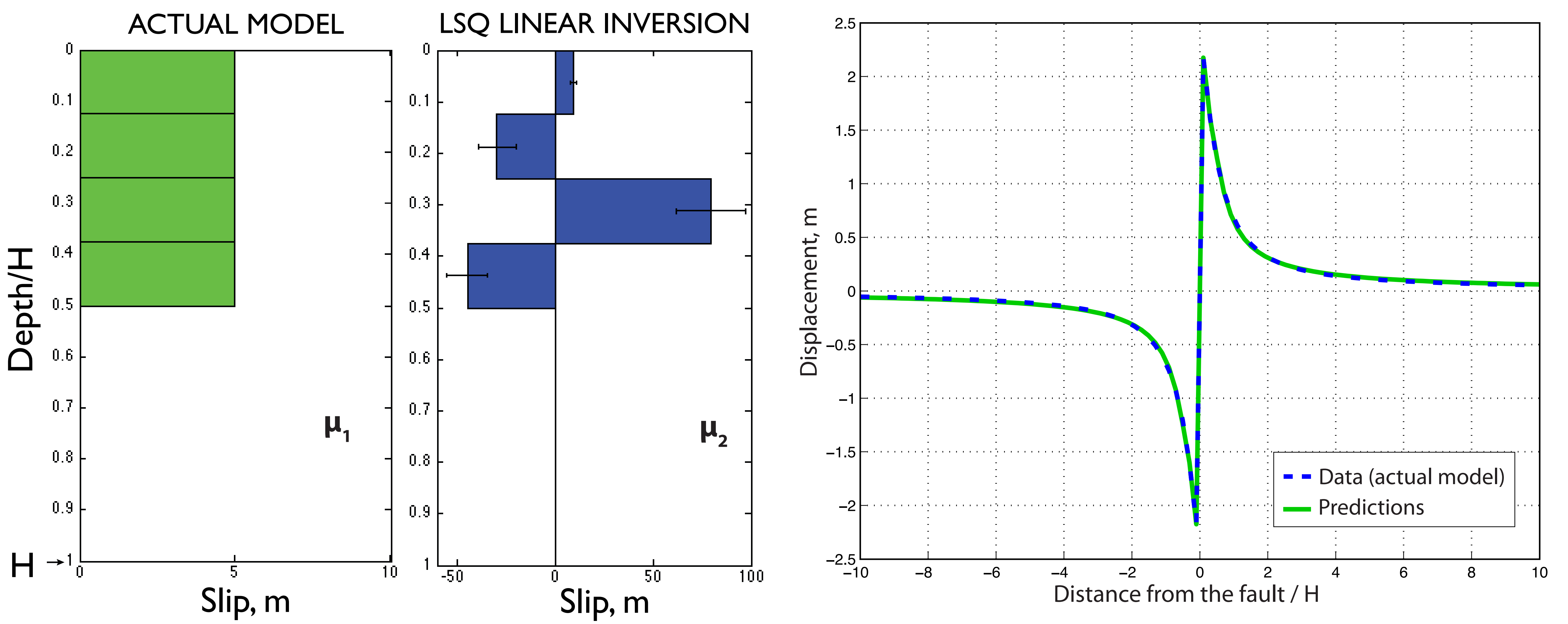


Prediction uncertainty due to the earth model

Motivational example: A two-dimensional infinite strike-slip fault:

Assuming a limited depth extent



Prediction uncertainty due to the earth model

Infinite strike-slip fault:

Assuming a limited depth extent